Testing Co-feature in Cointegration System----the Case of Electricity Consumption and Economic Growth in China

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Abstract:- Co-integration techniques have been used in energy issue extensively. The study of short-run dynamics in perspective of spectral domain is another promising direction. However none study has cover the two directions together in energy study. In the paper, we investigate the vector error correction model considering co-feature. The identification procedure and algorithms are overviewed briefly in line with (Hecq. A., 2004). Then a simple case study for electricity consumption and economic growth in China is given for model comparison. It is found that model considering co-feature can potentially improve the model estimation efficiency. To our knowledge, although literature addressing energy and economic growth in cointegration theory is pouring in recently, there has been no study considering co-feature in the same time. So our paper can be seen as a first step in applying this kind of model in the field.

Key words: Cointegration theory, Co-feature, Empirical analysis, Electricity consumption

1 Introduction

Cointegration techniques are now routinely applied by economists to extract r meaningful longrun relationships among a set of n non stationary time series yt = (y_{1t}, \ldots y_{nt})'. Interpreting the issue in its dual form, this means there also exist (n–r) common trends, hence only (n-r) common permanent shocks driving the economy. To evaluate the presence of such long-run co-movements, the Johansen maximum likelihood approach (1995 inter alia) is widely used.

Beyond cointegration, a number of papers have concentrated on modelling the common serial correlation feature among stationary time series. Indeed, alike cointegration is associated with long-run relationships, common dynamics is a sign of co-movements in the short-run. These common sources of transmission mechanisms, namely the so called common cycles, allow to extract common transitory shocks that can often be linked to business cycle co-movements. Additional advantages of considering these short-run restrictions are the large reduction of the number of parameters that need to be estimated and their role for forecasting (see Vahid and Issler, 2002 and Hecq, Palm and Urbain, 2004 on this latter issue).

Testing the relationship between energy consumption and economic growth has being a widely discussing field since 1980th. After the seminal work of Kraft (1978), there are lots of empirical literatures addressing the issue using cointegration method ( Akarca A.T., Long.C.V. 1980; Alice Shiu, Pun-Lee Lam, 2004). Our work in () discusses the long-run relationship between electricity and GDP in China during 1978 to 2004. In other work, we address the same issue in perspective of short-run relationship ( Wang Jing Yu Enhai ,Yuan Jiahai, 2006).

Obviously, as far as both long-run and short-run co-movements are of interest for the researcher, a natural practice consists in first estimating super-consistently the cointegrating vectors. In a subsequent step, it will be easy to perform a test for common serial correlation by considering the cointegrating relationships as given.

In the paper, we are to discuss the two-step procedure for testing co-feature in cointegrating system and interested in testing whether there exists
co-feature as cointegration pr-exists between electricity consumption and economic growth in China.

2 Model Representation

We consider the n-dimensional vector autoregressive model of order p for the I(1) variables $y_t = (y_{1t}, \ldots, y_{nt})'$

$$y_t = \Pi_0 y_{t-1} + \Pi_1 y_{t-2} + \ldots + \Pi_p y_{t-p} + \mu_t \tag{1}$$

Decomposing the matrix lag polynomial $\Pi(L) = \Pi(1)L + \Gamma(L)(1-L)$, and defining $\Delta = (1-L)$, we obtain the vector error correction model

$$\Delta y_t = \Theta D_t + \alpha \beta y_{t-1} + \sum_{j=1}^{p} \Gamma_j \Delta y_{t-j} + u_t, \quad t = 1, \ldots, T \tag{2}$$

Serial correlation common feature (SCCF hereafter, see Engle and Kozicki, 1993) holds for the VECM in (2), if there exists a $(n \times s)$ matrix $\delta$, whose columns span the cofeature space, such that $\delta' (\Delta y_t - \Theta D_t) = \delta' u_t$ is a $s$-dimensional vector mean innovation process with respect to the information available at time $t$. Consequently, SCCF arises if there exists a matrix $\delta$ such that the conditions $\delta' \Gamma_j = 0_{(s \times n)}, j = 1, \ldots, p - 1$ and $\delta' \Pi(1) = \delta' \alpha \beta = 0_{(s \times n)}$ are jointly satisfied.

When cycles are not perfectly synchronized (WF hereafter), alternative specifications have been proposed by Vahid and Engle (1997) and Cubadda and Hecq (2001). These two latter models have the advantage to have a nice interpretation in terms of delays of adjustment to shocks in the multivariate Beveridge-Nelson representation. Instead in the WF, we have the null hypothesis $\delta' (\Delta y_t - \Theta D_t) = \delta' \beta y_{t-1} + \delta' u_t$, where $\delta' = \delta' \alpha$.

