A New Method for Electric Utility Resource Planning

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Abstract: This paper presents an interval-based multi-attribute decision making (MADM) approach in support of the decision process with imprecise information. The proposed decision methodology is based on the model of linear additive utility function but extends the problem formulation with the measure of composite utility variance. A sample study concerning with the evaluation of electric generation expansion strategies is provided showing how the imprecise data may affect the choice toward the best solution and how a set of alternatives, acceptable to the decision maker (DM), may be identified with certain confidence.

Key Words- Decision Making, Power Generation, Electric Utilities, Resource Planning

1 INTRODUCTION

MADM methods have been widely used in strategic planning of electric utilities which provides an efficient decision analysis framework to help the DM of electric utilities in selecting the best resource strategy with regard to the chosen attributes. A useful MADM model should be able to display tradeoffs among different attributes, quantitative and qualitative, economic as well as non-economic, and quantify the preferences held by different interests. In many MADM problems, however, the information available to the DM is often imprecise due to inaccurate estimates of attribute values and inconsistent human judgments on attribute priorities. As such, the preference with regard to the ranking of different resource strategies determined using traditional MADM methods, which is based solely on the point value estimate, may not be adequate to distinguish between the outcomes from competing alternatives.

This paper presents an interval-based MADM approach in support of the decision making process with imprecise Information. Section 2 introduces a structured procedure for the construction of linear additive utility model, which is the best known and most used MADM model in electric utility planning studies, to facilitate the process of eliciting preference functions and weighting parameters. In Section 3, a sample case study is provided showing how the proposed decision methodology can be used in electric utility generation expansion analysis so as to increase the level of confidence for the selection of best resource strategy by examining a range of acceptable alternatives. Section 4 discusses how the composite utility variance can be estimated properly by the technique of propagation of errors, accounting for individual errors from inaccurate attribute measurements and inconsistent priority judgments. Thus, given a desired confidence interval, the likely range of composite utility values can be determined, leading to the interval-based MADM approach. Concluding remarks are given in Section 5.

2 CONSTRUCTIONS OF LINEAR ADDITIVE UTILITY MODELS

2.1 Linear Additive Utility Function

One popular approach in dealing with MADM problems is defining an appropriate formulation that transforms an n-dimensional vector performance to a scalar performance measurement, usually termed as multi-attribute utility function (MUF). In general, the MUF model is comprised of the single utility functions or preference functions associated with the chosen attributes and the weighting parameters that reflect the relative importance of these attributes toward the overall planning goal or objective. Conceptually, the composite utility value is a nonlinear function of single utility functions and weighting parameters. However, a special form of

MUF model, known as linear additive form, can be used if the condition of additive utility independence of attributes holds, thus greatly simplifying the procedure of model establishment. Less formally, this means that the contributions of an individual attribute to the composite utility is independent of other attribute values. Equation (1) below gives a general expression of linear additive utility model.

$$U(x) = \sum_{i=1}^{n} w_i U_I(x_i) \tag{1}$$

where $U(x)$ is the composite utility of each alternative characterized by the vector of attributes x =[$x1$,..., xn], $U_i(x_i)$ is the single utility function with respect to the ith attribute, w_i is an appropriate weighting parameter for the ith attribute, representing its relative importance in comparison to other attributes and satisfying $\sum w_i = 1$. Linear additive utility models have been used for a variety of decision problems in electric utility planning, including generation resource acquisition assessment, energy-conservation program evaluation, selecting new generation technologies, integrated resource planning, and transaction selection in a competitive market [1, 2, 3, 4, 5]. Since the attributes considered in these studies, such as project investment, energy production cost, system reliability, environmental impact and the flexibility in resource development, cover quite different fields of interest, the condition of additive utility independence is generally satisfied. There are two terms that are of concern in the construction of linear additive utility models: the single utility functions and the weighting parameters. These are usually determined through interviews with utility planners, performed by the DM, using techniques of decision analysis. In the following, a structured procedure is presented to facilitate the assessment of the single utility functions associated with individual attributes and the tradeoff among the conflicting attributes.

