Conservative Averaging Method and its Application for One Heat Conduction Problem

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Abstract: In this paper the description of the method of conservative averaging for partial differential equations in two-layer domain is given. As numerical example the non-linear heat conduction in electrical wire with insulation is considered. This problem is solved numerically.

Key-Words: partial differential equations, discontinuous coefficients, conservative averaging, non-classical boundary conditions, heat conduction, electrical wire, insulation temperature.

1 Introduction
Conservative averaging method was developed for partial differential equations with discontinuous coefficients. Because of its publication in non-English language and hard accessible papers [1] [2], we give here a short general conception of the method. Application of this method to heat conduction in wire with insulation is demonstrated.

2 Description of Conservative Averaging Method for Rectangular Two-layer Domain in Cartesian coordinates
Size of this paper allow us to describe this method for second type boundary condition (5) only.

2.1 Initial Problem
Let us look at domain \( D \) where \((x, y) \in D \subset R^{n+1}, x \in R, y = (y_1, ..., y_n) \in R^n \).

Domain \( D \) consists of several sub domains (for simplicity we can image \((n + 1)\) - dimensional parallelepiped, see Fig.1):

\[
D = G_0 \cup G \cup H
\]

\[
G_0 = \{(x, y) | x = (\delta, 0), y \in D\}
\]

\[
G = \{(x, y) | x > 0, y \in D\}
\]

With notation \( x > 0 \) we understand that domain \( G \) is located to the right from domain \( G_0 \). Shared border-hyper plane of domains \( G_0 \) and \( G \) we denote as \( H \):

\[
H = G_0 \cap G = \{(x, y) | x = 0, y \in D\}.
\]

Right border of the domain \( G_0 \) we denote as \( H_0 \):

\[
H_0 = \{(x, y) | x = -\delta, y \in G_0\}.
\]

We will use notation \( x = 0 \) and \( x = -\delta \) for hyper planes \( H \) and \( H_0 \).

Objective is to find function \( U_0(x, y) \) (continuous in domain \( G_0 \)) and function \( U(x, y) \) (continuous in domain \( G \)) that fulfills following equations:

a) differential equations:

\[
\frac{\partial}{\partial x} \left( k_0 \frac{\partial U_0}{\partial x} \right) + L^0(U_0) = -F_0(x, y), \tag{1}
\]

\[
\frac{\partial}{\partial x} \left( k \frac{\partial U}{\partial x} \right) + L(U) = -F(x, y), \tag{2}
\]

where \( L \) and \( L_0 \) are differential operators;

b) conjugation conditions:

\[
U_0 \bigg|_{x=0} = U \bigg|_{x=\delta}, \tag{3}
\]

\[
k_0 \frac{\partial U_0}{\partial x} \bigg|_{x=0} = k \frac{\partial U}{\partial x} \bigg|_{x=\delta}; \tag{4}
\]

c) boundary condition on border \( H_0 \):

\[
-k_0 \frac{\partial U_0}{\partial x} \bigg|_{x=\delta} = \phi^0(y); \tag{5}
\]
d) boundary conditions on the rest borders that are not necessary to concretize at this moment:
\[ \tilde{f}(U) = \tilde{\Psi}(x, y), \quad (x, y) \in \partial D = \partial D \setminus H_0. \quad (6) \]

Here continuous on \( \bar{D} \) (see (3)) function \( \tilde{U}(x, y) \) is defined to be equal to function \( U_0(x, y) \) in domain \( \bar{G}_0 \) and to be equal to \( U(x, y) \) in domain \( \bar{G} \).

We require that all derivatives of equation (1) are continuous in domain \( G_0 \) and derivatives of equation (2) – in domain \( G \). Derivative \( \partial \tilde{U}/\partial x \) is continuous function in domain \( D \) except hyper plane \( H \) where it has the first kind discontinuity and condition (4) is fulfilled. Note that value \( \delta \) is comparatively small but still finite – averaging will be not made by passage to the limit \( \delta \to 0 \).

We assume that coefficient \( k_0 \) and coefficients of the differential operator \( L^0 \) are not dependent on argument \( x \) but could be dependent on argument \( y \). Besides, operator \( L^0 \) is linear and it do not contain derivatives regarding the argument \( x \).

