

# MMSE Channel Estimation Scheme Based on Two-Dimensional Hadamard Transform for OFDM Systems

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*Abstract:* In this paper, a novel minimum-mean-square-error (MMSE) channel estimation algorithm for OFDM systems is proposed. The algorithm adopts two-dimensional Hadamard transform (TDHT) instead of the conventional Fourier transform, and more noise interference can be filtered with the proposed scheme. Both analytical and simulation results show that the performance of the proposed algorithm is better than that with Fourier Transform. The performance of the proposed scheme is even close to that with perfect channel estimation scheme. Furthermore, the computation of TDHT takes only complex additions, and thus the complexity of which is much lower than the scheme with Fourier transform since complex multiplications are not needed. Consequently, the proposed channel estimation scheme is more practical for applications in OFDM systems than the scheme with Fourier transform.

*Key-Words:* OFDM, MMSE, Channel Estimation, Two-Dimensional Hadamard Transform

## 1 Introduction

In recent years, significant research interest has been given to orthogonal frequency division multiplexing (OFDM) due to its advantages in high-bit-rate mobile systems over multi-path fading channels. The OFDM scheme has been exploited for asymmetric digital subscriber lines (ADSL), digital audio broadcasting (DAB), digital video broadcasting (DVB), HDTV terrestrial broadcasting, and the IEEE 802.11 standard [1][2].

In an OFDM system, channel estimation is necessary since wireless channel is dynamic and frequency selective for wide band mobile systems [3][4][5]. Estimation of a channel may be based on least square (LS) algorithm or Minimum Mean Square Error (MMSE) algorithm. In general, the MMSE schemes are generally more accurate [6]. Numerous MMSE estimation algorithms based on discrete Fourier transform (DFT) have been proposed to reduce the effects of noise [7][8]. An MMSE estimation based on Hadamard Transform has also been studied recently [9].

In this paper, a two-dimensional Hadamard transform (TDHT) incorporated with MMSE weighting is proposed. With TDHT, signal energy is concentrated on the lower end of the transform domain, while the noise energy on transform domain remains uniformly distributed. Therefore after an MMSE filtering, most

of the noise interference can be eliminated. Both analytical and simulation results show that the performance of the proposed algorithm is better than that with Fourier transform. The performance of the proposed scheme is even close to the performance of perfect channel estimation scheme. It is also worth noticing that since only addition operations are needed for the implementation of Hadamard Transform and Inverse Hadamard Transform, the computation required is extremely small.

The rest of this paper is organized as follows. In Section 2, the system model and the traditional channel estimation algorithm are introduced. The MMSE channel estimation based on Hadamard Transform is proposed in Section 3. Next, the Performance and computational complexity of the proposed algorithm are analyzed in Section 4, and then simulation results are given in Section 5. Finally, conclusions are drawn in Section 6.

## 2 System Model and MMSE Channel Estimation with Fourier Transform

### 2.1 System Model

Fig. 1 presents the system model used in this study. Existing approaches based on Fourier Transform also

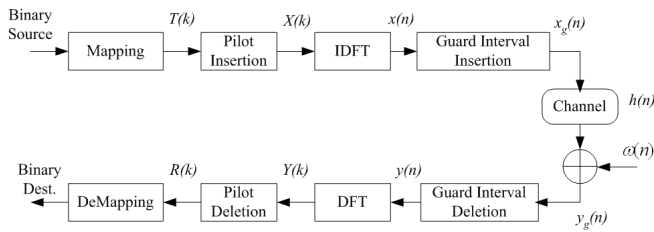


Figure 1: Baseband model of a pilot-aided OFDM system

use the same system. The binary source data are grouped and mapped into complex symbols  $\{T(k)\}$ . After pilot insertion, the modulated data  $\{X(k)\}$  in frequency domain is converted into time domain with normalized Inverse Discrete Fourier Transform (IDFT):

$$\begin{aligned} x(n) &= IDFT\{X(k)\} \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi kn}{N}}, \quad n = 0, 1, \dots, N-1 \end{aligned} \quad (1)$$

where  $N$  denotes the number of sub-carriers. Guard interval is introduced to prevent ISI, and the duration of the interval,  $N_g$ , is assumed to be shorter than the delay profile of the channel. The resultant signal is given by

$$x_g(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, \dots, -1 \\ x(n), & n = 0, 1, \dots, N-1 \end{cases} \quad (2)$$

