Proceedings of the 6th WSEAS International Conference on Multimedia Systems & Signal Processing, Hangzhou, China, April 16-18, 2006 (pp157-162)

M-Channel Linear Phase Cosine Modulated Filter Banks with Perfect Reconstruction

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Abstract— A class of cosine modulated filter banks in which all the sub-band filters have linear phase is presented. It has only M bands compared to the $2M$ channel structures so far reported in the literature. The filter bank uses only one prototype filter for both analysis and synthesis side and structurally guarantee perfect reconstruction property. The formulation is based on DCT-II. The necessary and sufficient conditions for the perfect reconstruction and linear phase are derived.

Key-Words: - Filter bank; Cosine modulation; Linear phase; Perfect reconstruction; Discrete cosine transform

1. Introduction

Maximally decimated multi-rate filter banks [1]-[3] are used in sub band processing of audio, image and video signals, analog to digital converters and signal compression systems. In a filter bank the analysis filters first channelise the signal to be processed. The extracted sub band signals are then decimated and processed by a processing unit according to the application in hand. This is either stored or transmitted. In the receiver side a synthesis filter bank reverses this process and reconstructs the original signal

When the sub-band filters are modulated versions of a prototype filter, such systems are called modulated filter banks. Cosine modulated filter banks (CMFB) are well known and widely used in practical multirate applications due to its inherent design ease and computationally efficient implementations based on fast discrete cosine transform algorithms. The Perfect reconstruction conditions (PR) for a CMFB were established in [4]-[8]. The formulation in [5]-[6] does not give linear phase analysis and synthesis filters. Besides eliminating phase distortion linear phase property permits use of simple symmetric extension methods to accurately handle the boundaries of finite length signals. Furthermore it leads to better sub band processing performance and more efficient implementation . To get linear phase sub filters the structure proposed in [7]-[8] uses a peculiar $2M$ -band structure with sine and cosine modulation.

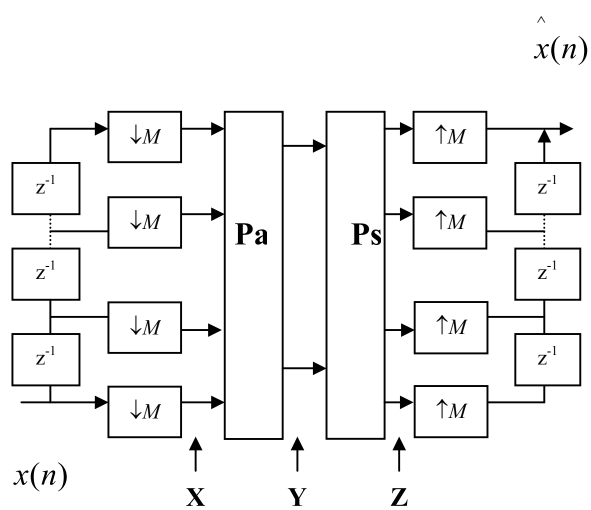

Fig.1. Proposed filter bank structure

In this paper the PR conditions for a M -Channel CMFB with linear phase sub filters are derived using the general framework given in [9]. The proposed filter bank uses only M -channels, all the filters are linear phase, uses single modulation using an orthogonal transform and only one prototype filter.

This paper is organized as follows: In section 2, classical CMFB is reviewed. Section 3, covers the general theory of the proposed linear phase CMFB. Section 4 presents the prototype filter design and results and section 5 concludes the paper.

Notations: Bold letters indicate vectors and matrices. Matrix $\mathbf{I}_m$ represents a $m \times m$ identity matrix .The notation A^T represents transpose operation and tilde operation on a polynomial $E(z)$ is defined as $\tilde{E}(z) = E_*(z^{-1})$,where $*$ denotes conjugation of the coefficients [1]. $\mathbf{J}$ is the reversal matrix as

$$\mathbf{J}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2. Cosine modulated filter banks

An M -Channel maximally decimated filter bank is PR if the analysis poly-phase matrix is invertible [1]. That is $\mathbf{R}(z)\mathbf{E}(z) = cz^{-l}\mathbf{I}$, $c \neq 0$, $l \geq 0$ where $\mathbf{R}(z)$ and $\mathbf{E}(z)$ are the synthesis and analysis poly-phase matrices respectively. In a CMFB all the channel filters are modulated versions of a prototype filter. A typical CMFB generates the analysis filters, $h_k(n)$, and the synthesis filters $f_k(n)$ by the modulation of a linear phase low pass prototype filter $h(n)$ as follows [1].