Let us see two examples of operators \( L \) and \( L^0 \):

1) operators for 1D heat transfer problem:
\[ L^0(U_0) = -c_0 \rho_0 \frac{\partial U_0}{\partial t}, \quad (7) \]
\[ L(U) = -c \rho \frac{\partial U}{\partial t}; \quad (8) \]

2) operators for steady-state 3D heat transfer:
\[ L^0(U_0) = \frac{\partial}{\partial y} \left( k_1 \frac{\partial U_0}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_2 \frac{\partial U_0}{\partial z} \right), \quad (9) \]
\[ L(U) = \frac{\partial}{\partial y} \left( k_1 \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_2 \frac{\partial U}{\partial z} \right). \quad (10) \]

Such estimate is valid:
\[ \max_{x \in [-\delta, 0]} \left| \frac{\partial U_0}{\partial x} \right| \leq k \max_{x \in [-\delta, 0]} \left| \frac{\partial U}{\partial x} \right| + \delta \max_{x \in [-\delta, 0]} \left| \frac{\partial^2 U_0}{\partial x^2} \right|. \]

It shows that values of the solution \( U_0 \) changes comparatively little in the direction of argument \( x \) if coefficient \( k_0 \) is significantly larger than \( k \) or if \( \delta \) is small (or both conditions take place). That means that function \( U_0(x, y) \) could be approximately substituted by constant regarding argument \( x \) with accuracy \( O \left( \frac{k}{k_0} \delta + \delta^2 \right) \). Otherwise substitution by constant in the \( x \)-direction could be inappropriate – polynomial or exponential approximation should be used.

\subsection{2.2 Transformed problem with non-classical boundary condition}

We will convert differential equation (1) to non-classical boundary condition for the equation (2). It means that we will have other one problem instead of the problem (1)-(6). To make difference between these two problems clearer, new solution of the equation (2) we denote as \( u(x, y) \) instead of the function \( U(x, y) \).

We rewrite equation (2):
\[ \frac{\partial}{\partial x} \left( k \frac{\partial u}{\partial x} \right) + L(u) = -F(x, y), \quad (x, y) \in G \quad (11) \]

and introduce the integral averaged value function:
\[ u_0(y) = \frac{1}{\delta} \int_{-\delta}^{\delta} U_0(x, y) dx. \quad (12) \]

After integrating equation (1) over the segment \( [-\delta, 0] \) and taking into account conjugation condition (4), we get:
\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} - k_0 \frac{\partial U_0}{\partial x} \bigg|_{x=0} + \delta \frac{\partial u_0}{\partial x} = -\phi_0, \quad (13) \]

where \( f_0(y) = \frac{1}{\delta} \int_{-\delta}^{\delta} F_0(x, y) dx \).

Equation (13) is not a valid boundary condition yet because it has two unknown functions: \( U_0(\delta, y) \) and \( u_0(y) \). We use boundary condition (5) to exclude function \( U_0 \) from the equation (13):
\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} + \delta \frac{\partial u_0}{\partial x} = -\left( \phi^0 + \tilde{\phi}_0 \right). \quad (15) \]

\subsection{2.2.1 Approximation with constant}

To move forward, we need to approximate function \( U_0(x, y) \) in respect to argument \( x \). Accuracy of this depends on particular task. We start with simplest one – approximation with constant. Taking into account expressions (3) and (12), we get:
\[ U_0(x, y) = u_0(y) = u(0, y). \quad (16) \]

New boundary condition is found if we put this equality into equation (15):
\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} + \delta \frac{\partial u_0}{\partial x} = -\left( \phi^0 + \tilde{\phi}_0 \right). \quad (17) \]

So the transformed problem consists of differential equation (11), non-classical boundary condition (17) on the border \( H \) and original boundary conditions (6) on the other borders. New problem is defined in the sub domain \( G \) of the initial domain \( D \).

To be correct we need also to add boundary condition on the border \( \partial H \):
\[ l^0(u) = \psi_0(y), \quad y \in \partial H, \quad (18) \]
where operator \( l^0 \) is defined in domain \( \partial D \cap G_0 \) and is equal to operator \( \tilde{l} \) here; \( \psi_0 \) is analogue of averaged value integral (12) for the function \( \Psi_0 \). \( \Psi_0 \) in its turn is \( G_0 \) part of the function \( \tilde{\Psi} \).

If \( \psi^0 \) is not identical to zero then boundary condition (5) and assumption of independency of \( x \) are true only for \( k_0 = \infty \). That means that approximation of argument \( x \) of function \( U_0 \) by constant is applicable when values of coefficient \( k_0 \) is relevantly large.

### 2.2.2 Linear Approximation

If we apply linear approximation to the function \( U_0(x, y) \) in direction of \( x \) axis

\[ U_0(x, y) = u_1(y) + xu_2(y) \quad (19) \]

and use conjugation condition (3) and boundary condition (5), we get

\[ U_0(x, y) = u(0, y) + \frac{x}{k_0} \psi^0(y). \quad (20) \]

Now we apply integral (12) to the expression (20) and put the result into equation (15) to get boundary condition on the border \( H \):

\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} + \partial L^0(u|_{x=0}) = -\phi_1, \quad (21) \]

where \( \phi_1 = \psi^0 + \frac{\delta^2}{6} L^0 \left( \frac{\psi^0}{k_0} \right). \quad (22) \]

After finding solution \( u(x, y) \) we can use formula (20) to restore function \( U_0(x, y) \) approximately.

### 2.2.3 Approximation with Polynomial of the Second Order

We assume such representation for the function \( U_0 \):

\[ U_0(x, y) = u(0, y) + \frac{\delta}{\delta} u_1(y) + \left( \frac{x}{\delta} \right)^2 u_2(y). \quad (23) \]

Conjugation condition (4) then gives:

\[ k \frac{d^2 u}{d x^2} \bigg|_{x=0} = \frac{k_0}{\delta} u_1(y) \quad (24) \]

From the condition (5) and integral (12), we get:

\[ k \frac{\partial u_1}{\partial x} \bigg|_{x=0} = \psi^0, \quad (25) \]

\[ u_0 = u|_{x=0} - \frac{1}{2} u_1 + \frac{1}{4} u_2. \quad (26) \]

After writing down \( u_1 \) and \( u_2 \) in terms of \( u_0 \), we can transform equation (24) in the following form:

\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} = \frac{3k_0}{\delta} u|_{x=0} - u_0 + \frac{\psi^0}{2}. \quad (27) \]

Now we need only to express \( u_0 \) from this equation and put it into (15) to get boundary condition on the border \( H \):

\[ k \frac{\partial u}{\partial x} + \partial L^0(u - \frac{\partial k}{\partial x} \frac{\partial u}{\partial x}) = -\phi_2 \quad (28) \]

where \( \phi_2 = \phi^0 + \frac{\delta^2}{6} L^0 \left( \frac{\psi^0}{k_0} \right). \quad (29) \]

For the calculating purposes it could be useful to leave boundary condition in the form of system. Then it would contain equations (27) and (15). If we put formula (27) into (15), we could use following form for the second equation of the system as well:

\[ \partial L^0(u_0) + \frac{3k_0}{\delta} u|_{x=0} - u_0 = -\frac{\psi^0}{2} + \frac{\delta^2}{6} u_0. \quad (30) \]

We can also use formulas (27) and (30) together as the system of equation on the boundary \( x = 0 \).

### 2.2.4 Exponential Approximation

We could assume another representation for function \( U_0 \):

\[ U_0(x, y) = u(0, y) + (e^{-qy/\delta} - 1)u_1(y) + (1 - e^{-qy/\delta})u_2(y) \quad (31) \]

where parameter \( q > 0 \) is arbitrary (positive) constant. It could be freely chosen taking into account physical and geometrical properties of particular problem. Simplest way is to take \( q = 1 \).

We get boundary condition on the border \( H \) using similar mathematical modification as in previous case:

\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} + \partial L^0(u|_{x=0} - C_1 \frac{\partial k}{\partial x} \frac{\partial u}{\partial x}|_{x=0}) = -\phi_3 \quad (32) \]

where \( \phi_3 = \psi^0 + \delta \phi^0_0 + C_2 \delta^2 L^0 \left( \frac{\psi^0}{k_0} \right). \quad (33) \]

Constants \( C_1, C_2 \) depend on choice of parameter \( q \):

\[ C_1(q) = \frac{e^{2q} - (q - 1) + q + 1}{q^2 (e^{2q} - 1)}, \quad (34) \]

\[ C_2(q) = \frac{e^{2q} - 2qe^q - 1}{q^2 (e^{2q} - 1)}. \quad (35) \]

Again, we can use system on the boundary \( x = 0 \). First equation of the system is (15) and the second is

\[ k \frac{\partial u}{\partial x} \bigg|_{x=0} = \frac{k_0}{C_1 \delta} u|_{x=0} - u_0 + \frac{C_2}{C_1} \phi^0, \quad (36) \]

or

\[ \partial L^0(u_0) + \frac{k_0}{C_1 \delta} u|_{x=0} - u_0 = -\left( 1 + \frac{C_2}{C_1} \right) \phi^0 + \delta \phi^0_0. \quad (37) \]
3 Conservative Averaging Method in Cylindrical Coordinates

3.1 Initial Problem

Let us take domain \( D = G_0 \cup G \cup H \) with points \((r, y) \in \overline{D} \subset \mathbb{R}^4, r \in \mathbb{R}, y = (\varphi, z, t) \in \mathbb{R}^3\).

\[
G_0 = \{(r, y) \mid r \in (r_0, r_1), y \in D\},
\]

\[
G = \{(r, y) \mid r \in (r_1, r_2), y \in D\},
\]

\[
H = \overline{G_0} \cap \overline{G} = \{(r, y) \mid r = r_0, y \in D\},
\]

\[
H_0 = \{(r, y) \mid r = \tilde{r}_0, y \in G_0\}.
\]

Argument \( r \) represent radius in this case. Similarly as in previous section, we denote domain (surface)

\( H_0 \) as \( r = \tilde{r}_0 \) and domain \( H \) as \( r = r_0 \).

We have

a) differential equations for domains \( G_0 \) and \( G \):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial U_0}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial U}{\partial r} \right) + L(U) = -F(r, y),
\]

where \( L \) is linear differential operator that is not dependent on argument \( r \);

b) conjugation conditions:

\[
U_0 \big|_{r=r_0} = U \big|_{r=r_0},
\]

\[
\frac{k_0}{r} \frac{\partial U_0}{\partial r} \big|_{r=r_0} = k \frac{\partial U}{\partial r} \big|_{r=r_0},
\]

c) boundary condition at \( r = \tilde{r}_0 \):

\[
\frac{\partial U}{\partial r} \big|_{r=r_0} = \varphi_0(y).
\]

Condition in such form is used when domain is a ring or cored cylinder. Boundary condition is homogeneous for \( \tilde{r}_0 = 0 \):

\[
\frac{\partial U}{\partial r} \big|_{r=0} = 0;
\]

d) boundary conditions on other borders:

\[
\tilde{L}(\tilde{U}) = \tilde{\Phi}(r, y), (r, y) \in \partial D = \partial D \setminus H_0
\]

where function \( \tilde{U}(r, y) \) is defined similarly as in previous section.

3.2 Transformed problem

Let us apply conservative averaging method to the subdomain \( G_0 \) that is located closer to the center of the domain \( D \). Solution of transformed problem we denote as \( u(r, y) \). It is defined in the domain \( G \).

We rewrite equation (40) with new notation:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial u}{\partial r} \right) + L(u) = -F(r, y).
\]

Again, we introduce integral averaged value function that is different from equation (12) because of cylindrical coordinates:

\[
u_0(y) = \frac{2}{r_0^2 - \tilde{r}_0^2} \int_{\tilde{r}_0}^{r_0} r U_0(r, y) dr.
\]

For simplicity we will consider only the case \( \tilde{r}_0 = 0 \) in further transformations.

We integrate equation (38) over the segment \([0, r_0]\) and take into account conjugation condition (41):

\[
\frac{2k}{r_0} \frac{\partial u}{\partial r} \big|_{r=r_0} + L^0(u_0) = -f_0(y),
\]

where \( f_0(y) = \frac{2}{r_0^2} \int_{r_0}^{r_0} F_0(r, y) dr \).

We will use equality (47) to obtain non-classical boundary condition for the equation (45) on the border \( r = r_0 \).

3.2.1 Approximation with constant

Let us assume that function \( U_0 \) is constant over argument \( r \). Taking into account conjugation condition (40), we find boundary condition on the border \( r = r_0 \):

\[
\frac{2k}{r_0} \frac{\partial u}{\partial r} \big|_{r=r_0} + L^0(u_0) = -f_0(y).
\]

Approximation with linear function leads to the same equation because of boundary condition (43) at the point \( r = 0 \).

3.2.2 Approximation with polynomial

We assume such representation for the function \( U_0 \):

\[
U_0(r, y) = u_1(y) + ru_2(y) + r^2 u_3(y).
\]

Conjugation conditions (40), (41) and boundary condition (43) allow us to find unknown functions \( u_1, u_2, u_3 \). This allows to rewrite the expression for the temperature \( U_0 \) as follows:

\[
U_0(r, y) = u_0 + \left( 2r^2 - 1 \right) \left( u_{r=r_0} - u_0 \right).
\]

Here \( u_0(y) = u \big|_{r=r_0} - \frac{r_k}{4k_0} \frac{\partial u}{\partial r} \big|_{r=r_0} \).

The equality (47) gives the first boundary equation:

\[
\frac{8k_0}{r_0^2} \left( u \big|_{r=r_0} - u_0 \right) + L^0(u_0) = -f_0.
\]

Derivation of function (51) by argument \( r \) and putting the result into conjugation condition (41)
gives the second equation on the boundary \( r = r_0 \):
\[
\frac{k}{r} \frac{\partial u}{\partial r} \bigg|_{r=r_0} = \frac{4k}{r_0} (u_{r=r_0} - u_0).
\]  
(54)

The new formulation of the problem consists of differential equation (45), boundary conditions (53), (54) on the border \( r = r_0 \) and original boundary conditions on the other borders of the domain \( G \).

After solving this problem we can find approximation for the function \( U_0(r, t) \) from (51).

### 3.2.3 Exponential Approximation

We assume following representation for function \( U_0 \):
\[
U_0(r, y) = u_1(y) + (e^{-r/r_0} - 1)u_2(y) + (1 - e^{-r/r_0})u_3(y)
\]  
(55)

We can obtain unknown functions \( u_1, u_2, u_3 \) and write down function \( U_0 \) after using boundary condition (43) and both conjugation conditions:
\[
U_0(r, y) = u_{r=r_0} + \frac{r_0 k}{\sinh(1)k_0} \frac{\partial u}{\partial r} \bigg|_{r=r_0} \times \frac{\alpha}{\alpha_0} \left( \cosh(r/r_0) - \cosh(1) \right)
\]  
(56)

Integral averaged value function in this case is given by expression
\[
u_0(y) = u_{r=r_0} - \frac{r_0 k}{\alpha_0 k_0} \frac{\partial u}{\partial r} \bigg|_{r=r_0}
\]  
(57)

where constant \( \alpha_0 = \left( e^2 - 1 \right) / \left( e^2 - 4e + 5 \right) \).

This is the first equation on the border \( r = r_0 \). We find the second one by using equality (47):
\[
\frac{2\alpha_0 k_0}{r_0^2} (u_{r=r_0} - u_0) + L^0(u_0) = -f_0.
\]  
(59)

After finding solution we can approximately reconstruct values in the domain \( G_0 \) by using formula (56). Function \( U_0 \) could be expressed also in terms of \( u_0 \) that is more convenient for calculation purposes:
\[
U_0(r, y) = u_{r=r_0} + \beta_0(r) (u_{r=r_0} - u_0)
\]  
(60)

where \( \beta_0(r) = \frac{2e}{e^2 - 4e + 5} \left( \cosh \frac{r}{r_0} - \cosh(1) \right) \).

### 4 Computation of Heat Transfer in Cylindrical Wire with Insulation

Calculation of heat transfer and heat emission in the electrical wires is a vital issue in many industries. Electric circuitry is used in cars, houses, consumer electronics, etc. Damage and accidents are possible if insulation melts because of heightened temperature in the wires. Thermal properties of conductor and insulation significantly differ that makes calculation of critical temperatures more difficult. Besides, wires frequently are combined in bundles. In this section, a single round wire is considered. Since length of the wire is much larger than diameter, 1D model is used as in [3] and [4].

#### 4.1 Statement of the Problem

We have two-layer domain that consists of conductor and insulation. With \( r_0 \) we denote radius of metallic wire, with \( r_1 \) – radius of the wire with insulation. We have

- a) heat conduction equation for conductor \( r \in (0, r_0) \):
\[
c_0 \frac{\partial U_0}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( rk_0 \frac{\partial U_0}{\partial r} \right) + F_0(r, t),
\]  
(62)

- and homogeneous non-linear heat conduction equation for the insulation \( r \in (r_0, r_1) \):
\[
c \frac{\partial U}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( rk \frac{\partial U}{\partial r} \right);
\]  
(63)

- b) conjugation conditions at \( r = r_0 \):
\[
U_0 \bigg|_{r=r_0} = U \bigg|_{r=r_0},
\]  
(64)

\[
k_0 \frac{\partial U_0}{\partial r} \bigg|_{r=r_0} = k \frac{\partial U}{\partial r} \bigg|_{r=r_0},
\]  
(65)

- c) boundary conditions:
\[
r = 0: \quad \frac{\partial U_0}{\partial r} \bigg|_{r=0} = 0;
\]  
(66)

\[
r = r_1: \quad \frac{\partial U}{\partial r} \bigg|_{r=r_1} + h(U \bigg|_{r=r_1} - \theta) = 0
\]  
(67)

- d) initial conditions:
\[
t = 0: \quad U_0 = \theta, \quad U = \theta.
\]  
(68)

Here \( c_0, c, k_0, k, h \) – coefficients which correspond to heat capacity, heat conductivity and heat exchange; \( \theta \) – temperature of environment.

#### 4.2 Transformed problem

We use conservative averaging method to exclude wire from definition domain – we inspect heat transfer only in the insulation. Calculation of temperature in the wire was discussed also in paper [3], where authors used finite volume method for both regions – conductor (wire) and insulation. The new solution transformed by conservative averaging method we denote as \( u(r, t) \). This statement of problem consists of differential equation (63) (replacing \( U \) by \( u \)), boundary condition (67), initial condition (68) and boundary condition on the border \( r = r_0 \) that we obtain from
previous section – formula (49) if we use approximation by constant, formulas (53)(54) for polynomial approximation and formulas (57)(59) – for exponential one.

4.3 Numerical solution
Since it is not possible to obtain analytical solution for the stated problem because of non-linearity, we solve it in numerical way. We constructed standard difference scheme with second order approximation regarding to \( r \) and first order approximation for \( t \).

As an example we take wire of copper with polyvinylchloride (PVC) insulation. Coefficients in this case are following:

a) heat conductivity coefficient \( k_0 \) and \( k \) (W/m K)

\[
k_0 = 401, \quad k = 0.2;
\]

b) specific heat capacity coefficient \( c_0, c \) (W/kg K)

\[
c(T) = 381 + 0.17T,
\]

\[
c(T) = 920 - 1.3T + 0.074T^2;
\]

c) heat generation is achieved by electric current; only direct current and ohm resistance is considered:

\[
f_0(T) = \frac{I^2 \rho_{\text{env}}}{A^2} \left( 1 + \alpha \rho (T - \theta) \right)
\]

where \( I \) – electric current, \( A \) – cross sectional area of the metallic conductor, \( \alpha \rho \) – copper resistance

\[
\alpha = \frac{\alpha_2}{(1 + \alpha_2 (\theta - 20))}, \quad \alpha_2 = 3.9 \cdot 10^{-3},
\]

\[
\rho_{\text{env}} = \rho_2 \alpha_2 / \rho_0, \quad \rho_0 = 1.75 \cdot 10^{-8},
\]

\[
\rho_2 = \text{specific resistance of copper at reference temperature 20°C};
\]

d) heat convection coefficient \( h(T) \) was calculated using given computer subroutine for laminar unforced convection to air of horizontal cylindrical surface. Range of coefficient’s values are 3–20 if temperature range is 0–100°C and temperature of environment \( \theta = 0°C \). All mentioned coefficients are valid in the temperature range of our interest. Temperature is given in centigrade degrees.

\[\theta = 0°C, \quad I = 20\,\text{A}.\]

Graphics of solutions matches even if we use different approximations for function \( U_0 \). Difference starts only by fifth significant digit (see Table).

<table>
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<th>( r = 0 )</th>
<th>( r = 10^{-3} )</th>
<th>( r = 10^{-2} )</th>
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<td>Polynomial approx.</td>
<td>42.6648</td>
<td>42.6647</td>
<td>42.6648</td>
</tr>
<tr>
<td>Exponential approx.</td>
<td>42.6647</td>
<td>42.6648</td>
<td>42.6648</td>
</tr>
</tbody>
</table>

Note that conservative averaging method does not make solution \( U \) linear as it is visually observed in Fig. 2. As an example, we can take outer radius 3 times larger: \( r_1 = 3.6 \cdot 10^{-3} \) (Fig. 2). We also numerically verified that we could choose such coefficients for stated problem that choice of approximation of \( U_0 \) by polynomial or exponential instead of constant is significant. But such parameters are useless from practical point of view.

5 Conclusions
Conservative averaging method can be applied to stationary and non-stationary problems. Constant approximation is sufficient in many practical heat transfer problems for the one material with good heat conductivity.

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References: