Frequency Granger Causality Test in Cointegration System by Wavelet Analysis

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Abstract:- Frequency domain Granger causality test is an important field in economics however by now no simple tools exist for the calculation. In the paper, we propose a novel method based on wavelet analysis to test the impact of specific frequency fluctuation of series in establishing the cointegration and Granger causility among series. The method is rather time domain oriented and can rely on the standard procedure of error correction model (Johansen & Juselius 1990; Johansen 1995). A simple case study for electricity consumption and economic growth in China is reported in the paper to validate our method.
***Key words**: Granger causility; Frequency domain; Wavelet analysis; Empirical study*

1 Introduction

Granger causality test has become a rather standard technique in economic research(Johansen & Juselius 1990; Johansen 1995). As to time domain analysis, the error correction model has become a prototype tool. Concerning frequency domain analysis, Yao and Hosoya (2000) has proposed a method based on Wald test to test one-way Granger effect for cointegrated vector time series. However, though mathemetically perfect, the method proposed by Yao and Hosoya is very complicated in computation and can not rely on existing mature computation toolbox such as Eviews ect. So the application of the method is rare.

In the paper, we are to propose a novel method for frequency Granger causality test. In general sense, our method is rather time domain oriented. So it is possible to rely on existing toolbox to realize the comuputation of the method, which gives it an adavantage for economist not familiar with complex computation toolbox to use it without difficulty.

2 Time-frequency Wavelet Analysis of Time series

2.1 Discrete wavelet analysis

wavelet analysis is a kind of localized time-frequency signal analysis with fixed window and alterable form. Suppose that $\psi(t) \in L^2(R)$ (square integralable real number space) with its Fourier transformation being $\widehat{\psi}(\omega)$. When $\widehat{\psi}(\omega)$ satisfies the admissiability condition

$$\int_R \frac{\left|\widehat{\psi}(\omega)\right|^2}{|\omega|} d\omega < \infty \tag{1}$$

$\psi(t)$ is called as wavelet base or mother function.

Breaking function $f(t)$ downing in any $L^2(R)$ space on the transformation of mother wavelet is called continous wavelet transform, which is expressed as

$$WT_f(a,\tau) = < f(t), \psi_{a,\tau}(t) > = \frac{1}{\sqrt{a}} \int_R f(t) \overline{\psi(\frac{t-\tau}{a})} dt \tag{2}$$

where a is scale factor and τ is translation factor.

When analysing signals of a non-stationary nature, it is often beneficial to be able to acquire a correlation between the time and frequency domains of a signal. The Fourier transform, provides information about the frequency domain, however time localised information is essentially lost in the process. The problem with this is the inability to associate features in the frequency domain with their location in time, as an alteration in the frequency spectrum will result in changes throughout the time domain. In contrast to the Fourier transform, the wavelet transform allows exceptional localisation in both the time domain via translations of the mother wavelet, and in the scale (frequency) domain via dilations. This is why we use wavelet analysis for pretreat time series before testing Granger causility because the series to be dealt with is non-stationary.

In practice, for conveniece of calculation it is needed to discretize the continous wavelet. Generally, it is to discretize the scale factor a and translation factor τ. To discretize a and τ, assuming that $a_0 > 1$ and $b_0 \neq 0$, then $a = a_0^{-m}$, $b = nb_0 a_0^{-m}$, thereby transforming continous wavelet into discrete one

$$\psi_{m,n}(x) = a_0^{m/2}\psi(a_0^m x - nb_0)\ m,n \in Z \qquad (3)$$

In digitization realiztion, discrete wavelet transformation is completed by famous Mallat algorithms, which is

$$\begin{cases} c_k^j = \sum_{l\in Z} c_l^{j+1}\overline{h}_{l-2k} \\ d_k^j = \sum_{l\in Z} c_l^{j+1}\overline{g}_{l-2k} \end{cases} \qquad (4)$$

the reconstruction algorithm can be expressed as

$$c_k^{j+1} = \sum_{l\in Z} c_l^j h_{k-2l} + \sum_{l\in Z} d_l^j h_{k-2l} \qquad (5)$$

where $\{h_k\}_{k\in Z} \in l^2(Z)$ are the coeffecients of low-pass filter and $\{g_k\}_{k\in Z} \in l^2(Z)$ are the coeffecients of high-pass filter. c_k^j is the approximation of c_l^{j+1} and d_k^j is the detailment of c_l^{j+1}.