2.2 Assessment of Single Utility Functions

The single utility function, $U_i (x_i)$, represents the utility values which the DM attaches to each attribute and reflects his/her attitude toward taking a risk. To obtain a comparable basis, the utility value is often defined on a normalized scale as the attribute varies between its lower and upper bounds. The single utility function is usually evaluated by the certainty equivalence method as described in [6]. However, it has been realized that the convergence procedure in assessing a certainty equivalent is time-consuming and cumbersome [7]. Sometimes, it may

be hard for the DM to determine a single value that would represent confidently his/her attitude. Instead, it would be more convenient for the DM to specify a boundary or several candidates around the true certainty equivalent. The DM's preferences may also be measured by a ratio-scale method [8]. But this method seems to work well only when there are a small number of alternatives. We attempt to improve the assessment procedures for single utility functions by incorporating the pair-wise comparison analysis into the certainty equivalent method. The revised procedure for the assessment of single utility functions will now be described as follows.

First, we need to identify the range of attribute values. For electric utility resource planning, these are usually obtained from detailed studies including production costing simulation, investment optimization, reliability evaluation and environmental impact analysis for all alternatives. *Next*, we assess three certainty equivalent values for xi with respect to $U_i(x.5)$, $U_i(x.75)$, and $U_i(x.25)$, respectively. To avoid the tedious convergence procedure, the DM may select a few candidate values, which are thought to be around the true certainty equivalent. By comparing each pair of these candidates for their closeness to the expected certainty equivalent, the judgment matrix can be formed from which the priority vector can be obtained by solving the corresponding eigenvalue problem. The certainty equivalent is then calculated as the weighted average of these candidates

$$\bar{c} = c^T \cdot p \tag{2}$$

Where, $c = [c_{1, c2,} cm]$ is the vector of candidates and $p = [p_{1, p2,} pm]$ is the corresponding priority vector. These three (x_i , $U_i (x_i)$) pairs, together with the end point utility values of 1 and 0, gives us five points on the single utility function for the ith attribute. *Finally*, we can fit a curve through these points to determine the corresponding equation for $U_i (x_i)$.

The above assessment procedure may be better than the traditional certainty equivalent method since the value of certainty equivalent is determined by examining several candidates on a compromise basis and therefore would increase the level of confidence in the resulting preference functions. This revised certainty equivalent method may be more reliable than the ratio-scale method because the DM's preference is evaluated over the entire range of attribute values.

In many MADM applications, the single utility function $U_i (x_i)$ in (1) may be replaced by the normalized attribute value r_i , reflecting a risk-

neutral attitude of the DM. Such a special form of linear additive utility model can be expressed by

$$U_d(x) = \sum_{i=1}^{n} w_i \left| \frac{x_i - x_i^*}{x_i^r} \right| = \sum_{i=1}^{n} w_i r_i \qquad (3)$$

Where x_i and r_i are the measured and normalized values of ith attribute, x_i^r is the range of variation of measured attribute values with x_i^* as the optimal (maximal for benefit attributes and minimal for cost attributes).

Unlike Equation (1), where the best solution is the alternative for which the measured composite utility value is maximum, the most favored alternative determined by Equation (3) represents a minimum distance from the ideal point on the direction preferred by the DM. Thus, without any confusion, the term "Composite Utility" or U(x) can be replaced by the term "Composite Distance" or $U_d(x)$ whenever appropriate.

2.3 Assessment of Attribute Priorities

A number of weighting-selection methods are available for MADM analysis, among them ratio questioning and indifference tradeoff methods is most frequently used because they represent a good combination of reliability and easy-to-use . Both methods need the input from the DM to prioritize attributes, but the ratio method directly asks for the relative importance between each pair of attributes while the indifference method indirectly infers the weighting information from tradeoff judgments.

The AHP based ratio-questioning method is applied in this paper for the assessment attribute priorities. First of all, it is a system approach taking care of various concerns for the preference of conflicting attributes. Secondly, it can compensate for the inconsistent human judgments by asking redundant questions and then retrieving the weighting parameters on a compromise basis using eigenvector prioritization method. Additionally, it can also incorporate the influence of the range of attribute values on the preference, a major feature of indifference tradeoff method, into the assessment process with properly revised ratio questions.