Assuming that the channel is subject to frequency-selective slow fading, the received signal becomes

$$y_g(n) = x_g(n) \otimes h(n) + \omega(n) \quad (3)$$

where  $h(n)$  is the impulse response of the fading channel,  $\omega(n)$  is additive white Gaussian noise (AWGN) with zero mean and variance of  $\sigma_T^2$ , and  $\otimes$  denotes  $N$ -points circular convolution. Then the channel response can be expressed as

$$h(n) = \sum_{k=0}^{M-1} \alpha_k \delta(n - \tau_k) \quad (4)$$

where  $\alpha_k$  is complex gain of the  $k$ -th path,  $\tau_k$  is the delay of the  $k$ -th path, and  $M$  is the number of channel paths.

Assuming perfect synchronization, after normalized Discrete Fourier Transform,

$$\begin{aligned} Y(k) &= DFT\{y(n)\} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) e^{-j \frac{2\pi kn}{N}}, \quad k = 0, 1, \dots, N-1 \end{aligned} \quad (5)$$

the received signal in frequency-domain may be expressed as

$$Y(k) = X(k) \cdot H(k) + W(k) \quad (6)$$

where  $H(k) = DFT\{h(n)\}$  is a channel transfer function in frequency-domain, and  $W(k) = DFT\{\omega(n)\}$  are AWGN noise samples in frequency-domain, with zero mean and variance of  $\sigma_F^2 = \sigma_T^2 [10]$ .

## 2.2 MMSE Channel Estimation with Fourier Transform

To obtain the transmitted signal  $X(k)$  at the receiver, it is important to have a good estimate of channel matrix  $H(k)$ . Assume transmitted pilot symbols to be  $P_T(k)$ , and received pilot symbols to be  $P_R(k)$ . Then, with an LS estimation scheme at pilot sub-carriers the channel matrix may be estimated as

$$H_{LS} = H(k) + W(k)/P_T(k) = P_R(k)/P_T(k) \quad (7)$$

Therefore, estimation at other sub-carriers may be achieved by interpolation. It can be proved that best performance is obtained when the pilots are uniformly distributed among sub-carriers [11]. However, the channel noise reduces the precision of channel estimation, and subsequently deteriorates the performance of the system [3][5]. To deal with noise interference, the channel impulse response may be obtained from the channel transfer function with DFT, and filtered in the Fourier Transform domain. That is,

$$H_{MF} = F_R Q_{MF} F H_{LS} \quad (8)$$

where  $F$  is the DFT matrix,  $F_R$  is the IDFT matrix, and  $Q_{MF}$  is diagonal weighting matrix with MMSE channel estimation [7][8].

It is noted that, the computation for a Fourier Transform is relatively large since complex multiplications are required.

## 3 MMSE Channel Estimation with two-dimensional Hadamard Transform

Consider a system with  $N(N = 2^n)$  sub-carriers, in which  $M(M = 2^m)$  pilots are uniformly distributed. Assuming we deploy enough pilots, the LS estimation of channel transfer function,  $H_{LS}$ , can be considered as an over-sampling of the channel.  $L(L = 2^l)$  LS pilot channel estimation vectors in adjacent OFDM symbols can be assembled into an  $M \times L$  matrix in frequency and time domain for two-dimensional