2.2 Multi-resolution analysis

by multi-resolution analysis, the signal in every resolution is the sum of an approximate signal and detail signal. When continously adding the detail signal into approximation signal ,the original signal is reconstruted.

Multi-resolution analysis is a series of close subspace $\{V_j\}$, $j \in Z$ meets with the following conditions:

Concordant monotoly: $\ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \cdots$

Gradual perfectibility: $\bigcap_{j\in Z} V_j$ ={0}, $\bigcup_{j\in Z} V_j = L^2(R)$

Flex regularity: $f(t) \in V_j \Leftrightarrow f(2^j t) \in V_0$

Translation invariability:

$f(t) \in V_0 \Rightarrow f(t-n) \in V_0$, for all $n \in Z$

Existence of orthogonality: $\ni \phi \in V_0$,

$\{\phi(t-n)\}_{n\in Z}$ is the orthogonality base of V_0

$$V_0 = \overline{span\{\phi(t-n)\}}, \int_R \phi(t-n)\phi(t-m)dt = \delta_{m,n} \qquad (6)$$

where condition of existence of orthogonality can be relaxed to existence of Rieze base. According to the properties of multi-resolution analysis, the equation below can be satisied

$$\phi(t) = \sum_{k=-\infty}^{\infty} h[k]\sqrt{2}\phi(2t-k) = \sqrt{2}\sum_{k=-\infty}^{\infty} h(k)\phi(2t-k) \qquad (7)$$

which indicates the relationship between two ordinal scales. Its frequency version is

$$\Phi(\omega) = \frac{1}{\sqrt{2}} H(e^{j\frac{\omega}{2}})\Phi(\frac{\omega}{2}) \qquad (8)$$

where $H(e^{j\omega})$ is the discrete fourier transformation of $h[n]$.

Although Fourier analysis and wavelet analysis can both distill the wave components of series in different frequencies, Fourier analysis is good at dealing with stationary series. For non-stationary series, the wavelet is better than Fourier. And also, wavelet is well in localized time-frequency unite analysis.

3 Frequency Granger Causality Test Based on Wavelet Analysis

3.1 Granger causality test in cointegrated system

When both series are integrated of the same order, we can proceed to examine for the presence of cointegration. The Johansen Maximum likelihood procedures are used for the test (Johansen and Juselius, 1990). Any long-tem cointegrating relationship found between the series will contribute an additional error-correction term to the ECM. The Johansen procedure is a vector autogressive (VAR) based test on restriction imposed by cointegration in the unrestricted VAR. The null hypothesis in consideration is Ho, that there are a different number of cointegration relationship, against H1, that all series in the VAR are stationary. The ECM used in this paper is specified as follows:

$$\Delta y_t = \alpha_2 + \beta_2 ECM_{t-1} + \sum_i \alpha_{21}(i)\Delta y_{t-i} + \sum_i \alpha_{22}(i)\Delta x_{t-i} + \varepsilon_{2t} \quad (9)$$

$$\Delta x_t = \alpha_1 + \beta_1 ECM_{t-1} + \sum_i \alpha_{11}(i)\Delta y_{t-i} + \sum_i \alpha_{12}(i)\Delta x_{t-i} + \varepsilon_{1t} \quad (10)$$

Where ΔY_t, ΔX_t are the differences in the variables that capture their short-run disturbances, $\varepsilon_{1t}, \varepsilon_{2t}$ are the serially uncorrelated error terms, and ECT_{t-1} is the error-correction term (ECT), which is derived from the long-run cointegration relationship and measures the magnitude of the past disequilibrium.

In each equation, change in the endogenous variable is caused not only by their lags, but also by the previous period's disequilibrium in level, i.e. ECT_{t-1}. Given such a specification, the presence of short and long-run causality could be tested. Consider Eq.(1), if the estimated coefficients on lagged values of Xt are statistically significant, then the implication is that Xt Granger causes Yt in the short-run. On the other hand, long-run causality can be found by testing the significance of the past disequilibrium term.

