Reducing the power consumption in wireless network by the use of Chvatal-Gomory strategy

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Abstract

This aim of this paper is to present a new methodology to reduce the power consumption in wireless applications. This new formulation adapts the hub and spoke model with constrains in the arcs to model the wireless channel and the use of Chvatal-Gomory strategy to generate a benefit of 8000% in the CPU time and a power saving of 30 dB allowing faster hub reallocation. Also comparison with standard methods and complexity studies are carried out.

Keywords: *Hub and Spoke model, Computer efficiency and Wireless Networks*

1.- Introduction

Traditional wireless networks such as cell-phone networks—rely on centrally located towers. Smaller devices send signals to one of the towers, which communicate with other towers or with other devices in the local network. This is called a “hub and spoke” architecture because it resembles a wagon wheel; the tower acts as the hub and sends spokes to all the local devices. These networks need high power, and every new device attached is a drain on the capacity of the hub [1].

Usually, Wireless Sensor Network (WSN) resembles a mesh rather than the hub-and-spoke arrangement used for cell phones; instead of linking each sensor directly to a central communication point, the nodes send data only to neighbours within radio range, saving power [2]. The literature reports several formulations for the hub and spoke model with a large number of versions; additional good references can be found in [3]. The aim of this paper is to describe a method to obtain an optimum solution to Hub and Spoke models in sensible times. It firstly in the system description explains the main characteristics of Hub and Spoke models. Secondly, it presents the mathematical framework. Third a description new method, which improves the algorithm as computer efficiency; case studies conducted in LINGO are then presented, performance times are compared with and without the new method. Finally, a few concluding remarks are made.

2.- System description

Figure 1 present a system description corresponding to the simplest method is to directly connect pairs of nodes in a wireless network. To improve this intuitive method, hub nodes are introduced: communications take place through a connection shaped by one or a set of hub nodes with their associated cities of nodes. Both network designs – totally connected and Hub and Spoke networks–have $N(N-1)$ origin/destine pairs, where N is the number of network nodes. A totally connected network needs one connection per each origin/destine pair; that is, $N(N-1)$ connections. If a network with one hub is designed and connected with all other hub nodes, then $2(N-1)$ connections are needed to service all origin/destine pairs.

If it is desired to assign p nodes as hubs in a network, then the model is known as p-Hub. There are variations or extensions to the p-Hub models. The most studied variation is known as Incapacitated Single Allocation p-Hub Median Problem, USApHMP for short. In this variation, exactly p hubs must be allocated among the possible N nodes and each of the other nodes is assigned with exactly one hub (Single Allocation). The purpose is to minimize the total transport cost.

3.- Hub and Spoke mathematical formulation

It now follows the quadratic and linear versions of the CCSApHMP model. Within the model *p* is assigned through a simple manner, each hub is connected to one hub only and archway capacities are restricted.

Capacitated Cost Single Allocation *p*-Hub Median Problem. Quadratic formulation

Original Problem:

$$Q_0) \qquad Min \quad \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{j=1}^{j} F_{ij} H_{ij} (\alpha * d_{ik} + \beta * d_{kl} + \gamma * d_{lj}) * Y_{ik} * Y_{jl} \tag{1}$$

subject to

$$\sum_{k=1}^{n} X_k = p \tag{2}$$

and

$$\sum_{k=1}^{n} Y_{ik} = 1, \quad \forall i = 1 \ldots n \tag{3}$$

$$Y_{ik} \le X_k, \quad \forall i = 1, \ldots n, k = 1 \ldots n \tag{4}$$

$$\sum_{i \ne k}^{n} (\sum_{j=1}^{n} H_{ij} Y_{ik} - H_{ii} * Y_{ik}) + H_{kk} * X_k + \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{l \ne k} H_{ji} * Y_{ik} * Y_{jl} \le M_k, \quad \forall k = 1 \ldots n \tag{5}$$

and

$$\sum_{i=1}^{n}\sum_{j=1} (H_{ij} + H_{ji}) * Y_{ik} * Y_{jl} = V_{kl}, \quad \forall k < l \tag{6}$$

$$\sum_{j \ne i} (H_{ij} + H_{ji}) * (1 - X_i) = Q_i \quad , \qquad \forall i = 1 \ldots n \tag{7}$$

and

$$X_k \in \{0,1\}, \quad \forall k = 1 \ldots n \tag{8}$$

$$Y_{ik} \in \{0,1\} \quad \forall i = 1 \ldots n, k = 1 \ldots n \tag{9}$$

Inputs:

H_{ij} :demand or flow between origin node *i* and destine node *j*.

d_{ij} : distance between node *i* and node *j*.