3 COMPOSITE UTILITY VARIANCE

3.1 Variance of Production Cost

Due to the uncertainties in the availability of generating units (i.e., unexpected outages) and the load variations over a certain time interval, the production cost of an electric power system is not a single deterministic value, but a random variable. Its mean is the expected production cost and its probability distribution depends on the load patterns and the stochastic characteristics of the forced outages of the generating units in the system. The outputs of conventional production costing models include the expected energy production costs, system reliability indices and generation-related emissions.

In recent publications, the variance of production costs has been discussed extensively recognizing that the measure of production cost variance would be very useful input to the decision making process in comparing different generation expansion alternatives. Some efficient methods have been introduced for estimating this variance. In this paper, we are not going to verify or compare these methods but rather to incorporate the concept of production cost variance, as the error parameter, into the corresponding equations to estimate the composite utility variance or the composite distance variance.

3.2 Variance of Priority Assessment

The AHP technique has been proved to be a reliable tool for the assessment of attribute priorities in MADM analysis. Statistical studies have been carried out to model the subjective errors in the creation of judgment matrix . It has shown that the error associated with each judgment ratio can be approximated by a log-normally distributed error factor $\varepsilon \approx \left(0, \sigma^2\right)$. Study results also shown that a good estimate of the error parameter σ^2 can be calculated by

$$\sigma^2 = \frac{2}{(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} y_{ij}^2 \qquad (4)$$

Where n is the size of the judgment matrix, $y_{ij} = \ln(\frac{a_{ij}}{w_{ij}})$, aij represents the judgment ratio [i,j] of judgment matrix, wij = wi/wj and w = [w1 , w2, ... wn] is the priority vector. It can be noted that the error parameter σ^2 is calculated directly from the judgment matrix, since it involves only the judgment ratio aij and the priority ratio wij. The variances of the priorities associated with each judgment matrix at any layer of the hierarchy can then be estimated as

$$\sigma_{wi}^2 = \frac{n^2 - 1}{n} \left[\sum_{i=-1}^{n} w_i^2 - w_i^2 \right] \sigma^2 w_i^2 \qquad (5)$$

For a three-layer fully connected hierarchy, the variances of composite priorities from the bottom layer to the top layer, i.e., the attribute weighting parameters, can then be calculated as

$$
\begin{bmatrix} (\sigma_{1,3}^{1,1})^2 \\ (\sigma_{2,3}^{1,1})^2 \\ (\sigma_{3,3}^{1,1})^2 \\ (\sigma_{4,3}^{1,1})^2 \end{bmatrix} = \begin{bmatrix} (p_{1,3}^{1,2})^2 (p_{1,3}^{2,2})^2 (p_{1,3}^{3,2})^2 (p_{1,3}^{4,2})^2 \\ (p_{2,3}^{1,2})^2 (p_{2,3}^{2,2})^2 (p_{2,3}^{3,2})^2 (p_{2,3}^{4,2})^2 \\ (p_{3,3}^{1,2})^2 (p_{3,3}^{2,2})^2 (p_{3,3}^{3,2})^2 (p_{3,3}^{4,2})^2 \\ (p_{4,3}^{1,2})^2 (p_{4,3}^{2,2})^2 (p_{4,3}^{3,2})^2 (p_{4,3}^{4,2})^2 \end{bmatrix} \begin{bmatrix} (\sigma_{1,2}^{1,1})^2 \\ (\sigma_{2,2}^{1,1})^2 \\ (\sigma_{3,2}^{1,1})^2 \\ (\sigma_{4,2}^{1,1})^2 \end{bmatrix}
$$

$$
+ \begin{bmatrix} (\sigma_{1,3}^{1,2})^2 (\sigma_{1,3}^{2,2})^2 (\sigma_{1,3}^{3,2})^2 (\sigma_{1,3}^{4,2})^2 \\ (\sigma_{2,3}^{1,2})^2 (\sigma_{2,3}^{2,2})^2 (\sigma_{2,3}^{3,2})^2 (\sigma_{2,3}^{4,2})^2 \\ (\sigma_{3,3}^{1,2})^2 (\sigma_{3,3}^{2,2})^2 (\sigma_{3,3}^{3,2})^2 (\sigma_{3,3}^{4,2})^2 \\ (\sigma_{4,3}^{1,2})^2 (\sigma_{4,3}^{2,2})^2 (\sigma_{4,3}^{3,2})^2 (\sigma_{4,3}^{4,2})^2 \end{bmatrix}
$$

(6)

where $p_{i,j}^{k,l}$ and $\sigma_{i,j}^{k,l}$, are the priority and standard deviation of ith factor at layer j with respect to kth factor at layer l, respectively.

Table1- System Simulation Results under Different Expansion Strategies

Expansion Strategy	Policy Description	Cost (c/kWh)	SO2 (Ton/GWh)	EUSE (%)	CGR (c/kWh)
1	Purchase No DSM Low EUSE	3.56	3.3803	0.1026	2.67
2	Purchase DSM Low EUSE	3.38	3.3716	0.1263	2.93
3	Purchase No DSM Base EUSE	3.49	3.3083	0.1707	2.07
4	Purchase DSM Base EUSE	3.31	3.3154	0.2034	2.13
5	Purchase No DSM High EUSE	3.34	3.3210	0.2861	3.32
6	Purchase DSM High EUSE	3.27	3.3097	0.2957	3.55
7	Controls No DSM Low EUSE	3.75	1.4868	0.1026	2.67
8	Controls DSM Low EUSE	3.54	1.5155	0.1243	2.93
9	Purchase No DSM Base EUSE	3.67	1.4988	0.1549	2.96
10	Controls DSM Base EUSE	3.48	1.5251	0.2001	5.53
11	Controls No DSM High EUSE	3.54	1.5172	0.2935	3.32
12	Controls DSM High EUSE	3.45	1.5272	0.2637	7.70

4 SAMPLE STUDY: EVALUATION OF GENERATION EXPANSION STRATGIES

4.1 Problem Formulation

The sample system used here is based primarily on a moderate-sized U.S. electric utility. The utility long-range generation resource expansion strategies include three main policy approaches: SO2 emissions, demand-side management (DSM) and system reliability. Emissions policy considers the allowance purchase policy versus the use of scrubbers and fuel switching. DSM policy options are between go and no-go decisions. Approaches to system reliability include choices among high, base case and low capacity reserve margins. In all, these three approaches result in (2*2*3=) 12 alternative generation resource expansion strategies, and cost of energy supply, system reliability, environmental impact and resource flexibility are the four major attributes considered in the decision process for the selection of most desired resource strategy. The utility system performance under different expansion strategies has been studied using the electric generation expansion analysis system (EGEAS) package. The simulation results give actual project costs in millions of dollars, the amounts of SO2 emissions, the expected unsaved energy (EUSE) in kilowatt-hour, and the numbers and capacities of gas-fired and coal-fired units during the planning period. Since the base energy requirement change with the DSM impacts, the values of different attributes are normalized with respect to the total energy requirement as shown in Table (1). Cost as give here includes the annual levelized investment costs, fuel costs operating and maintenance costs, as well as the cost of allowances. A rather indirect measure of resource flexibility is used here, the ratio of coal to gas capacity, in view of the ease with which gas plants can be changed and the possibility of conversion of gas power plants to other types such as combined cycle power plants. The equivalent cost of the coal-to-gas ratio (CGR) is the ratio of coal to gas capacities. This is normalized to reflect an average cost that is equal to the average cost of different alternatives.

4.2 Decision Model Establishment

After obtaining the simulation results of the 12 alternative generation expansion strategies, we can assess the single utility functions and the weighting parameters using the structured procedure described in Section 2 and then assemble them into the linear additive utility model as defined in Equation (1).

4.3 Assessment of Single Utility Functions

Let us consider the utility function for reliability as an example. The values of attribute EUSE are calculated to

be in the range between 0.1% and 0.3%. The lower bound of EUSE represents the best system performance in terms of the reliability of electric energy supply and therefore we have U(0.1%) = 1.0. Conversely, the upper bound of EUSE indicates the worst situation of system reliability and thus we have U(0.3%) = 0.