The null hypothesis of the F test is:

H0 $\alpha_{22}(i) = 0, i = 1,2\cdots p$, and the statistics is

$$F = \frac{(RSS_R - RSS_0)/J}{RSS_0/(T-K)} \sim F(J, T-K) \quad (11)$$

3.2 Frequency Granger test based on wavelet analysis

In reequency domain time series can be seen as the sum of specific frequnecy components. So the Granger causality among series is the result of the sum of different frequency components. Because of the difference of fluctuation in different frequency components, the influcence of different on the Granger causality is different also. So the natural idea is to remove specific frequency component in the series firstly, then to test whether the Granger causality is changed in the reconstructed series. The decompostion and reconstruction of wavelet analysis provides such the tool. For example, consider the Granger causality of two series, electricity consumption and economic growth. If by deleting specific frequency component in electricity consumption, for example four year/cycle component, the unilieteral Granger cause running from electricity consumption to economic growth is not significant, it is evident that the four year/cycle component of electricity consumption is important in establishing the Granger cause from electricity consumption to economic growth. Otherwise, the frequency component is of no influence to the

Granger cause. On the other hand, if by deleting the specific frequency component of both series, the bilaterial Granger cause of the two series changes dramatically, it is evident that this frequency wave has significant influence on the Granger cause of the series. Finally, if by deleting the same component in two series, the cointegration relationship of the two series does not exist anymore, it means that the specific component is rightly the main component to establish the long-run equilibrium for the two series.

4 Case Study: Electricity Consumption and Economic Growth in China

In this section, we use the test of Granger cause between electricity consumption and economic growth in China to validate our idea.

In the paper ,wavelet haar is choosen as the mother wavelet of the analysis and according to the requirement the scale of resolution is determined as four. Thereby, the high frequency component of the lowest level corresponds to wave of two year/cycle , and four year, eight year, sixteen year in turn.

Because wavelet analysis is sensitive to the choose of mother wavelet, we adopt bior and rbio wavelet also to analyze the frequency charactersitic of the series, almost same results are obtained.

Sample the electricity consumption and economic growth(gross domestic product GDP) data for analysis, the time-frequency characterstic of the two series are shown in figure 1 and 2.

We are to deleting the two year, four year, eight year and sixteen year/cycle component of the series and reconstruct them. Then we proceed to test the Granger cause of the reconstructed series according to the idea explained in section 3. according our research in (Yuan Jiahai, Wang Jing, Hu Zhaoguang 2006), there exists cointegration between electricity consumption and GDP in China during 1978 to 2004 and there exists unilateral Granger cause running from electricity consumption to GDP.

We make three battery of tests in such way: 1) deleting the two year, four year, eight year and sixteen year/cycle component of electricity consumption and reconstruct it, then test Granger cause with untreated GDP series; 2) deleting the two year, four year, eight year and sixteen year/cycle component of GDP and reconstruct it, then test Granger cause with untreated electricity consumption series; 3) deleting the two year, four year, eight year and sixteen year/cycle component of two series and reconstruct them, then test Granger cause with reconstructed series. The results of the test are reported in table 1 to table 3. The calculation is done in existing tools: Eviews for the Granger causality test and MATLAB WAVELET TOOLBOX for wavelet analysis.

From table 1, we can see that by deleting the two year, four year and eight (and above) year/cycle wave component of electricity consumption, the Granger cause running from electricity consumption to GDP become insigficant, which implies that, the fluctuation of electricity consumption, be it high frequency or low frequency, has significant influence on GDP. From table 2, we can see that by deleting the two year, four year and eight (and above) year/cycle wave component of GDP, except for four year/cycle component, the Granger cause is unchanged, which implies that, the influence of GDP to electricity consumption, mainly works in long-run. From table 3, it is evident that the long run fluctuation components of the two series (eight year and above /cycle) play leading role in establishing their long run relationship. Especially, deleting the eight and sixteen year cycle components of the two series leads to the vanishing of cointegration between them.

To sum up, the fluctuation of electricity consumption, be it short term or long term, has significant impact on GDP; that of GDP, only long run low frequency component has significant impact on electricity consumption, which is evidence that the business cycle is correlated with fluctuation in electricity consumption; the cointegration relationship is mainly established by the low frequency components of the two series, which is in line with the conclusion of (Wang Jing, Yu En-hai, Yuan Jia-hai 2006).