Costs*:*

α :unitary cost to collect per distance unity and demand unit.

β :transfer unitary cost per distance unity and demand unit.
γ :Distribution unitary cost per distance unity and demand unit.
F_{ij} :Unitary cost to satisfy a demand unit using node connection *i* and *j*.
Variables:
Q_i :Capacity of connection between spoke i and its linked hub.
V_{kl} :Capacity of connection hub(*k*) to hub(*l*).
Decision Variables
Y_{ik} : if node *i* is connected to a hub in node *k* and 0 otherwise.
X_k : 1 if hub located in el node *k* and 0 otherwise.

4.- Proposed Algorithm

The methodology proposed in this paper is based in the Chvatal-Gomory method [3] and [5], to improvement the formulation of the problem, transforming the structure of the polyhedron in other more similar to the hull convex of the set of integer points. This strategy transforms weak inequalities in strong constraints by round down the coefficients of the inequalities and the right size too. Although this strategy is widely used in integer linear problems, it is thought that it can also be useful for this problem because the search zone is reduced, despite being it a non linear problem. In both forms, restriction (2) works as a complex parameter and it cannot be changed because it is associated to the available number of hubs in the system. A simplified version is obtained if the number of hubs outside the system were the same as the number of hubs in the network; on the contrary, if p = 1 a more complex problem is obtained.

The methodology proposed solves the problem in two stages, well defined. In the first stage, a transformation in the Chvatal–Gomory sense of the inequalities is made and then the variables to represent the flow between arcs are relaxed of be integer and are only constrained to be between zero and one. In the second stage, the results of the decision variable that represent if a node is assigned like hub or not, are fixed in the value obtained in the first stage, the variables that represent the flow between arcs are integer again and a transformation in the Chvatal–Gomory sense of the inequalities is made, such as in the first stage, then the problem is solved to optimality.

Next we explain the Chvatal-Gomory method and it adaptation to the:
Capacitated Cost Single Allocation *p*-Hub Median Problem, Quadratic formulation
Problem:

Thus, the inequality:

$$\sum_{j=1}^{n} a_j x_j \leq b \qquad (10)$$

is transform in:

$$\sum_{j=1}^{n} \lfloor a_j \rfloor x_j \leq \lfloor b \rfloor \qquad (11)$$

It can be demonstrated that the inequality (2) is always valid for the original problem and that it is stronger than (1). These inequalities were applied to inequalities of the (5) type in the quadratic formulae. The transformed inequality (12) is as follows:

$$\sum_{i \neq k}^{n}(\sum_{j=1}^{n}\lfloor H_{ij} \rfloor Y_{ik} - \lfloor H_{ii} \rfloor * Y_{ik}) + \lfloor H_{kk} \rfloor * X_k + \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{l \neq k}\lfloor H_{ji} \rfloor * Y_{ik} * Y_{jl} \leq \lfloor M_k \rfloor, \forall k = 1 \ldots n \quad (11)$$

The new transformed inequality (12) is solved in two phases, as is explained next.

In the quadratic formulae the integer variable X determines which nodes are assigned to hubs and the integer variable Y defines which archways are used as links. This is the reason why the integer variable X is "stronger" than variable Y. That is, if a group of nodes p is assigned as hubs, the problem is to find the best archways to be used as links, which is a less complex problem than the original one. This is so because the space of possible solutions is smaller. This idea was used to prove solutions to problems derived from the original and arising from relaxing either integer variables X or Y.
The option with better results was to solve the problem in two stages: first, the problem was solved by relaxing variable X from being integer. In all solved instances, this variable obtained 0 or 1 values. The problem was then solved by fixing variable X with previously obtained values and variable Y as 0-1 variable. The optimum solutions were obtained match with solutions of the original problem, without the two stages method, but solution times were much smaller, ten times smaller. The method is now presented:
Let Q_1 the problem arising from changing restrictions (5) for (12) in Q_0.
Let Q_2 the problem arising from changing restriction (9) for the following (13) in Q_1.