$$h_k(n) = 2h(n)\cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) + (-1)^k \frac{\pi}{4}\right) \tag{1}$$

$$f_k(n) = 2h(n)\cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) - (-1)^k \frac{\pi}{4}\right) \tag{2}$$

$$0 \leq k \leq M-1, \quad 0 \leq n \leq N-1,$$

PR is obtained when $h(n)$ satisfies the following constraint [5]-[8].

$$\tilde{G}_k(z)G_k(z) + \tilde{G}_{k+M}(z)G_{k+M}(z) = \frac{1}{2M} \tag{3}$$

where $G_n(z)$ is the nth type 1 polyphase component of $H(z)$ .But the individual filters do not have linear phase even though the prototype filter is symmetric.

The linear phase CMFB reported in [7]-[8] has two subsystems in which first subsystem has M+1 channels and the second has M-1 channels or vice-versa. These use cosine modulation for one subsystem and sine modulation for the next. Also shifting of the window in [7]-[8] for the sine modulated filters introduces additional complexity in the handling of finite length signals [10]. The redundancy in this filter bank is evident.

3. *M*-channel CMFB with linear-phase sub-band filters

Schuller and Smith[9] have given a compact representation of classical filter banks. Fig. 1 shows the proposed M -channel filter bank using that structure. The input is represented as a row vector composed of sampled input sequences

$\mathbf{x}(n) = [\, x(nM), x(nM+1)..x(nM+M-1)\,]$

where n, may be viewed as the index of sequences $x(nM+m), m = 0,1,...M-1.$.With this definition it is clear from Fig.1 that the output of the block $\mathbf{P_a}$ is $\mathbf{y}(n) = [y_0(n), y_1(n),.........y_{M-1}]$.

Let the Z transforms of $\mathbf{x}(n)$ and $\mathbf{y}(n)$ be $\mathbf{X}(z) = [X_0(z), X_1(z),.......X_{M-1}(z)]$, $\mathbf{Y}(z) = [Y_0(z), Y_1(z),......Y_{M-1}(z)]$ respectively. The block denoted as $\mathbf{P_a}$ in Fig.1 is an $M \times M$ matrix formed from the analysis filters. The filter length is assumed to be integer multiple of M . Arbitrary length can be incorporated by padding zeros while forming the block elements. The matrix $\mathbf{P_a}$ has the form

$$\mathbf{P_a} = \begin{bmatrix} Ea_{0,0}(z) & Ea_{0,1}(z) & \cdots & Ea_{0,M-1}(z) \\ Ea_{1,0}(z) & Ea_{1,0}(z) & \cdots & Ea_{0,M-1}(z) \\ \vdots & \vdots & \vdots & \vdots \\ Ea_{M-1,0}(z) & \cdots & \cdots & Ea_{M-1,M-1}(z) \end{bmatrix} \tag{4}$$

The matrix elements are polynomials defined as

$$Ea_{n,k}(z) = \sum_{m=0}^{L-1} h_k(n+mN)z^{-(L-1-m)} \tag{5}$$

where L is a positive integer such that filter length $N = LM$ .Similarly the matrix $\mathbf{Ps}$ for the synthesis bank is

$$\mathbf{Ps} = \begin{bmatrix} Es_{0,0}(z) & Es_{0,1}(z) & \cdots & Es_{0,M-1}(z) \\ Es_{1,0}(z) & Es_{1,0}(z) & \cdots & Es_{0,M-1}(z) \\ \vdots & \vdots & \vdots & \vdots \\ Es_{M-1,0}(z) & \cdots & \cdots & Es_{M-1,M-1}(z) \end{bmatrix} \tag{6}$$

with the elements

$$Es_{k,n}(z) = \sum_{m=0}^{L-1} f_k(n+mN)z^{-m} \tag{7}$$

Let $\mathbf{Z}$ be the z transform of the vector at the output of the block $\mathbf{Ps}$, then the analysis is represented as $\mathbf{Y} = \mathbf{XPa}$ and reconstruction process as $\mathbf{Z} = \mathbf{YPs}$. The sub-band filters in a modulated filter bank have the form $h_k(n) = h(n).Ta(n,k)$ where $Ta(n,k)$ is the modulation kernel with $0 \le n \le LM-1$, $0 \le k \le M-1$. Let $Ta(n,k)$ be a block M transform and $\mathbf{Fa}$ be a $M \times M$ matrix such that $\mathbf{Pa} = \mathbf{Fa.Ta}$ and $\mathbf{Ps} = \mathbf{Ts.Fs}$, $\mathbf{Ts}$ and $\mathbf{Fs}$ being the inverses of $\mathbf{Ta}$ and $\mathbf{Fa}$ respectively. The block transform chosen is

$$Ta_{nk} = c_k \sqrt{\frac{2}{M}} \cos\left(\frac{\pi}{M} k \left(n + 0.5\right)\right) \tag{8}$$

$c_k = \frac{1}{\sqrt{2}}$ for $k = 0$, $c_k = 1$ for $k \ne 0$

$0 \le k \le M-1,\ 0 \le n \le M-1$

The modulated analysis filters considered are

$$h_k(n) = c_k \sqrt{\frac{2}{M}} h(n) \cos\left(\frac{\pi k}{M}\left(n - \frac{N+M-1}{2}\right)\right) \tag{9}$$

For simplicity consider an even channel filter bank with filters of length $2mM$ where m is a positive integer. It is easy to verify that all the analysis filters have linear phase.

The Type I 2M-polyphase representation of the prototype filter H(z) is

$$H(z) = \sum_{n=0}^{2M-1} G_n\left(z^{2M}\right) z^{-n} \tag{10}$$

The Type I polyphase matrix of the analysis section is [1],

$$\mathbf{E(z)} = \mathbf{C} \begin{bmatrix} \mathbf{g}_0(z^2) \\ \mathbf{g}_1(z^2) z^{-1} \end{bmatrix} \tag{11}$$

where $\mathbf{g}_0(z)$ =diag($G_0(z)\ G_1(z)\ ....G_{M-1}(z)$)

$\mathbf{g}_1(z)$ =diag($G_M(z)$ ....$G_{2M-1}(z)$

$$[C\ ]_{kn} = \sqrt{\frac{2}{M}} c_k \cos\left(\frac{\pi k}{M}\left(n - \left(\frac{N+M-1}{2}\right)\right)\right) \tag{12}$$

$k = 0,1...M-1$

$n = 0,1........2M-1$

Therefore from (4) and (5)

$$\mathbf{Pa} = z^{-(2m-1)}[\tilde{\mathbf{g}}_0(z^2)\ \ z\tilde{\mathbf{g}}_1(z^2)]\mathbf{C}^T \tag{13}$$

and

$$\mathbf{Fa} = z^{-(2m-1)}[\tilde{\mathbf{g}}_0(z^2)\ \ z\tilde{\mathbf{g}}_1(z^2)]\mathbf{C}^T\mathbf{Ta}^{-1} \tag{14}$$

$$\text{but } \mathbf{C}^T\mathbf{Ta}^{-1} = \begin{bmatrix} \mathbf{0}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & \mathbf{J}_{\frac{M}{2}} \\ \mathbf{J}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{I}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \end{bmatrix} \tag{15}$$

Let $z^{-(2m-2)}\tilde{G}_k(z^2) = P_k(z)$

Therefore $\mathbf{Fa}$ is sparse and takes a diamond structure of the form

$$\mathbf{Fa} = \begin{bmatrix} 0 & 0 & P_M(z) & z^{-1}P_0(z) & 0 & 0 \\ & \iddots & 0 & 0 & \ddots & 0 \\ P_{M+\frac{M}{2}-1} & 0 & & & 0 & z^{-1}P_{\frac{M}{2}-1}(z) \\ P_{M+\frac{M}{2}}(z) & 0 & & & 0 & z^{-1}P_{\frac{M}{2}}(z) \\ & \ddots & 0 & 0 & \iddots & \\ 0 & & P_{2M-1}(z) & z^{-1}P_{M-1}(z) & & 0 \end{bmatrix} \tag{16}$$