3 Test Statistics

3.1 The two-step approach

As a shortcut, the expression

$${\text{CanCor}}\left\{ \Delta y_t, \Delta y_{t-1}, \ldots, \Delta y_{t-p+1} \right\}$$

will summarize the reduced rank regression procedure used in the Johansen approach. That means that one extracts the squared canonical correlations between $\Delta y_t$ and $y_{t-1}$, both sets concentrated out the effect of deterministic terms and lags of $\Delta y_t$. In order to test for the significance of the $r$ largest eigenvalues, one can rely on Johansen’s trace statistic (3) or on one of the modified version to account for small samples like in (4).

$$LR_r = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \tag{3}$$

$$LR_r = -(T - np) \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \tag{4}$$

Once $\beta$ has been found in a first step and superconsistently estimated by $\hat{\beta}$, we can implement the common feature test statistics. We make again the distinction between the SCCF and the WF specifications. These are respectively based on the following reduced rank regressions:

$$\text{SCCF}: {\text{CanCor}}\left\{ \Delta y_t, (\Delta y'_{t-1}, \ldots, \Delta y'_{t-p+1}, y'_{t-1}\hat{\beta}) \right\} | D_t$$

$$\text{WF}: {\text{CanCor}}\left\{ \Delta y_t, (\Delta y'_{t-1}, \ldots, \Delta y'_{t-p+1}) \right\} | (D_t, y'_{t-1}\hat{\beta})$$

Both procedures allow to obtain the squared canonical correlations, namely the eigenvalues $\hat{\lambda}_{SCCF}$ or $\hat{\lambda}_{WF}$ used to test for rank reductions. For the VECM of order $p-1$, the significance of the $s$ smallest eigenvalues is evaluated through the following likelihood ratios:

$$LR_{SCCF} = -T \sum_{i=s+1}^{n} \ln(1 - \hat{\lambda}_{SCCF}^i) \sim \chi^2(v_s), s = 1, \ldots, n - r$$

$$LR_{WF} = -T \sum_{i=s+1}^{n} \ln(1 - \hat{\lambda}_{WF}^i) \sim \chi^2(v_s), s = 1, \ldots, n$$

Alternatively, information criteria can be used.
For \( p \) fixed and \( r \) given we can obtain these information criteria for different values of reduced rank \( n-s \) using

\[
AIC(p, r, s) = -\frac{2}{T} \log \text{lik} + \frac{2}{T} \times r \times (np+n-r)
\]

(7)

\[
HQ(p, r, s) = -\frac{2}{T} \log \text{lik} + \frac{2 \ln \ln T}{T} \times r \times (np+n-r)
\]

(8)

\[
HQ(p, r, s) = -\frac{2}{T} \log \text{lik} + \frac{\ln T}{T} \times r \times (np+n-r)
\]

(9)

Where the log likelihood is nothing else than the log of the determinant of the reduced rank residuals covariance matrix under common feature restrictions.

### 3.2 Test algorithm

To illustrate the approach, we consider a cointegrated VAR with an intercept, one lag in its VECM form (\( p = 2 \) in the VAR) and common factor restrictions similar to (2) such that

\[
\Delta y_t = u + \delta_{s}^{\perp} \Psi_1 \beta y_{t-1} + \delta_{s}^{\perp} \Psi_2 \Delta y_{t-1} + \epsilon_t
\]

(10)

where \( \delta_{s}^{\perp} \) is the orthogonal complement of the cofeature matrix, namely \( \delta_{s}^{\perp} \delta = 0_{svn} \) and \( \text{rank}[\delta_{s}^{\perp} : \delta] = n \). Hansen and Johansen (1998) impose SCCF restrictions by premultiplying (10) by the partitioning matrix \( B \),

\[
B = \begin{pmatrix}
(\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} \\
\delta_{s}^{\perp} \in \mathbb{N}
\end{pmatrix}
\]

\[
(\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} \Delta y_t = u^{**} + \Psi_1^{\perp} \beta y_{t-1}^{*} + \Psi_2 \Delta y_{t-1}^{*} + (\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} \epsilon_t
\]

(11)

\[
\delta_{s}^{\perp} \Delta y_t = u^{**} + \delta_{s}^{\perp} \epsilon_t
\]

(12)

where \( u^{**} = (\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} u \) \( u^{**} = \delta u \) are vector column of size respectively \( n-s \) and \( s \). Solving (11) and (12) gives

\[
(\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} \Delta y_t = (u^{**} - \omega \epsilon^{**}) + \Psi_1^{\perp} \beta y_{t-1}^{*} + \Psi_2 \Delta y_{t-1}^{*}
\]

(13)

where \( \omega = \text{cov}((\delta_{s}^{\perp} \delta_{s}^{\perp})^{-1} \delta_{s}^{\perp} \epsilon_t, \delta_{s}^{\perp} \epsilon_t) \var{\delta_{s}^{\perp} \epsilon_t}^{-1} \).

The algorithm is as follows: (i) estimate \( \beta \) without constraints in a first step, i.e. the usual Johansen approach; (ii) fixing the matrix \( \beta \) to its estimated value, estimate \( s \) and \( \delta \); (iii) obtain the \( n-s \) common dynamic factors \( \Psi_0 = (\Psi_1^{\perp}, \Psi_2) \) using the duality principle of canonical correlations; (iv) estimate \( \delta \) in (10) by multivariate least squares; (v) reestimate \( \beta \) and keep on iterating until convergence is reached.

In the WF case there are not cross-equation restrictions similar to (10), so the constrained model is simply

\[
\Delta y_t = u + a \beta^{*} y_{t-1} + \delta_{s}^{\perp} \Psi_2 \Delta y_{t-1} + \epsilon_t
\]

(14)

Imposing WF restrictions is convenient because this allows to consider both cointegration and common feature test statistics without the constraint \( r + s \leq n \). To solve (14), we start by estimating \( \beta \) by ML and we fix it to find the number of common feature vectors \( s \). We estimate the \( n-s \) dynamic common factors forming \( \Psi_2 \) in (14) and we use this constraint to reestimate \( \beta \) using the program

\[
\text{CanCor} \{\Delta y_t, y_{t-1} \} \text{CanCor} \{1, \Delta y_{t-1}, \Psi_2 \}
\]

This sequence is iterated until convergence is reached.

### 4 Case Study: Electricity Consumption and Economic Growth in China

#### 4.1 The data, cointegration and co-feature analysis

Following the study of (Yuan Jiahai et al 2006), we discuss the VECM model considering co-feature for electricity consumption in China. Our study covers the time period of 1978 to 2004. According to Yuan Jiahai et al 2006, there exists a long-run cointegration with the cointegration equation being,

\[
\text{LELEC} = 0.88 \times \text{LGDP}
\]
Since we have found that there exists one common trend, the identification procedure for this bi-variable system is rather simple. According to the test procedure discussed in section 3, we proceed on co-feature test. The test results are reported in tab 1.

The LR test, AIC and HQ criterion have all witnessed the existence of one co-feature vector, except for SC witnessing two co-feature vectors. Because (Vahid F. Issler 2002) gives out the monte-carlo simulation that HQ criterion is apt to over-estimate while the estimation of HQ is more reliable, we can confidently state that there exists only one co-feature vector in the system, with the co-feature equation being,

\[ \Delta ELEC = 0.4678 \times \Delta DGDP + 0.034 \times 0.025D \]

where D is a dummy variable as D=1 for T>1998 and D=0 for other periods, to capture the structure shift in the economy.

### 4.2 the analysis of model estimation accuracy

We compare the model estimation results of VECM model in (Yuan Jiahai et al 2006) without considering co-feature and the VECM model considering co-feature to investigate the feasibility of CVECM in energy study. Figure 1 gives the comparison of in sampling fitting of the both models. In the figure, line “actual” is the actual electricity consumption during the period, “vecmf” is the fitting of VECM model while the line “cvecmf” is the fitting of CVECM model. It can be seen that the fitting accuracy during 1978 to 1990 of both models is almost the same. However, after 1990, CVECM is systematically better than VECM, which implies that, the consideration of co-feature can enhance the efficiency of model estimation and lead to the improvement of prediction accuracy.

### Tab 1-test results of co-feature in electricity consumption and GDP

<table>
<thead>
<tr>
<th>Null</th>
<th>eigenvalue</th>
<th>LR</th>
<th>5% significance value</th>
<th>P-value</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 0*</td>
<td>0.678</td>
<td>23.9918*</td>
<td>12.5</td>
<td>0.04</td>
<td>-9.85*</td>
<td>-8.65</td>
<td>-10.34*</td>
</tr>
<tr>
<td>S 1</td>
<td>0.145</td>
<td>2.35567</td>
<td>3.85</td>
<td>0.85</td>
<td>-9.21</td>
<td>-8.75*</td>
<td>-10.18</td>
</tr>
</tbody>
</table>

Fig 1 the prediction of VECM and VECM model considering co-feature (CVECM)
5 Concluding Remarks

In the paper, we investigate the vector error correction model considering co-feature. The identification procedure and algorithms are overviewed briefly. Then a simple case study for electricity consumption and economic growth in China is given for model comparison. It is found that model considering co-feature can potentially improve the model estimation efficiency. To our knowledge, although literature addressing energy and economic growth in cointegration theory is pouring in recently, there has been no study considering co-feature in the same time. So our paper can be seen as a first step in applying this kind of model in the field.

References: