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A Circuit Theory Based Load Flow Tracing Method Considering Counter-Flow Contribution

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Abstract:--A new method to trace the power flow based on the converged AC power flow solution is proposed in this paper. The method is formulated by using the transmission network structure, the equivalent-current-injection and the load- equivalent-admittances from the engineering viewpoint where all the electrical circuit theories are satisfied. Four steps are used to trace the relationship between each line flow and each generator injection power without any assumptions, and the power distribution of each generator to each load can also be determined. Besides, the line loss also can be allocated to each generator. This tracing algorithm can calculate power contributions effectively, and can be integrated into the existent tariffs of charging for transmission losses and services.

Index Terms:--Equivalent Current Injection, Equivalent Load Impedance, Generator Contribution, Line Loss Allocation.

1 Introduction

Recent years have seen a worldwide trend towards deregulation and unbundling of services provided by utilities throughout the world, especially in power market. This trend separates the traditional vertical-integrated power system structure into generation, transmission and distribution independently companies. And, the goal is always lowering of the average consumer price and introduction of competition. While competition is introduced in generation and retail, it is widely agreed that transmission network is a natural independent and monopoly. Therefore, the transmission network company should remain neutrality and centrally controlled to make the market is operating fairly.

Transparency is one of the most important ingredients in the operation of the transmission system. It is necessary to find accurate and indisputable answers to questions as "which GENCOs are using this transmission line?" or "which GENCOs are supplying this load?" In other words, the network operating company or ISO must find out the capacity usage of individual transactions happening at the same time, and then a fair use-of-transmission charge can be allocated to individual GENCO. This problem relates to how to allocate the total cost of transmission between all electrical users in an equitable and non-discriminatory manner, which also provides a correct, market-based economical signal to every participant at the same time. In order to determinate usage-of-transmission, we need to know the contributions of individual participants to the line flow and loss, and how participants utilize the system by the AC power flow tracing.

Several papers have been published on this subject [1]-[10]. Ref. [1]-[5] traced the contributions of generators and line flows using a proportionality assumption. In [6]-[8], the authors used the state graph concept to represent the system structure according to line flow directions. With the reactive power being usually much smaller than the active power, the contributions were either neglected or approximated solely on the basis of line active power [2, 3, 6, 7]. On the other hand, neglecting the effect of active power flow when dealing with reactive power is not persuasive either. Ref. [1], [4] and [5] discussed the reactive flow tracing by using the same concept as active flow tracing method. In [7], the authors proposed a decomposition of complex power flows in relation to individual transaction, but the flows can't allocate to each participant independently. The couplings between the active and reactive power have not been well addressed. Methods were also proposed to determine the active and reactive power contribution by tracing the real and imaginary current networks [8], which is accurate only for the reactive flow.

Besides the line flow, the transmission loss allocation is another important issue for the electricity market. There is no accurate theory on the contribution of individual generator to branch active and reactive loss owing to the impossibility of superposition. In [8]-[10], the active/reactive loss is proportional to the active/reactive power generation. For Ref. [8], the load contributions to generators will include the inductive-compensation power and the generation contribution to load will include the charging capacitive power and capacitive-compensation power. And, the branch power loss will be allocated to the charging capacitive power and compensation power, if losses and fixed construction cost are to be allocated to both generators and loads. Ref. [9] used DC power flow model to allocate system loss to each existing transaction with assumption on bus voltages and angles. Ref. [10] proposed mathematical models of MVA load flow and loss allocation, its basic theory of active and reactive power wheeling cost analysis and the transmission pricing under the pool-mode environment.

Counter flow is the flow component that goes in the opposite direction of the net flow. In many proportional tracing methods using the DC flow model, the counter

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flows were not even considered. Some theorems discussed this issue from a cost viewpoint which insisted that counter flows pay for the cost and the toll even if congestion is involved. This information can be used to debate the most appropriate procedure for dividing up the embedded cost of counter flow [11]. However, inspecting the contributions in relieving the congestion and postponing the system reinforcement, counter flows should even be well rewarded [12], in regard to the fact that they are paying for line and facility usage fees.

In this paper, a new formulation is developed from the engineering standpoint to trace the AC power flow. This method considers both contributions of the active-power and the reactive-power injection where power flow equations and the basic circuit theory are satisfied. That is, a circuit theory based method is proposed where all the electrical theories are satisfied. With the exact calculation, contribution of counter flow can be well presented and re-measured. Besides, the proposed method allocates the line loss according to the true contributions where counter flow can be fairly treated. The IEEE 9-bus test system was presented to illustrate the proposed approach, and the results shown the effectiveness and applicability of the new theorem.

In this paper, a new formulation is called Upstream Tracing Method (UTM), which is developed from the engineering standpoint to trace the AC power flow. This method considers both contributions of the active-power and the reactive-power injection where power flow equations and the basic circuit theory are satisfied. That is, a true flow based method is proposed where all the electrical theories are satisfied. With the exact calculation, contribution of counter flow can be well presented and re-measured. Test systems are presented to illustrate the proposed approach, and the results show the effectiveness and applicability of the new tracing method.

2 Theory Derivation

The proposed method to trace power is based on a converged AC load flow solutions. There are four matrices involved, describing the relationships among various variables.

<u>Admittance Matrix with Load Modification</u>

Let a N×N network shown in Figure 1 be originally represented by

$$\left[I^{ori} \right] = \left[y_{ij}^{ori} \right]_{N \times N} \left[V^{sol} \right] \tag{1}$$

The equivalent load admittance can be considered instead of the original current injection. With the conversion of load into admittance, only the generator injection exists in the modified matrix. We have

$$\begin{aligned} \overline{I}_i^{eq} &= \frac{P_i - jQ_i}{(\overline{V}_i^{sol})^*} = \frac{P_i - jQ_i}{\overline{V}_i^{sol} \cdot (\overline{V}_i^{sol})^*} \times \overline{V}_i^{sol} \\ &= y_{ii}^{eq} \cdot \overline{V}_i^{sol} \end{aligned} \quad , \; i \in L \tag{2}$$

where $P + jQ$ and $|V^{sol}|$ are the load power and the bus voltage of the converged load flow solution. L is the number of load buses.

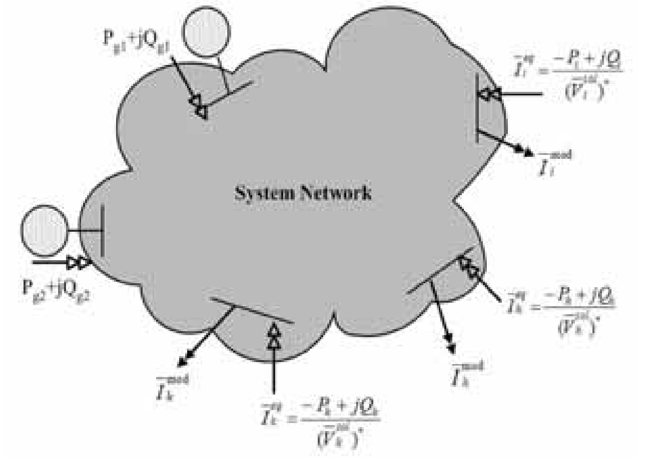

Figure 1 Load Equivalent Current Modifications

Adding each equivalent load admittance to the corresponding diagonal element of the original admittance matrix in Eq. (1). We have a modified system of

$$\begin{aligned} \left[I^{mod} \right] &= \left[y_{ii}^{ori} \right]_{N \times N} \left[V^{sol} \right] + \left[I^{eq} \right] \\ &= \left[y_{ii}^{ori} \right] \left[V^{sol} \right] + \left[y_{ii}^{eq} \right] \left[V^{sol} \right] \\ &= \left[Y^{mod} \right] \left[V^{sol} \right] \end{aligned} \tag{3}$$

where $y_{ii}^{mod} = y_{ii}^{ori} + y_{load,i}^{eq}$.

Cartesian coordinate formulation can be used by

$$\begin{bmatrix} I_R^{mod} \\ - \\ I_I^{mod} \end{bmatrix} = \begin{bmatrix} Y_G^{mod} & | & Y_B^{mod} \\ -- & - & -- \\ Y_B^{mod} & | & -Y_G^{mod} \end{bmatrix} \begin{bmatrix} E^{sol} \\ - \\ F^{sol} \end{bmatrix} \tag{4}$$

By inverting Eq.(14), we have

$$\begin{bmatrix} E^{sol} \\ - \\ F^{sol} \end{bmatrix} = \begin{bmatrix} J_1^{mod} & | & J_2^{mod} \\ -- & - & -- \\ J_3^{mod} & | & J_4^{mod} \end{bmatrix} \begin{bmatrix} I_R^{mod} \\ - \\ I_I^{mod} \end{bmatrix} \tag{5}$$

where

$$\begin{bmatrix} J_1^{mod} & | & J_2^{mod} \\ -- & - & -- \\ J_3^{mod} & | & J_4^{mod} \end{bmatrix} = \begin{bmatrix} Y_G^{mod} & | & Y_B^{mod} \\ -- & - & -- \\ Y_B^{mod} & | & -Y_G^{mod} \end{bmatrix}^{-1}$$

<u>Matrix Formulation</u>

For a N-bus system, the injection-power on bus k can be formulated by

$$P_k + jQ_k = (E_k I_{k,R} + F_k I_{k,I}) + j(F_k I_{k,R} - E_k I_{k,I}) \tag{6}$$

Eq.(6) can be expend to a N-bus matrix form by

$$\begin{bmatrix} P \\ -- \\ Q \end{bmatrix} = \begin{bmatrix} E & | & F \\ -- & - & -- \\ F & | & -E \end{bmatrix} \begin{bmatrix} I_R \\ -- \\ I_I \end{bmatrix} \quad (7)$$

where E and F are the real and imaginary parts of bus voltages, and I_R and I_I are the real and imaginary parts of bus injection-currents, respectively. Eq. (7) can be rewritten as

$$\begin{bmatrix} I_R \\ - \\ I_I \end{bmatrix} = \begin{bmatrix} T_1^{inv} & | & T_2^{inv} \\ -- & - & -- \\ T_3^{inv} & | & T_4^{inv} \end{bmatrix} \begin{bmatrix} P \\ - \\ Q \end{bmatrix} \quad (8)$$

where the matrix dimension is 2N×2N.

Let the transmission line be represented by a π-model as shown in Figure 2 with line charging, the current I_{ij} can be derived by

$$\begin{aligned} I_{ij,R} + jI_{ij,I} &= [g_{ij}(E_i - E_j) - b_{ij}(F_i - F_j) - B_c F_i] \\ &+ j[g_{ij}(F_i - F_j) + b_{ij}(E_i - E_j) + B_c E_i] \end{aligned} \quad (9)$$

We have the matrix form for a M line-section network that

$$\begin{bmatrix} I_{ij,R} \\ - \\ I_{ij,I} \end{bmatrix} = \begin{bmatrix} Y_{c,G}^f & | & Y_{c,B}^f \\ -- & - & -- \\ Y_{c,B}^f & | & -Y_{c,G}^f \end{bmatrix} \begin{bmatrix} E \\ - \\ F \end{bmatrix} \quad (10)$$

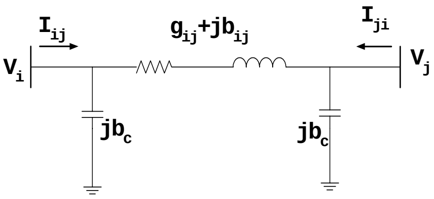

Figure 2 Transmission line π model and the forward/ backward current

Since current I_{ji} is not equal to current I_{ij}, a similar matrix can be written by

$$\begin{bmatrix} I_{ji,R} \\ - \\ I_{ji,I} \end{bmatrix} = \begin{bmatrix} Y_{c,G}^b & | & Y_{c,B}^b \\ -- & - & -- \\ Y_{c,B}^b & | & -Y_{c,G}^b \end{bmatrix} \begin{bmatrix} E \\ - \\ F \end{bmatrix} \quad (11)$$

For line i~j, we know that

$$P_{ij} + jQ_{ij} = (E_i I_{ij,R} + F_i I_{ij,I}) + j(F_i I_{ij,R} - E_i I_{ij,I}) \quad (12)$$

$$P_{ji} + jQ_{ji} = (E_j I_{ji,R} + F_j I_{ji,I}) + j(F_j I_{ji,R} - E_j I_{ji,I}) \quad (13)$$

a matrix forms can also be expanded by

$$\begin{bmatrix} P_{ij} \\ - \\ Q_{ij} \end{bmatrix} = \begin{bmatrix} H_E^f & | & H_F^f \\ -- & - & -- \\ H_F^f & | & -H_E^f \end{bmatrix} \begin{bmatrix} I_{ij,R} \\ - \\ I_{ij,I} \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} P_{ji} \\ - \\ Q_{ji} \end{bmatrix} = \begin{bmatrix} H_E^b & | & H_F^b \\ -- & - & -- \\ H_F^b & | & -H_E^b \end{bmatrix} \begin{bmatrix} I_{ji,R} \\ - \\ I_{ji,I} \end{bmatrix} \quad (15)$$

<u>Contribution matrix and tracing</u>

Cascading Eq. (5), (8), (10), (14) to produce Eq.(16), and Eq.(5), (8), (11), (15) to yield Eq.(17). It can get

$$\begin{bmatrix} P_{ij} \\ - \\ Q_{ij} \end{bmatrix} = \begin{bmatrix} D_1^f & | & D_2^f \\ -- & - & -- \\ D_3^f & | & D_4^f \end{bmatrix} \begin{bmatrix} P^{\mathrm{mod}} \\ - \\ Q^{\mathrm{mod}} \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} P_{ji} \\ - \\ Q_{ji} \end{bmatrix} = \begin{bmatrix} D_1^b & | & D_2^b \\ -- & - & -- \\ D_3^b & | & D_4^b \end{bmatrix} \begin{bmatrix} P^{\mathrm{mod}} \\ - \\ Q^{\mathrm{mod}} \end{bmatrix} \quad (17)$$

where $\left[D^f \right] = \left[H^f \right] \left[Y_c^f \right] \left[J^{\mathrm{mod}} \right] \left[T^{inv} \right]$

$$\left[D^b \right] = \left[H^b \right] \left[Y_c^b \right] \left[J^{\mathrm{mod}} \right] \left[T^{inv} \right]$$

are called the sending/receiving contribution matrix (SCM/RCM) with dimension 2M×2N.

<u>Line Flow Contribution Tracing</u>

We can find the active and reactive power flow contribution from the g^{th} generator on the k^{th} line section by

$$P_{line,k}^g = D_{1,kg}^f P_g + D_{2,kg}^f Q_g \quad (18)$$

$$Q_{line,k}^g = D_{3,kg}^f P_g + D_{4,kg}^f Q_g \quad (19)$$

$$k = 1,...., M \quad g \in NG$$

where NG is set of all generators. D_{kg}^f is the element of the k^{th} row and the g^{th} column of SCM. The converged line flow of the load flow program can be found by

$$P_{line,k} + jQ_{line,k} = \sum_{g \in NG} P_{line,k}^g + j\left(\sum_{g \in NG} Q_{line,k}^g \right) \quad (20)$$

Now we have a mathematical formulation for not only the generator's active power tracing but also the reactive power load contribution tracing. Every generator's contribution to the designated d^{th} load can be found as

$$\begin{aligned} P_{load,d}^g &= \sum_{m \in \Omega} P_{line,m}^g \\ &= \sum_{m \in \Omega_f} (D_{1,mg}^f P_g + D_{2,mg}^f Q_g) + \sum_{m \in \Omega_b} (D_{1,mg}^b P_g + D_{2,mg}^b Q_g) \end{aligned}$$

(21)

$$Q^g_{load,d} = \sum_{m\in\Omega} Q^g_{line,m} = \sum_{m\in\Omega_f}(D^f_{3,mg}P_g + D^f_{4,mg}Q_g) + \sum_{m\in\Omega_b}(D^b_{3,mg}P_g + D^b_{4,mg}Q_g) \quad (22)$$

where Ω is the set of all line sections connected to d^{th} load bus. It can be divided into Ω_f and Ω_b for forward and backward contributions, respectively.

3 Loss Allocation

The line loss attribution from each generator can be calculated by using SCM and RCM of Eq. (16) and Eq. (17). For instance, line flow on section i~j attributed to the g^{th} generator is

$$\begin{aligned} P^g_{line,ij} &= D^f_1 P_g + D^f_2 Q_g \\ Q^g_{line,ij} &= D^f_3 P_g + D^f_4 Q_g \end{aligned} \quad (23)$$

$$\begin{aligned} P^g_{line,ji} &= D^b_1 P_g + D^b_2 Q_g \\ Q^g_{line,ji} &= D^b_3 P_g + D^b_4 Q_g \end{aligned} \quad (24)$$

the active and reactive power loss contribution of the g^{th} generator can be calculated by adding Eq.(24) to Eq.(23) as

$$P^g_{loss,ij} = P^g_{line,ij} + P^g_{line,ji} = (D^f_1 + D^b_1)P_g + (D^f_2 + D^b_2)Q_g \quad (25)$$

$$Q^g_{loss,ij} = Q^g_{line,ij} + Q^g_{line,ji} = (D^f_3 + D^b_3)P_g + (D^f_4 + D^b_4)Q_g \quad (26)$$

4 Numeric Test

Extensive tests were conducted for the IEEE 9-bus, and the IEEE 30-bus systems. All tests showed good results. For the IEEE 9-bus system is used for example to show the accuracy and the effectiveness of the proposed theory.

Figure 3 shows active and reactive flows of the IEEE 9-bus system obtained from a load flow program. The line charging suseptance of each transmission line is considered without any assumption.

Line flow Tracing

Table 1 shows the active and reactive line flow tracing of the proposed method. The proposed UTM Method result in a different tracing result with showing the delivered power may have positive and negative contribution components in the same line. For instance, the line net flow on 6~7 is 24.1MW, GENCO 1 and GENCO 3 contribute positive flow, but GENCO 2 contribute a negative component as a counter flow provider. It provides an opportunity to reduce net flow and improve system security.