$P_k(z)$ can also be expressed as

$$P_k(z) = \sum_{l=0}^{m-1} h(2lM+k)z^{-2(m-1-l)} \tag{17}$$

Since $\mathbf{Ts} = \mathbf{Ta}^T$, for perfect reconstruction, only the conditions for the existence of $\mathbf{Fa}^{-1}$ are to be investigated. A direct inverse leads to IIR synthesis filters. To get FIR solutions certain conditions are to be imposed on the prototype filters as is shown below. Equation (16) can be written as $\mathbf{Fa} = \hat{\mathbf{J}}\,\mathbf{FD}$, where

$$\mathbf{F}=\begin{bmatrix} P_{M+\frac{M}{2}-1} & 0 & & & 0 & P_{\frac{M}{2}-1}(z) \\ 0 & \ddots & 0 & 0 & ⋰ & 0 \\ & 0 & P_M(z) & P_0(z) & 0 & \\ & 0 & P_{2M-1}(z) & P_{M-1}(z) & 0 & \\ 0 & ⋰ & 0 & 0 & \ddots & 0 \\ P_{M+\frac{M}{2}}(z) & 0 & & & 0 & P_{\frac{M}{2}}(z) \end{bmatrix} \quad (18)$$

$$\mathbf{D}=\begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & z^{-1} & & \\ & & & & \ddots & \\ & & & & & z^{-1} \end{bmatrix} \quad (19)$$

$$\hat{\mathbf{J}}=\begin{bmatrix} \mathbf{J}_{\frac{M}{2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\frac{M}{2}} \end{bmatrix} \quad (20)$$

Both $\mathbf{D}$ and $\hat{\mathbf{J}}$ have simple inverses. . For a linear phase prototype filter

$$P_k(z) = z^{-2(m-1)} P_{2M-1-k}(z^{-1}) \quad (21)$$

The determinant of $\mathbf{F}$ is

$$z^{-M(m-1)}\left(\prod_{k=0}^{\frac{M}{2}-1}(P_{M-k-1}(z)\tilde{P}_{M-k-1}(z) - P_k(z)\tilde{P}_k(z))\right) \quad (22)$$

For perfect reconstruction with FIR synthesis filters the determinant should be a monomial and $\mathbf{Fa}.\mathbf{Ta}.\mathbf{Ts}.\mathbf{Fs} = cz^{-d}\mathbf{I}$ where d is the delay to make the system causal and c a non-zero constant . Applying these conditions and solving for the inverse and using (21),

$$\mathbf{F}^{-1} = z^{-d}\begin{bmatrix} P_{\frac{M}{2}}(z) & 0 & & & 0 & -P_{\frac{M}{2}-1}(z) \\ 0 & \ddots & 0 & 0 & ⋰ & 0 \\ & 0 & P_{M-1}(z) & -P_0(z) & 0 & \\ & 0 & -P_{2M-1}(z) & P_M(z) & 0 & \\ 0 & ⋰ & 0 & 0 & \ddots & 0 \\ -P_{M+\frac{M}{2}}(z) & 0 & & & 0 & P_{M+\frac{M}{2}-1}(z) \end{bmatrix}$$

(23

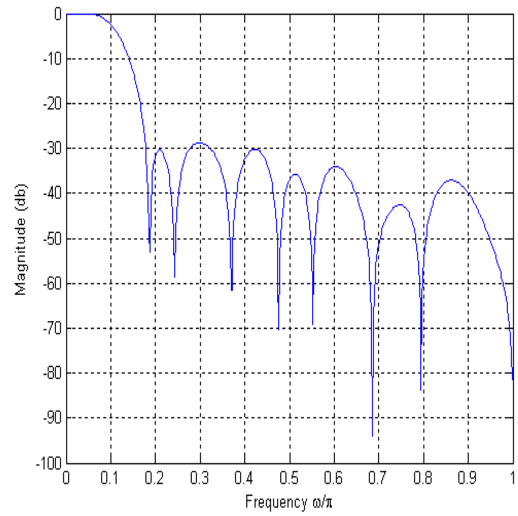

Fig.2 Frequency response of the prototype filter

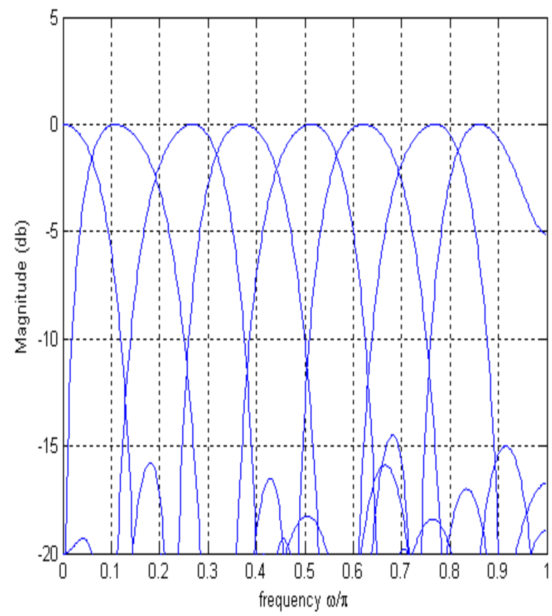

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Fig.3. Frequency responses of analysis filters