Next, we need to assess three certainty equivalent values in the range of EUSE measurements with respect to $U_i(x.5)$, $U_i(x.75)$, and $U_i(x.25)$.

The candidates for x.5, with respect to U(x.5) = 0.5, are selected to be c = [0.16, 0.18, 0.20, 0.22](%). By comparing each pair of these candidates for their closeness to the expected certainty equivalent, the judgment matrix [A] is formed

$$[A] = \begin{bmatrix} 1 & 1/3 & 1/7 & 1/3 \\ 3 & 1 & 1/5 & 1 \\ 7 & 5 & 1 & 5 \\ 3 & 1 & 1/5 & 1 \end{bmatrix}$$

These ratio scales reflect the independent assessments that: · Candidate 1 (0.16% EUSE level) is slightly less likely than candidates 2 and 4(0.18% and 0.22% EUSE levels), but very much less likely than candidate 3 (0.20% EUSE level)· Candidate 2 is much less than candidate 3 and is as likely as candidate 4· Candidate 3 is much more likely than candidate 4.

Solving the eigenvalue problem associated with the above judgment matrix yields the priority vector p = [0.0624, 0.1514, 0.6348, 0.1514]. The certainty equivalent x.5 is then computed as the weighted-average of these candidates

$$x_{.5} = [0.16, 0.18, 0.20, 0.22] * [0.0624, 0.1514, 0.6348, 0.1514]^T = 0.1975\%$$

In a similar manner, the candidates for x.75 and x.25, with respect to U(x.75) = 0.75 and U(x.25) =0.25, are selected to be [0.12, 0.13, 0.14, 0.15] (%) and [0.25, 0.26, 0.27, 0.28] (%),respectively. The corresponding certainty equivalents for these two reliability levels are determined to be x.75 = 0.1293% and x.25 = 0.2643%.

These three (x_i , $U(x_i)$) pairs, along with the two end points, give us five points on the preference function of system reliability. We then fit these points by a third-order polynomial function, which represents the preference function for attribute EUSE This procedure is performed for all four attributes and the resulting single utility functions for cost, reliability (EUSE), SO2 emissions and flexibility (CGR) are expressed below by $U_1(x_1)$, $U_2(x_2)$, $U_3(x_3)$ and $U_4(x_4)$, respectively.

$U_1(x_1) = -2.7734 x_1^3 + 27.7246 x_1^2 - 93.9817 x_1 + 108.8031$
$U_2(x_2) = -238.2481 x_2^3 + 144.13 x_2^2 - 31.67 x_2 + 2.9599$
$U_3(x_3) = -0.1088 x_3^3 + 0.8082 x_3^2 - 2.36 x_3 + 3.037$
$U_4(x_4) = (0.077 - x_4) / 0.0563$
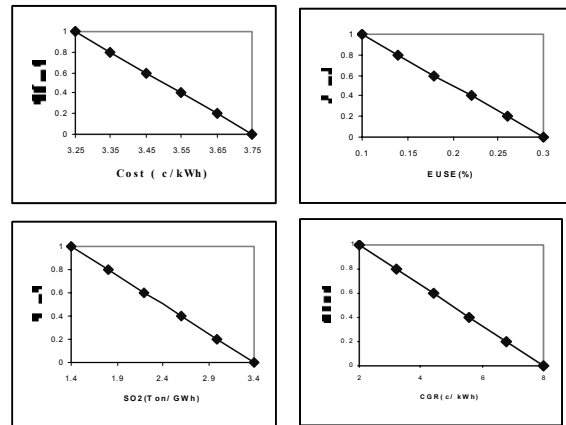
4.4 Assessment of Attribute Priorities

Table (2) defines the four hypothetical alternatives used for this sample case study. Instead of asking what is the relative importance of the cost of energy supply with respect to the system reliability, the ratio question now would become clear: How much more preferable is a specific savings in the cost of energy supply than a specific improvement in system reliability? This question is asked for each pair of hypothetical alternatives with respect to each player.