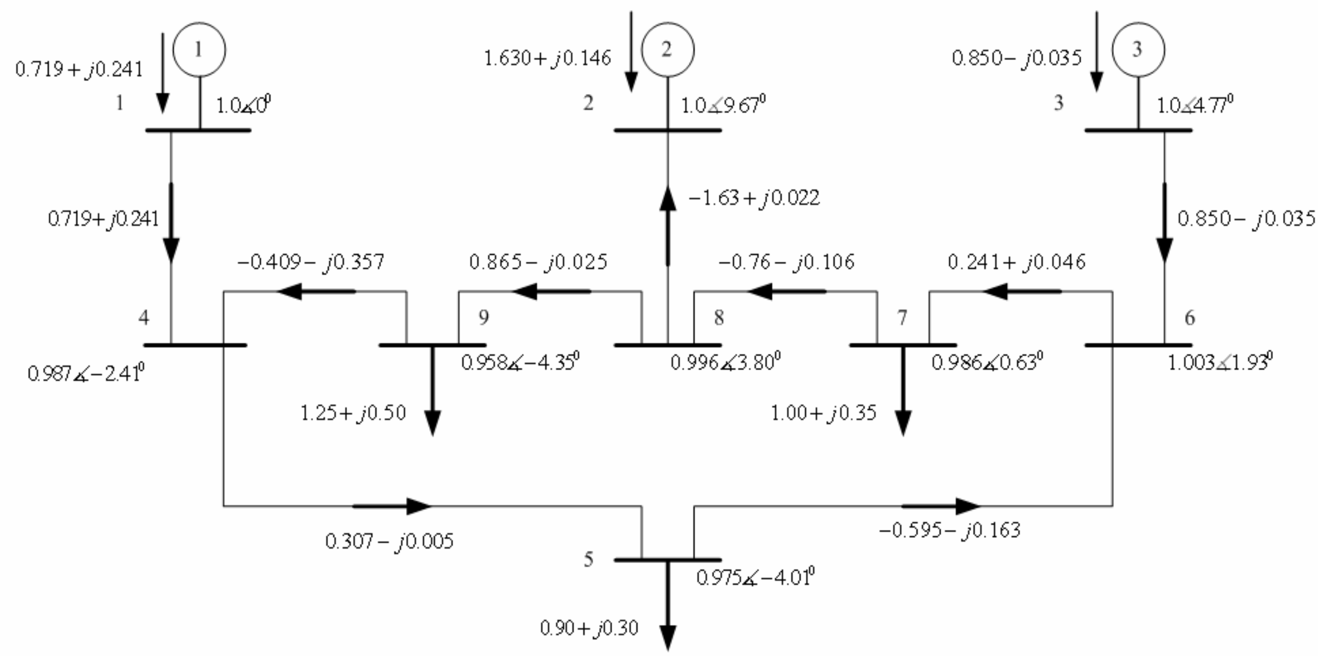

Figure 3 AC load flow result of IEEE 9-Bus system

Load Tracing

Table 2 shows active and reactive load attribution of UTM. Load bus 4, 6 and 8 with zero load consumption is shown in Fig.3 and the tracing result match this condition.

Loss Allocation

Table 3 shows line loss trace result of each generator. GENCO 2 and GENCO 3 act as a major loss reducer as the numeric result shown in the table.

Table 1 Active and Reactive Line flow tracing result (MVA base=100MVA)

Branch From To	Sending-End Load flow	GENCO 1's Contribution P / Q (pu.)	GENCO 2's Contribution P / Q (pu.)	GENCO 3's Contribution P / Q (pu.)
1~4	0.719/0.241	0.719/0.241	0.000/0.000	0.000/0.000
4~5	0.307/-0.005	0.325/0.065	0.126/-0.065	-0.144/-0.005
5~6	-0.595/-0.163	0.127/-0.030	-0.333/-0.131	-0.389/-0.002
3~6	0.850/-0.035	0.000/0.000	0.000/0.000	0.850/-0.035
6~7	0.241/-0.046	0.102/0.059	-0.331/-0.12	0.470/-0.025
7~8	-0.760/-0.106	-0.088/-0.078	-0.859/0.005	0.187/-0.033
8~2	-1.630/0.022	0.000/0.000	-1.630/0.022	0.000/0.000
8~9	0.865/-0.025	-0.101/-0.054	0.772/0.012	0.194/0.017
9~4	-0.409/-0.357	-0.379/-0.165	0.114/-0.149	-0.144/-0.043

Table 2 Active and Reactive loads tracing result

GENCO Bus / Load Bus	**1**		**2**		**3**	
	Contribution P/Q(pu.)	% P/Q	Contribution P/Q(pu.)	% P/Q	Contribution P/Q(pu.)	% P/Q
4	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00
5	0.19/0.12	21.11/40.00	0.46/0.14	51.11/46.67	0.25/0.04	27.78/13.33
6	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00
7	0.17/0.18	17.00/51.43	0.54/0.12	54.00/34.29	0.29/0.05	29.00/14.28
8	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00
9	0.26/0.19	20.80/38.00	0.65/0.20	52.00/40.00	0.34/0.11	27.20/22.00

Table 3 Comparison of active power loss distribution

Branch From To	Solution loss (MW)*1	Bus 1's Contribution (MW)	Bus 2's Contribution (MW)	Bus 3's Contribution (MW)
1~4	0.00	0.00	0.00	0.00
4~5	0.17	0.86	0.03	-0.72
5~6	1.45	2.57	-0.23	-0.89
3~6	0.00	0.00	0.00	0.00
6~7	0.09	2.06	-1.54	-0.43
7~8	0.51	1.27	-0.13	-0.63
8~2	0.00	0.00	0.00	0.00
8~9	2.46	2.23	0.51	-0.28
9~4	0.26	1.43	-1.18	0.01

*1：Load flow program solution.

5 Discussion and Conclusion

A novel formulation to trace the power flow for deregulated transmission systems was developed in this paper. The formulation was based on the basic circuit theories, the equivalent current injection, and the equivalent impedance method. The method can determine the amount of the active and reactive power output from a particular generator to a particular load. The loss allocation of each line, which is produced by each generator, can be also obtained. The proposed method possesses the advantages that

(1)the proposed method satisfies the power flow equations, power balance equations and the electric circuit theories simultaneously.
(2)the algorithms are true flow based and fool-proof without needing any additional assumptions such as proportional sharing or constant voltages, neither is it limited to incremental changes.
(3)the proposed method can calculate the contributions for both the active and reactive power. And, the delivered power may have positive and negative components in the same line.
(4)the proposed method can trace the line flow, load contribution and loss allocation.

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