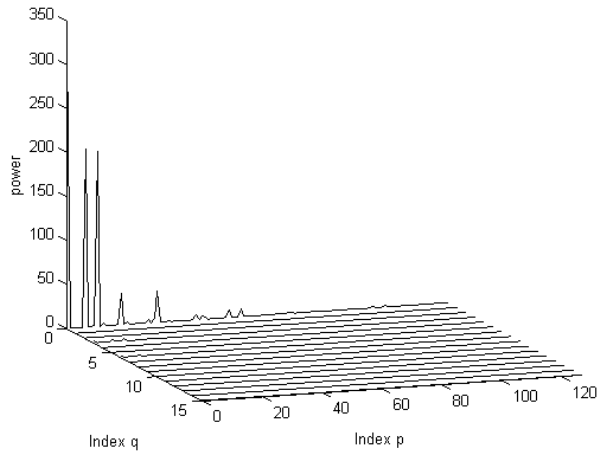


Figure 2: Energy distribution after two-dimensional Hadamard Transform

Hadamard Transform. After the first dimensional Hadamard Transform in frequency domain, energy of the channel tends to concentrate on the low end of the transform domain. Since HT is a linear transform, the energy of white Gaussian noise after HT remains uniformly distributed [9]. Since the property of channel remains similar among adjacent symbols, the energy of a channel will become concentrated further after the two-dimensional Hadamard Transform, while noise distribution remains unchanged. Fig. 2 shows an example that manifests this fact, in which the power of the channel focuses into a vector along  $q = 0$ , and therefore the noise can be thereby reduced after MMSE weighting. In order to represent the two-dimensional Hadamard Transform in a matrix form, the channel transfer function matrix in time and frequency domains need to be converted to a long vector. To this end,  $L(L = 2^l)$  LS adjacent estimation vectors of channel are combined to an  $(L \times M) \times 1$  vector

$$\tilde{H}_{LS} = [H_{LS,0}^T, H_{LS,1}^T, \dots, H_{LS,L-1}^T]^T \quad (9)$$

The first dimensional Hadamard Transform of the sub-vectors in  $\tilde{H}_{LS}$  has now become:

$$\tilde{H}_{FH} = T_M \cdot \tilde{H}_{LS} \quad (10)$$

where

$$T_M = \begin{bmatrix} W_1 & & & \\ & W_1 & & \\ & & \ddots & \\ & & & W_1 \end{bmatrix} \quad (11)$$

is an  $(L \times M) \times (L \times M)$  block diagonal matrix.  $W_1$  is the  $M \times M$  real symmetric core matrix of

one-dimensional Hadamard Transform, expressed as

$$W_1 = \frac{1}{\sqrt{M}} \begin{bmatrix} w_{0,0} & \cdots & w_{0,M-1} \\ \vdots & & \vdots \\ w_{M-1,0} & \cdots & w_{M-1,M-1} \end{bmatrix} \quad (12)$$

in which  $w_{j,k} = (-1)^{\sum_{i=0}^{m-1} b_i(j)b_i(k)}$ , is the  $(j, k)$ -th element of matrix  $W_1$ , where  $b_i(j)$  is the  $(i+1)$ -th bit of integer  $j$  in binary form.

Then,  $\tilde{H}_{FH}$  is interleaved for the second-dimensional Hadamard Transform, as

$$\tilde{H}_{IH} = K \cdot \tilde{H}_{FH} \quad (13)$$

where  $K$  is a sparse matrix, with the element

$$k_{i,j} = \begin{cases} 1, & \text{if } i = L * q + p \text{ and } j = M * p + q, \\ & \forall 0 \leq p \leq L - 1, 0 \leq q \leq M - 1 \\ 0, & \text{others} \end{cases} \quad (14)$$