if $P_{M-k-1}(z)\tilde{P}_{M-k-1}(z) - P_k(z)\tilde{P}_k(z) = 1$,

$$0 \le k \le \frac{M}{2} - 1 \quad (24)$$

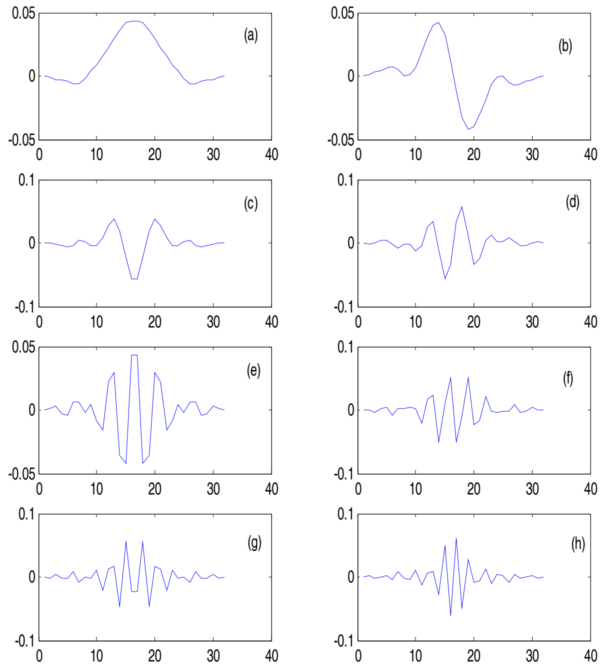

Fig.4. Impulse response of analysis filters.

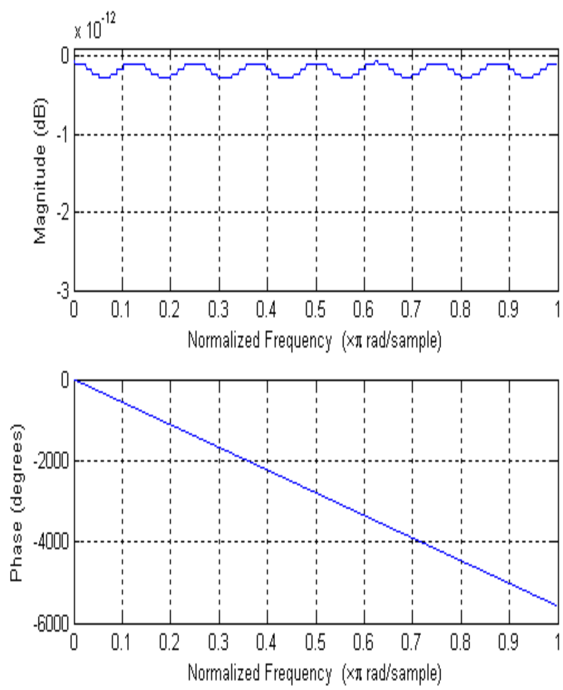

Fig.5 Transfer function

Since the prototype filter is symmetric from (21) and (23) the coefficients in both $\mathbf{F}$ and $\mathbf{F}^{-1}$ are the same and the matrix has coefficient symmetry. Therefore from (9) and (23) the synthesis filters are also modulated versions of the same prototype filter and have linear phase. Causality is achieved by introducing sufficient delay into (23). So the system is PR if (24) is satisfied.

4. Prototype filter design

Modified form of the algorithms described in [11] may be used for the prototype filter design with constraint (24). The frequency response of a linear phase prototype filter satisfying the PR condition (24), obtained for M=8, length=32 taps, is shown in Fig.2. Frequency response and impulse response of the analysis filters are shown in Fig 3 and Fig.4 respectively.. The overall transfer function of the filter bank is shown in Fig.5.

5. Conclusion

The theory and design of a class of M -channel cosine modulated filter banks with linear phase analysis and synthesis filters is presented. The filter bank uses only one prototype filter for both analysis and synthesis and structurally guarantees perfect reconstruction property. Since DCT-II is used for the formulation, efficient implementation and wide range of applications are possible. Results show that prototype filters with good response and PR can be obtained with the constraints derived.

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