Table2- Definition of Hypothetical Alternatives

Hypothetical Alternative	Cost of Energy (c/kWh)	SO2 Emissions (Ton/GWh)	EUSE (%)	CGR (c/kWh)
1	3.25	3.4	0.3	800
2	3.75	1.4	0.3	800
3	3.75	3.4	0.1	800
4	3.75	3.4	0.3	200

Table (3) gives the priority vectors for the second and third layers with respect to the least-cost planning goal. The priority vector of the second layer indicates the relative importance of each player in implementing the least-cost planning strategy in the order of utility, regulators, customers and general public. As for the composite priorities of attributes or the weighting parameters with respect to the planning objective, the minimization of energy production cost is ranked at the top followed by SO2 emissions, system reliability, and flexibility in resource development.

Table3- Composite Priority Vectors for Layer 2 & 3

Layer2		Layer3	
Player	Priorities	Attribute	Priorities
Utility	0.5738	Flexibility	0.0884
Customers	0.1310	Cost	0.4760
Regulators	0.2388	Reliability	0.1510
General Public	0.0563	Emissions	0.2846

4.5 Linear Additive Utility Model

Finally, by assembling the single utility functions and the weighting parameters into the linear additive utility formulation defined in (1), we obtain the following MADM model for this particular decision making problem.

$U(x) = 0.476U_1(x_1) + 0.151U_2(x_2) + 0.284U_3(x_3) + 0.088U_4(x_4)$

4.6 Decision Analysis and Interpretations

In traditional MADM analysis, the goodness of alternatives is measured based on the expected composite utility values by substituting the system simulation results and the weighting parameters directly into the established MADM model. Both the expected composite utility values and the ranking of 12 generation expansion alternatives are given in Table (4) (see the second column and the last column). Apparently, the best solution determined by the point estimate of traditional MADM analysis is the alternative for which the measured composite utility value is maximal. It can be noted that the best solution (CDL) suggested by MADM analysis is not the one with the least cost of energy supply due to the contributions of SO2 emissions and system reliability on the value of composite utility. The best solution supports the generation expansion strategy having SO2 controls, DSM measures, and low EUSE or high capacity reserve margin.

Table4- Estimated Composite Utility Values

Expansion Strategy	Point Estimate	Likely Range Estimate			Ranking
		a	b	c	
PNL	0.4522	0.4894 0.4151	0.4750 0.4296	0.5122 0.3923	12
PDL	0.5734	0.5914 0.5555	0.6012 0.5457	0.6129 0.5277	6
PNB	0.4772	0.4831 0.4712	0.4963 0.4581	0.5022 0.4522	10
PDB	0.6031	0.6074 0.5989	0.6361 0.5702	0.6403 0.5660	4
PNH	0.5075	0.5313 0.4837	0.5359 0.4792	0.5597 0.4553	9
PDH	0.5477	0.5795 0.5158	0.5834 0.5119	0.6153 0.4800	7
CNL	0.4671	0.4913 0.4429	0.5001 0.4341	0.5243 0.4099	11
CDL	0.6978	0.7172 0.6783	0.7306 0.6649	0.7501 0.6454	1
CNB	0.5377	0.5483 0.5270	0.5681 0.5072	0.5788 0.4965	8
CDB	0.6643	0.6691 0.6595	0.6954 0.6331	0.7002 0.6283	2
CNH	0.5805	0.6105 0.5504	0.6104 0.5505	0.6405 0.5205	5
CDH	0.6232	0.6352 0.6111	0.6551 0.5912	0.6671 0.5792	3

Note: P/C   Allowance Purchase/ Emission Control
     N/D   No DSM / DSM
     L/B/H   Low / Base / High EUSE Limit

5 CONCLUSIONS

An interval-based MADM approach has been developed to enhance the decision making process with imprecise information. The main contributions from this paper include:
· Providing a structured procedure to facilitate the evaluation of preference functions and the relative importance of attributes in the construction of linear additive utility models.
· Providing a confidence interval-based MADM decision approach to help the planner of electric utilities identify a desirable resource strategy by examining a range of acceptable alternatives.
Experience from the sample case study indicates that this enhanced MADM methodology can build insight on how the imprecise information may affect the choice toward the best solution and increase the level of confidence for the selection of a final resource strategy.

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