Consequently, the second-dimensional Hadamard Transform has now become

$$\tilde{H}_{SH} = T_L \cdot \tilde{H}_{IH} = T_L \cdot K \cdot T_M \cdot \tilde{H}_{LS} \quad (15)$$

where

$$T_L = \begin{bmatrix} W_2 & & & \\ & W_2 & & \\ & & \ddots & \\ & & & W_2 \end{bmatrix} \quad (16)$$

is also an  $(L \times M) \times (L \times M)$  block diagonal matrix, in which  $W_2$  is the  $L \times L$  core matrix of one-dimensional Hadamard Transform, expressed as

$$W_2 = \frac{1}{\sqrt{L}} \begin{bmatrix} \hat{w}_{0,0} & \cdots & \hat{w}_{0,L-1} \\ \vdots & & \vdots \\ \hat{w}_{L-1,0} & \cdots & \hat{w}_{L-1,L-1} \end{bmatrix} \quad (17)$$

in which  $\hat{w}_{j,k} = (-1)^{\sum_{i=0}^{l-1} b_i(j)b_i(k)}$ .

After the MMSE weighting in the two-dimensional Hadamard Transform domain, and the two-dimensional Inverse Hadamard Transform, the channel transfer function in frequency domain becomes

$$\tilde{H}_M = T_{M,R} \cdot K^T \cdot T_{L,R} \cdot Q \cdot T_L \cdot K \cdot T_M \cdot \tilde{H}_{LS} \quad (18)$$

where  $T_{L,R} = T_L$  is a block diagonal matrix for the second-dimensional Inverse Hadamard Transform,  $T_{M,R} = T_M$  is a block diagonal matrix for the first-dimensional Inverse Hadamard Transform,

$Q = \text{diag}(q)$  is an  $(L \times M) \times (L \times M)$  diagonal weighting matrix for MMSE weighting, in which  $q = [q_0, q_1, \dots, q_{L \times M - 1}]^T$ .

Let  $A = T_{M,R} \cdot K^T \cdot T_{L,R}$ ,  $B = T_L \cdot K \cdot T_M \cdot \tilde{H}_{LS}$ , then Eqn. (18) becomes

$$\tilde{H}_M = A \cdot Q \cdot B = A \cdot \text{diag}(B) \cdot q \quad (19)$$

With this representation, MSE of channel estimation becomes

$$\begin{aligned} \rho(q) &= \frac{1}{M \cdot L} E\{[\tilde{H} - \tilde{H}_M]^H \cdot [\tilde{H} - \tilde{H}_M]\} \\ &= \frac{1}{M \cdot L} E\{[\tilde{H} - A \cdot \text{diag}(B) \cdot q]^H \cdot [\tilde{H} - A \cdot \text{diag}(B) \cdot q]\} \\ &= \frac{1}{M \cdot L} \{E[\tilde{H}^H \cdot \tilde{H}] - E[q^H \cdot \text{diag}^H(B) \cdot A^H \cdot \tilde{H}] \\ &\quad - E[\tilde{H}^H \cdot A \cdot \text{diag}(B) \cdot q] \\ &\quad + E[q^H \cdot \text{diag}^H(B) \cdot A^H \cdot A \cdot \text{diag}(B) \cdot q]\} \end{aligned} \quad (20)$$

where  $\tilde{H} = [H_0^T, H_1^T, \dots, H_{L-1}^T]^T$  is  $(L \times M) \times 1$  correct channel estimation.

The second-order derivation of  $\rho(q)$  is

$$\nabla_q^2 \rho(q) = E[\text{diag}^H(B) \cdot A^H \cdot A \cdot \text{diag}(B)] \quad (21)$$

Since

$$A^H \cdot A = T_{L,R}^H \cdot (K^T)^H \cdot T_{M,R}^H \cdot T_{M,R} \cdot K^T \cdot T_{L,R} = I_{L \times M} \quad (22)$$

is an  $(L \times M) \times (L \times M)$  unit matrix, the second order derivative

$$\nabla_q^2 \rho(q) = E[\text{diag}^H(B) \cdot \text{diag}(B)] \quad (23)$$

is positive definite. Therefore,  $q$  has a unique solution which minimizes  $\rho(q)$ .