$$Y_{ik} \geq 0 \quad \forall i = 1 \ldots n, k = 1 \ldots n$$
$$Y_{ik} \leq 1 \quad \forall i = 1 \ldots n, k = 1 \ldots n \quad (12)$$

Problem Q_2 is solved by using standard non linear programming software.
Let X^* an X vector from the optimum solutions by solving Q_2.
Let Q_3 the resulting problem form eliminating in Q_1 restrictions (8) and replacing X^*.
Results are shown later in a results table. Instances of the Hub and spoke model were taken from the public library on-library [20]. Here the exhibited problems are not from the capacitated type, therefore a dimensional procedure was applied to fix archway capacity, which were possible and of medium to high complexity.

Capacitated Cost Single Allocation *p*-Hub Median Problem. Lineal formulation

M_k : node hub k capacity to collect and distribute incoming and own demands.
d_{ij} : distance between node i and node j.

$$C_{ijkl} = \beta d_{ik} + \alpha d_{kl} + \delta d_{lj}$$

F_k : cost to make a hub in node k.
Variables: V_{kl} : capacity of connection hub(k) to hub(l).
Q_i : capacity of connection between spoke i and its linked hub.
Decision Variables
X_k : 1 if hub located in el node k and 0 otherwise.
Y_{ik} : 1 if node i is connected to a hub in node k and 0 otherwise.
Z_{ijkl} : 1 if node i arrives to j through hub k and l, and 0 otherwise.

Consequently, the model is as follows:

$$Q_1) \qquad Min \quad \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{j=1}^{j} F_{ij} H_{ij} (\alpha * d_{ik} + \beta * d_{kl} + \gamma * d_{lj}) * Y_{ik} * Y_{jl} \tag{13}$$

subject to

$$\sum_{k=1}^{n} X_k = p \tag{14}$$

and

$$\sum_{k} Y_{ik} = 1 \quad \forall\ i \tag{15}$$

$$Y_{ik} \leq X_k \quad \forall\ i,k \tag{16}$$

$$\sum_{l} Z_{ijkl} = Y_{ik} \quad \forall\ i,k,j \tag{17}$$

and

$$\sum_{k} Z_{ijkl} = Y_{jl} \quad \forall\ i,l,j \tag{18}$$

$$\sum_{i \neq k}\sum_{j} H_{ij}\, Y_{ik} - \sum_{i \neq k} H_{ii}\, Y_{ik} + H_{kk}\, X_k + \sum_{j}\sum_{i}\sum_{l \neq k} H_{ji}\, Z_{ijkl} \leq M_k \quad \forall\ k \tag{19}$$

$$\sum_{l \neq k}\sum_{i}\sum_{j} (H_{ij} + H_{ji}) Z_{ijkl} = V_{kl} \quad \forall\ k,l \tag{20}$$

and

$$\sum_{i}\sum_{j \neq i} (H_{ij} + H_{ji})\, (1 - X_i) = Q_i \quad \forall\ l \tag{21}$$

$$X_k \in \{0,1\} \quad \forall k \tag{22}$$

and

$$Y_{ik} \in \{0,1\} \quad \forall i,k \tag{23}$$

$$Z_{ijkl} \in \{0,1\} \quad \forall i,k,l,j \tag{24}$$

The objective function, in linear formulation, minimize the transport demand total cost through the location of p hubs and the mathematical assignment of other nodes to the nearest hub and adding the costs of making those p hub nodes (F_k). The total cost is made up of collecting, transferring and distributing costs. Constraint (15) shows that the number of hubs to be located in the network is exactly *p*. Constraint (16) determines that each origin or destine node is assigned to a unique hub (single allocation). Constraint (17) establishes that a node *I* cannot be connected to a hub node *k* if there is no allocation of hub in node *k*.