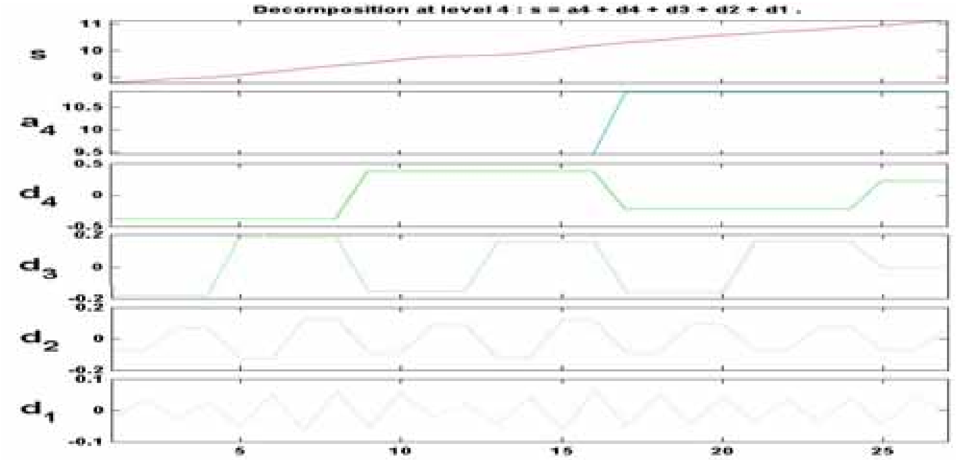

Fig 1 the time and frequency characteristic of logarithm series of GDP

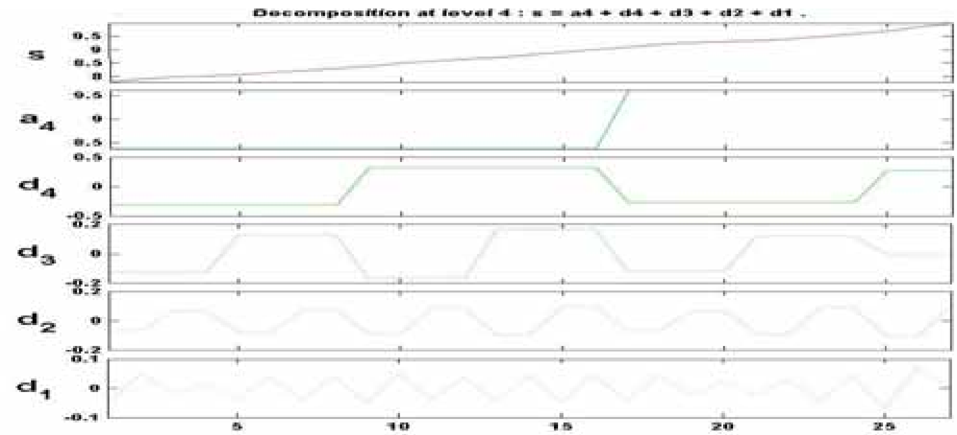

Fig 2 the time and frequency characteristic of logarithm series of electricity consumption

Tab 1-test results of Granger cause running from reconstructed electricity consumption series to GDP

Deleting component from electricity consumption	Short-run Granger cause	Long-run Granger cause
2 year /cycle	insignificant	insignificant
4 year/cycle	insignificant	insignificant
8 year and above/cycle	insignificant	insignificant

Tab 2-test results of Granger cause running from reconstructed GDP series to electricity consumption

Deleting component from GDP	Short-run Granger cause	Long-run Granger cause
2 year /cycle	insignificant	significant
4 year/cycle	significant	significant
8 year and above/cycle	insignificant	significant

Tab 3-test results of Granger cause with reconstructed GDP and electricity consumption series

Deleting component from GDP and electricity consumption	Short-run Granger cause	Long-run Granger cause
8 year /cycle component		
Electricity consumption to GDP	insignificant	insignificant
GDP to electricity consumption	insignificant	significant
8 year and above /cycle components		
Electricity consumption to GDP	insignificant	insignificant
GDP to electricity consumption	insignificant	insignificant

5 Concluding Remarks

In the paper, we propose a novel method based on wavelet analysis to test the impact of specific frequency fluctuation of series in establishing the cointegration and Granger causility among series. The method is rather time domain oriented and can rely on the standard procedure of error correction model. A case to test the Granger cause between electricity consumption and economic growth in China is investigated to validate our method.

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