If we compute the complex gradient of  $\rho(q)$  and force it to be zero, the MMSE weighting vector  $\tilde{q}$  can be obtained as

$$\tilde{q} = E[\text{diag}^H(B) \cdot A^H \cdot A \cdot \text{diag}(B)]^{-1} \cdot E[\text{diag}^H(B) \cdot A^H \cdot \tilde{H}] \quad (24)$$

Since  $T_L = T_{L,R} = T_{L,R}^H$ ,  $T_M = T_{M,R} = T_{M,R}^H$ ,  $K = (K^T)^H$ , and the channel transfer function and noise are uncorrelated, the weighting vector becomes

$$\tilde{q}_i = (E[|b_i|^2] - \delta_N^2) / E[|b_i|^2] \quad (25)$$

where  $\tilde{q}_i, b_i$  are the  $i$ -th items of vectors  $\tilde{q}, B$ , respectively, and  $\delta_N^2$  is the variance of noise.

In the descriptions above,  $L \times M \times 1$  vectors are combined to form an  $(L \times M) \times 1$  vector, and a sparse matrix is used between the first and the second dimensional Hadamard Transform for the convenience of analysis. In the implementation of the proposed algorithm, the first-dimensional Hadamard Transform can be performed in  $M \times 1$  vector form, as

$$H_{FH,l} = W_1 \cdot H_{LS,l}, \quad l = 0, 1, \dots, L-1 \quad (26)$$

After combining  $L$  vectors into an  $M \times L$  matrix  $\hat{H}_{FH} = [H_{FH,0}, H_{FH,1}, \dots, H_{FH,L-1}]$  and perform a transpose operation,

$$\hat{H}_{IH} = \hat{H}_{FH}^T \quad (27)$$

$M \times L \times 1$  vectors  $\hat{H}_{IH} = [H_{IH,0}, H_{IH,1}, \dots, H_{IH,M-1}]$  can be used for the second-dimensional Hadamard Transform:

$$H_{SH,m} = W_2 \cdot H_{IH,m}, \quad m = 0, 1, \dots, M-1 \quad (28)$$

Moreover, instead of using an  $(L \times M) \times (L \times M)$  diagonal matrix  $Q = \text{diag}(q_0, q_1, \dots, q_{L \times M - 1})$ , an  $L \times M$  matrix

$$\hat{Q} = \begin{bmatrix} q_0 & \cdots & q_{L \times (M-1)} \\ \vdots & \ddots & \vdots \\ q_{L-1} & \cdots & q_{L \times M - 1} \end{bmatrix} \quad (29)$$

is used for MMSE weighting, with element-by-element multiplication between  $\hat{Q}$  and  $\hat{H}_{SH} = [H_{SH,0}, H_{SH,1}, \dots, H_{SH,M-1}]$ . It is noted that the Inverse Hadamard Transform can also be implemented in a similar fashion as the Hadamard Transform.

## 4 Performance Evaluation and Complexity Analysis

### 4.1 MSE of Channel Estimation

When adopting a two-dimensional Hadamard Transform in MMSE channel estimation, MSE of channel estimation becomes

$$\begin{aligned} \rho(\tilde{q}) &= \frac{1}{M \cdot L} E\{(\tilde{H} - \tilde{H}_M)^H (\tilde{H} - \tilde{H}_M)\} \\ &= \frac{1}{M \cdot L} \text{tr}\{E\{(\tilde{H} - \tilde{H}_M)(\tilde{H} - \tilde{H}_M)^H\}\} \\ &= \frac{1}{M \cdot L} \{ \text{tr}(R_{\tilde{H}\tilde{H}}) - \text{tr}(R_{\tilde{H}\tilde{H}_M}) \\ &\quad - \text{tr}(R_{\tilde{H}_M\tilde{H}}) + \text{tr}(R_{\tilde{H}_M\tilde{H}_M}) \} \\ &= \frac{1}{M \cdot L} \{ \text{tr}(SR_{\tilde{H}_{LS}\tilde{H}_{LS}}^H S^H) - \text{tr}(SR_{\tilde{H}\tilde{H}_{LS}}^H) \\ &\quad - \text{tr}(R_{\tilde{H}\tilde{H}_{LS}} S^H) + \text{tr}(R_{\tilde{H}\tilde{H}}) \} \end{aligned} \quad (30)$$

where  $\text{tr}(a)$  denotes the rank of matrix  $a$ ,  $R_{AB}$  denotes the correlation matrix between  $A$  and  $B$ , and  $S = T_{M,R} \cdot K^T \cdot T_{L,R} \cdot Q \cdot T_L \cdot K \cdot T_M$ .