Constraints (18) and (19) show that there exists one pair of hubs that link origin node *I* with destine node *j*; additionally, they show that the flow from node *I* towards node *j* cannot be sent to hub node *k* if origin node *I* in not linked to hub in *k*. Something similar happens for the case of destine node. Constraint (20) establishes that a node k cannot be a hub if its capacity to collect and distribute demand is lower than the amount collected and distributed from and towards nodes assigned to hub *k*. Constraints (21) and (22) determine the capacity of the hub to hub connection as well as from hub to spoke, to assure the correct network functioning. Finally, (23), (24) and (25) define decision variables X_k, Y_{ik} and Z_{ijkl} as binary variables.

5.- Computational results

To evaluate the performance of the proposed method, we developed an experimental computational, using simulated problem instances which are solved using a Standard method (SM) code.
Considered problems have twenty nodes and three hubs. These are taken from the public internet library OR-Library [6]. For this case, the problems were solved using the software LINGO version 9, running in a PC, Pentium 4. This optimization software uses a program based in the Conjugated Gradient Method [4] to solve this kind of problems, which is a general purpose procedure for non linear problems. Every problem was solved following the original formula and then following the two stages method developed in this paper.
In the table 1, we show the result of CPU time, for different instances of the studied problem. We can see from the results in this table, there is a saving when using the proposed method. As we can see from the results in Table 1, there is saving when using the proposed method, although the conclusions pertain only to this set of data, corresponding to rather small instances and for one specific formulation.
Figure 3 present a comparison of iterations generated for SM and the proposed method, the iterations axis is compressed using logarithmic scale, to make visible the iterations generated by the proposed method which most of the times produce less iterations than the SM. Although the power saving can be estimated using the average number of iterations of SM and the proposed method divided by the iteration corresponding to the proposed method, producing a benefit of 30dB in terms of power which allow to reduce the size of the hub as the computers in the spoke. This benefit is generated since structure of the problem in the proposed method is easier to solve, because the new polyhedron is more compact and is more near to the convex hull, and can be implemented to introduce hub reallocation. This is a very interesting property in wireless sensor network were power save is required.

6.- Conclusions and future works

This formulation is being conducted in adapting the CCSApHMP model to the problem of designing wireless IP networks, where the channel capacity is of great importance to improve the economic-technical efficiency of its design and quality of service.
This model determines the capacity of inter mode archways for demand transport and introduce a benefit of 8000% in the CPU time and a power saving in the power of 30 dB, which allow faster hub reallocation.
The model formulation showed the great amount of variables needed for its solution, it is a complex formulation (NP-hard), which makes running time relatively high. Therefore it would be interesting to develop heuristic or Meta heuristic methods to find a better solution in less time.

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Table 1: Comparison of the solution time for the standard method versus the proposed method

	Standard method		Proponed method						
			First stage		Second stage		Total Methos proposed		
Instance	Solution Time	Iterat,	Solution Time	Iterat,	Solution Time	Iterat.	Solution Time	iterat.	% Save time
1	10:39	345	1:03	25	1:29	8	2:32	33	7618%
2	13:48	2501	0:38	41	1:19	327	1:58	368	8574%
3	14:40	225	1:00	23	0:23	8	1:24	31	9043%
4	18:40	7	0:14	6	0:28	8	0:43	14	9614%
5	23:01	1606	0:57	32	0:26	8	1:24	40	9390%
6	18:52	343	0:58	34	0:23	8	1:21	42	9279%
7	18:46	314	0:43	32	0:17	8	1:01	40	9451%
8	10:09	547	0:20	46	6:35	89	6:55	135	7973%
9	20:39	520	0:55	40	2:43	8	3:39	48	8232%
10	22:10	896	1:02	34	1:29	8	2:31	42	8859%
11	19:13	70	0:46	36	1:38	9	2:24	45	8747%
12	19:15	726	1:01	31	3:13	8	4:15	39	7793%
13	21:53	502	0:54	39	3:17	8	4:12	47	8080%
14	17:33	709	1:02	33	3:24	8	4:27	41	7464%
15	16:33	262	0:51	38	1:48	8	2:40	46	8384%
16	19:04	790	1:03	37	1:47	10	2:50	47	8508%
17	19:33	185	0:48	35	2:32	8	3:20	43	8287%
18	6:44	151	0:34	31	2:58	8	3:32	39	4737%
19	18:12	991	0:19	30	1:25	8	1:45	38	9034%
20	7:01	607	0:36	34	1:39	8	2:16	42	6775%

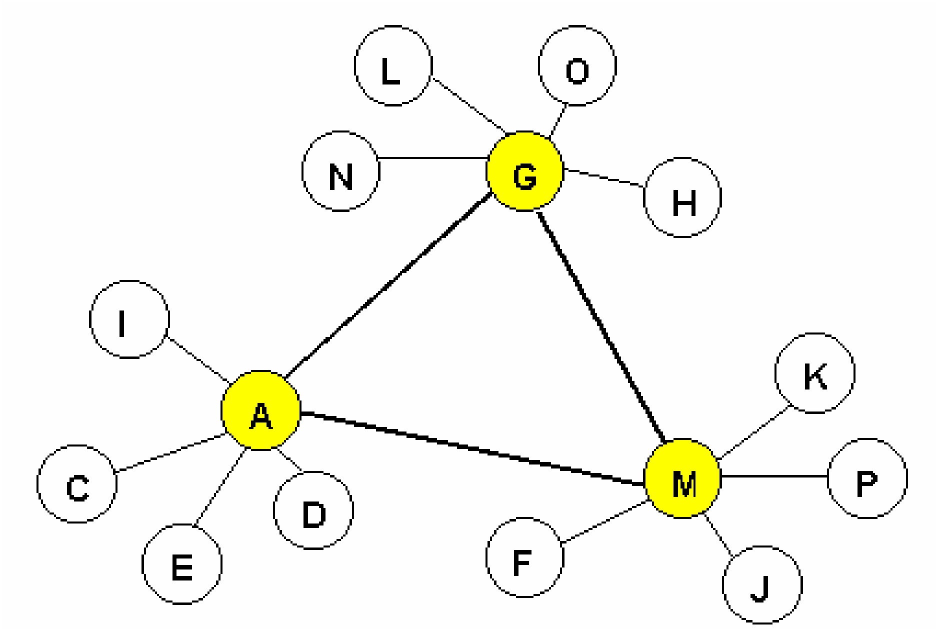

Figure 1: A network with multi hubs characteristics containing three hubs and 210 origin-destine pairs

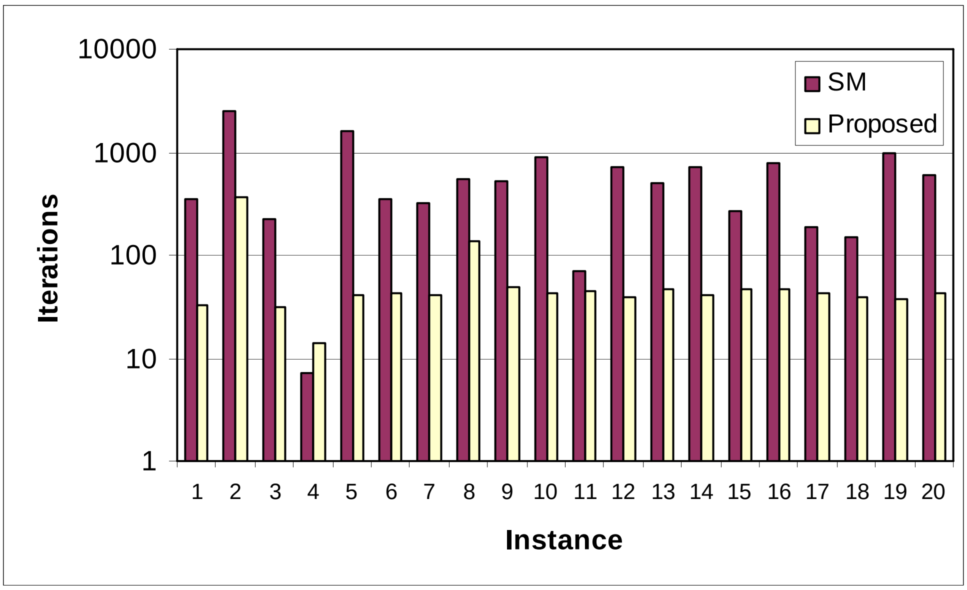

Figure 2: Comparison of Standard Method versus proposed method