Since  $\tilde{H}_{LS} = \tilde{H} + \tilde{N}$ , where  $\tilde{N}$  is additive Gaussian noise,

$$\begin{aligned} R_{\tilde{H}_{LS}\tilde{H}_{LS}} &= E\{(\tilde{H} + \tilde{N})(\tilde{H} + \tilde{N})^H\} \\ &= E\{\tilde{H}\tilde{H}^H\} + E\{\tilde{N}\tilde{N}^H\} \\ &= E\{\tilde{H}\tilde{H}^H\} + \frac{1}{SNR} U_M \\ &= R_{\tilde{H}\tilde{H}} + \frac{1}{SNR} U_M \end{aligned} \quad (31)$$

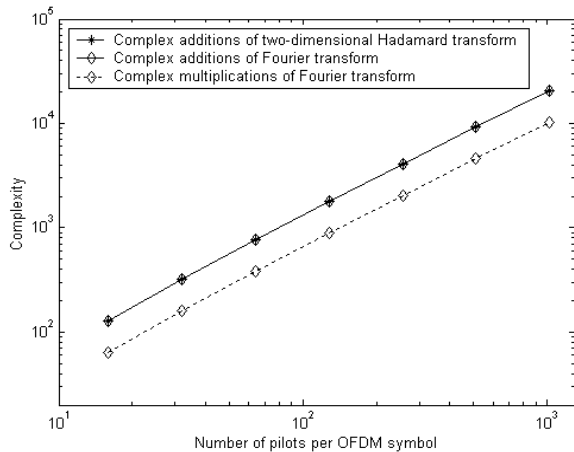


Figure 3: Complexity comparison

Consequently,  $\rho(\tilde{q})$  can be rewritten as

$$\rho(\tilde{q}) = \rho_n(\tilde{q}) + \rho_s(\tilde{q}) \quad (32)$$

where

$$\rho_n(\tilde{q}) = \frac{1}{M \cdot L} \text{tr} \left( \frac{1}{SNR} S S^H \right) \quad (33)$$

comes from the channel noise, while

$$\rho_s(\tilde{q}) = \frac{1}{M \cdot L} \{ \text{tr}(S R_{\tilde{H}\tilde{H}} S^H) - \text{tr}(S R_{\tilde{H}\tilde{H}}^H) - \text{tr}(R_{\tilde{H}\tilde{H}} S^H) + \text{tr}(R_{\tilde{H}\tilde{H}}) \} \quad (34)$$

comes from the inter-carrier interference (ICI) caused by time-varying fading.

## 4.2 Complexity Analysis

In general the computations for MMSE channel estimation consist of LS estimations, some type of Transform pair, and MMSE filtering. For LS estimation and filtering, the computational complexity is the same for channel estimation with Hadamard transform and with Fourier transform. Thus, the difference comes from individual Hadamard transform or Fourier transform itself. In this paper, the number of complex multiplications and the number of complex additions are used to measure computational complexity. For MMSE channel estimation with two-dimensional Hadamard Transform,  $2M \log_2 M$  complex additions are needed for one OFDM symbol. Since an  $M$ -point FFT requires  $(M/2) \log_2 M$  complex multiplications and  $M \log_2 M$  complex additions with the split-radix FFT algorithm when  $M$  is a power of 2, the MMSE channel estimation with Fourier Transform needs  $M \log_2 M$  complex multiplications

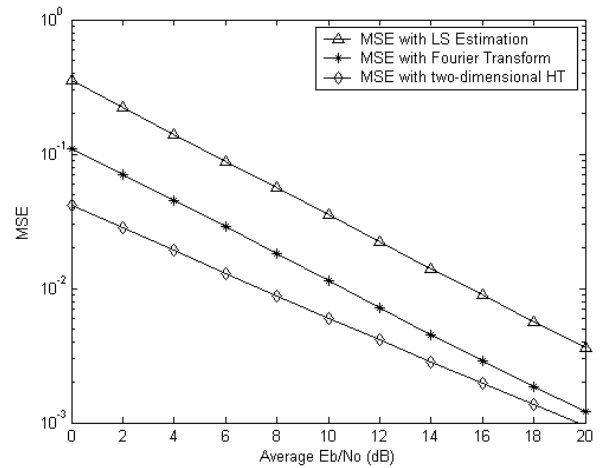


Figure 4: MSE of the two channel estimation algorithm

and  $2M \log_2 M$  complex additions. As shown in Fig. 3, the number of additions with a two-dimensional Hadamard Transform is the same as the number of additions with an FFT. However, the complex multiplications that needed in an FFT can be eliminated for Hadamard Transform. Therefore, the complexity of the proposed algorithm is significantly reduced comparing to FFT-based scheme.

## 5 Simulation Results

A QPSK-OFDM and a 16QAM-OFDM system are used in the simulation with carrier frequency of 1.8GHz and bandwidth of 5MHz. The vehicle speed is 10m/s, resulting in the maximum Doppler frequency of 60Hz. The total number of sub-carriers is 512, the number of uniformly distributed pilot sub-carriers is 64, and 16 adjacent symbols are used in channel estimation based on Hadamard Transform. Cubic interpolation is used to get the channel estimation of all sub-carriers in frequency domain from channel estimation of pilot sub-carriers. The channel model used in this research is the Rayleigh channel recommended by European Telecommunication Standards Institute (ETSI) for European 3G standard. The channel parameters are shown in Table 1[12].

It can be observed from Fig. 4 that with two dimensional Hadamard Transform, the MSE of MMSE channel estimation is smaller than that with Fourier Transform. Moreover, It is illustrated in Fig. 5 that with QPSK signal mapping in simulation, the BER with the proposed algorithm is smaller than that with the MMSE estimation based on Fourier Transform. The performance of the proposed channel estimation

Table 1: Channel parameters for simulation

Tap	Relative delay(ns)	Average power(dB)
1	0	0.0
2	310	-1.0
3	710	-9.0
4	1090	-10.0
5	1730	-15.0
6	2510	-20.0

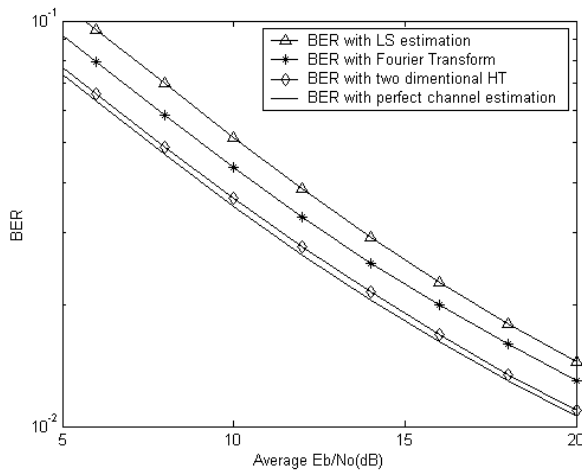


Figure 5: BER Performance with QPSK signal mapping

scheme is even close to that with perfect channel estimation.

## 6 Conclusions

The performance of channel estimation is extremely important for OFDM systems. In this paper, a novel MMSE channel estimation algorithm based on two-dimensional Hadamard Transform is developed. The performance of the proposed scheme is better than that with Fourier-transform-based MMSE estimation scheme, and the computation complexity is also significantly reduced. We expect that the proposed algorithm based on Hadamard transform will become more practical channel estimation scheme for OFDM systems.

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