

# The Minimum-Energy Broadcast Problem in Symmetric Wireless Ad Hoc Networks

FREDRICK MTENZI YINGYU WAN

Computer Science Department

Dublin Institute of Technology

School of Computing, DIT Kevin Street, Dublin 8

IRELAND

<http://www.comp.dit.ie/fmtenzi>

*Abstract:* - Minimizing energy consumption in communication is a crucial problem in wireless ad hoc networks, as in most cases the nodes are powered by battery only. The minimum-energy broadcast problem is studied in this paper, for which it is well known that the broadcast nature of the radio transmission can be exploited to optimize energy consumption. This problem has been studied in a lot of literature on different models. In this paper a symmetric network is considered. First we propose an approximation algorithm, which takes  $O(mn\alpha(m,n))$  time, where  $m$  is the number of links,  $n$  is the number of nodes and  $\alpha$  is the inverse of Ackerman's function. The algorithm delivers a broadcast tree with energy consumption being at most  $2H_{n-1}$  times of the optimal solution, where  $H_{n-1}$  is the  $(n-1)^{st}$  harmonic number. Since it has been proved that the minimum energy broadcast problem in general graph case (including symmetric case) cannot be approximated within a sub-logarithmic factor (unless  $P=NP$ ), so the algorithm is almost optimal. For a special case where each node is equipped with the same type of battery it improves the known  $O(\log^3 n)$ -approximation algorithm. Moreover for some asymmetric but nearly symmetric network, the algorithm can also be applied with  $O(\ln n)$  performance guarantee. Finally a special case is studied, where the degree of network is bounded by a constant  $\Delta$  and the ratio of the maximum transmission energy to the minimum transmission energy is bounded by another constant  $C$ . For the case we devise a  $2(C + H_{\Delta-1})$ -approximation algorithm.

*Key-Words:* - Broadcast, wireless network, ad hoc network, approximation algorithm, energy efficient

## 1 Introduction

In recent years, wireless ad hoc networks have received significant attention due to their potential applications in battlefield, emergency disaster relief, large sporting events or congresses [1,2,3,4]. Unlike wired networks or cellular networks, no wired backbone infrastructure is installed in wireless ad hoc networks. A communication is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. Each node in such a network has a limited energy resource (battery), and each node operates unattended. Consequently, energy efficiency is an important design consideration for these networks. The problem of minimizing the energy consumption has been studied intensively in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

The wireless ad hoc network is a distributed system intrinsically, so broadcast is an important communication primitive. In addition, many routing

protocols for such networks need a broadcast mechanism to update their states and maintain the routes between nodes [15]. In this paper, we focus on source-initiated broadcasting of data. Data are distributed from a source node to each node in a network. The main objective is to construct a minimum-energy broadcast tree rooted at the source node. Nodes belonging to a broadcast tree can be divided into two categories: relay nodes and leaf nodes. The relay nodes are those that relay data by transmitting it to other nodes (relaying or leaf), while leaf nodes only receive data. The total energy consumption of a broadcast tree is the sum of the transmission energy on all the relay nodes (including the source node). Due to the broadcast nature of the wireless channel, each node can transmit at different power levels and thus reach a different number of neighboring nodes. So a crucial issue is to trade off between reaching more nodes in a single hop using higher power and reaching fewer nodes using lower power.

The minimum-energy broadcast problem was introduced by Wieselthier et al. in [16] and has been studied in many other literatures [17, 18, 19, 20, 21, 22]. In [16] Wieselthier et al. proposed three heuristics: BLiMST(Broadcast Link-based MST), BLU(Broadcast Least-Unicast-cost) and BIP(Broadcast Incremental Power), and evaluated them through simulations. In [22] Wan et al. gave the first analytical results. For the geometric plane case, by exploring geometric structures of an Euclidean minimum spanning tree, they proved that the approximation ratio of BLiMST is between 6 and 12, and the approximation ratio of BIP is between  $\frac{13}{3}$  and 12. For the general graph case,

Wan et al. also proved that the problem is NP-hard, and even more, inapproximable within a factor of  $(1 - \varepsilon) \log \Delta$ , where  $\Delta$  is the maximal degree and  $\varepsilon$  is any arbitrary small positive constant unless  $NP \subseteq DTIME[n^{O(\log \log n)}]$  by an approximation-preserving reduction from the connected dominating set problem. In [19], Clementi et al. studied the minimum-energy broadcast problem on linear networks and proved it can be solved in polynomial time. The problem on higher dimensions was studied in [18] where the problem was proved to be NP-hard. In [17], Cagalj et al. also proved that for the graph case the problem is NP-hard and cannot be approximated better than  $O(\log \Delta)$ . The proof was done by a reduction from the set cover problem. Moreover Cagalj et al. proved the problem in two-dimensional Euclidean metric space is NP-Complete. A similar proof was given by Egecioglu et al. in [20]. In [21], Liang studied the general graph case. Liang proved the minimum-energy broadcast problem on general graphs is NP-complete by reduction from 3SAT. For any asymmetric network, Liang proposed an approximation algorithm. The main idea behind Liang's algorithm is to reduce the minimum-energy broadcast problem to the directed steiner tree problem on an auxiliary weighted graph. The approximation ratio of his algorithm is  $O(n^\varepsilon)$ , where  $\varepsilon$  is constant with  $0 < \varepsilon \leq 1$ . For a special case where each node is equipped with the same type of battery, Liang gave another approximation algorithm with better performance ratio, which is  $O(\log^3 n)$ .

In this paper we consider the minimum-energy broadcast problem on symmetric wireless ad hoc networks. The major contributions are as follows.

We propose an approximation algorithm. The time complexity of it is  $O(mn\alpha(m, n))$ , where  $m$  is the number of links,  $n$  is the number of nodes and  $\alpha$  is the inverse of Ackerman's function. The energy consumption of the broadcast tree delivered by the algorithm is at most  $2H_{n-1}$  times of the optimal solution, where

$$H_{n-1} = \sum_{1 \leq i \leq n-1} \frac{1}{i} \text{ is the } (n-1)^{\text{st}} \text{ harmonic number.}$$

Taking into consideration the known hard result on the approximability of the broadcast problem for the general graph case, our algorithm is almost optimal. Note that the special case studied in [21] where each node is equipped with the same type of battery is in fact a symmetric case. Our algorithm improves the  $O(\log^3 n)$ -approximation algorithm in [21]. In addition, for some asymmetric but nearly symmetric network, if the symmetry is evaluated by a constant  $K$ , our algorithm can also be applied with  $(K+1)H_{n-1}$  performance guarantee. Finally a special case is studied, where the degree of the network is bounded by a constant  $\Delta$  and the ratio of the maximum transmission energy to the minimum transmission energy is bounded by another constant  $C$ . For the case we devised a  $2(C + H_{\Delta-1})$ -approximation algorithm. Since  $C$  and  $\Delta$  are constant, the algorithm is constant approximation.

The rest of the paper is organized as follows. In Section 2 we introduce the network model and define the minimum-energy broadcast problem on the model. In Section 3 we propose an algorithm for the symmetric case and analyze the performance of the algorithm. We also generalize the algorithm to the nearly symmetric case. Then we devise a constant approximation algorithm for a special case in Section 4. Finally the conclusion is given in Section 5.

## 2 Preliminaries

### 2.1 Network Model

We first give a wireless ad hoc network model and then, based on it we develop a graph model, which will be used to define the minimum-energy broadcast problem formally.

In our model of a wireless ad hoc network, nodes are stationary. We do not consider mobility in this paper. We assume the availability of a large number of bandwidth resources, i.e., communication

channels. This is so because, in this paper, we are focused only on minimum-energy broadcast communication and do not consider issues like contention for the channel, lack of bandwidth resources etc. We also assume that nodes in a network are equipped with omni directional antennas. Thus by a single transmission of a transmitting node, due to the broadcast nature of wireless channels, all nodes that fall in the transmission range of the transmitting node can receive its transmission. This property of wireless media is called *Wireless Multicast Advantage*. Each node can adjust its transmission energy to communicate with different nodes. If a network is symmetric, then for any two nodes  $u$  and  $v$ , if  $u$  falls in the transmission range of  $v$  when  $v$  is transmitting with energy  $w$ , then  $v$  must fall in the transmission range of  $u$  when  $u$  transmits with the same energy.

Next, we model a symmetric wireless ad hoc network as an undirected graph  $G=(V,E)$ ,  $V$  is the set of nodes.  $E$  is the set of single-hop links. If node  $u$  and  $v$  can communicate with each other in single hop, there is an edge  $(u,v) \in E$ . And a weight  $w(u,v)$  is assigned to  $(u,v)$ , which corresponds the transmission energy for a unit transmission from  $u$  to  $v$  or from  $v$  to  $u$ . For any directed subgraph  $F$  of  $G$ , define the weight on node  $u \in V(F)$  as  $w_F(u) = \max\{w(u,v) : (u,v) \in E(F)\}$  and the weight of  $F$  as  $w_F = \sum_{v \in V(F)} w_F(v)$ , which represents the energy consumption in  $F$ .

## 2.2 The Minimum-Energy Broadcast Problem

Given a symmetric wireless ad hoc network and a source node, the minimum-energy broadcast problem is to broadcast a unit message from the source node to all the other nodes such that the sum of transmission energy at all nodes is minimized. Based on the graph model in section 2.1, the problem can be defined formally as follows.

**Definition 1** (*Minimum-Energy Broadcast Problem, MEBP*) Given a connected undirected graph  $G=(V,E)$  with an edge weight function  $w: E \rightarrow \mathbb{R}^+$  and a source node  $s \in V$ , construct a directed spanning tree (arborescence)  $T$  rooted at  $s$  and spanning all the other nodes such that  $w_T$  is minimized.

**Theorem 1** [22], [17] *MEBP is NP-Complete and cannot be approximated better than  $O(\log \Delta)$  unless  $P=NP$ , where  $\Delta$  is the degree of the network.*

## 3 Approximation Algorithm

### 3.1 Symmetric Network

In this section we devise an approximation algorithm for MEBP. The algorithm consists of two stages: firstly a weakly connected subgraph containing all the nodes is constructed. Then an arborescence rooted at  $s$  is constructed based on the subgraph.

Given a symmetric wireless ad hoc network  $G(V,E)$ , a greedy algorithm is proposed to construct a weakly connected subgraph  $G'=(V',E')$ , which is a directed graph satisfying  $V'=V$  and the undirected graph underlying  $G'$  is a subgraph of  $G$ .

The algorithm starts with an empty set of edges and constructs the subgraph gradually. At each step, a node is selected and some directed edges away from it are added to  $G'$ . The algorithm ends after  $G'$  is weakly connected.

A crucial issue of the algorithm is the rule to select nodes and edges. We use a greedy rule in the algorithm. Assume at some step,  $G'$  is not yet weakly connected, but contains some weakly connected components. For node  $v \in V$ , if we add some directed edges away from  $v$  to  $G'$ , the weight of  $v$  will increase while the number of weakly connected components in  $G'$  will decrease. Normally, the more the weight of  $v$  increases, the more the number of components decreases. Define the cost of such an operation as the ratio of the increased weight to the decreased number of components. Obviously for each node  $v$ , a best operation on it with the minimal cost exists, which corresponds the most efficient growing of  $G'$  through  $v$ . The greedy rule is to select the node with the global minimum cost and add edges according to the cost.

The detail of the algorithm is described as follows.

---

**Algorithm 1** Greedy algorithm to construct a weakly connected graph

---

**Input:** A symmetric wireless ad hoc network  $G(V,E)$

**Output:** A weakly connected subgraph

$G'(V', E')$  of  $G$

**Begin**

1. Let  $G' = (V, \emptyset)$ .
2. **while**  $G'$  is not weakly connected **do**
3. **for** every node  $v \in V$  **do**
4. compute the minimal cost  $c_v$  of  $v$
5. **end for**
6. Select the node with global minimum cost  $c_{\min} = \min\{c_v : v \in V\}$ .
7. Add directed edges corresponding to  $c_{\min}$  to  $G'$ .
8. **end while**

**End**

**Theorem 2** *The time complexity of Algorithm 1 is  $O(mn\alpha(m, n))$ , where  $n = |V|, m = |E|$  and  $\alpha$  is the inverse of Ackerman's function.*

*Proof:* To implement Algorithm 1, we use the data structure for disjoint sets [23] to compute the number of connected components. Using the path compression and union-by-rank heuristic for  $t$  Union-Find-Operations on  $s$  elements one obtains a running time of  $O(t\alpha(t, s))$ .

In Algorithm 1, the *while* iteration will run at most  $O(n)$  times. Let  $m_v$  be the number of edges incident to node  $v$ . For node  $v$ , step 4 can be carried out by  $O(m_v)$  Union-Find-Operations on the set of the weakly connected components, whose cardinality is no more than  $n$ . So in each *while* iteration, the total time of step 3 to 5 is

$$O(\sum_{v \in V} m_v \alpha(m_v, n)) \subseteq O(\alpha(m, n) \sum_{v \in V} m_v).$$

$\sum_{v \in V} m_v = 2m$  and step 3 to 5 are the dominating steps. So the work in each *while* iteration can be bounded by  $O(m\alpha(m, n))$ . The time complexity of the algorithm is  $O(m\alpha(m, n))$ .

Now we show the relationship between  $G'$  and the optimal solution of MEBP.

**Lemma 1** *The weight of the weakly connected graph  $G'$  is at most  $H_{n-1}$  times of the optimal solution of MEBP.*

*Proof:* Assume there are  $g$  *while* iterations in Algorithm 1. Let  $\text{cost}_i$  and  $n_i$  be the cost and the

decreased number of weakly connected components corresponding to the  $i^{\text{th}}$  iteration respectively. Then  $w_i = n_i \text{cost}_i$  is the increased weight of  $G'$  after the  $i^{\text{th}}$  iteration. From the algorithm, it is easy to see that:

$$\sum_{1 \leq i \leq g} n_i = n - 1 \quad (1)$$

and at the end of the algorithm, the weight of  $G'$  is:

$$w_{G'} = \sum_{1 \leq i \leq g} w_i = \sum_{1 \leq i \leq g} n_i \text{cost}_i \quad (2)$$

Let  $T_{OPT}$  be the optimal arborescence,  $w_{T_{OPT}}$  be the weight of  $T_{OPT}$ . Without loss of generality, let  $v_1, v_2, \dots, v_k$  be the internal nodes of  $T_{OPT}$  and  $n_{v_1}, n_{v_2}, \dots, n_{v_k}$  be the number of children of  $v_1, v_2, \dots, v_k$  respectively. Define

$\text{cost}_{v_i} = w_{T_{OPT}}(v_i) / n_{v_i}$  be the cost of  $v_i$ . Assume

$$\text{cost}_{v_1} \leq \text{cost}_{v_2} \leq \dots \leq \text{cost}_{v_k} \quad (3)$$

Since

$$w_{T_{OPT}} = \sum_{1 \leq i \leq k} w_{T_{OPT}}(v_i) = \sum_{1 \leq i \leq k} n_{v_i} \text{cost}_{v_i} \geq \sum_{1 \leq i \leq k} n_{v_i} \text{cost}_{v_1}$$

and

$$\sum_{1 \leq i \leq k} n_{v_i} = n - 1$$

so we get

$$\text{cost}_{v_1} \leq w_{T_{OPT}} / (n - 1)$$

From the greedy rule of Algorithm 1, it is easy to see that  $\text{cost}_{t_1} \leq \text{cost}_{v_1} \leq w_{T_{OPT}} / (n - 1)$ .

After the first iteration, there are  $n - n_1$  components in  $G'$ . By a similar analysis, we can get that:

$$\text{cost}_{t_2} \leq w_{T_{OPT}} / (n - n_1 - 1),$$

and more for  $1 \leq i \leq g$

$$\text{cost}_{t_i} \leq w_{T_{OPT}} / (n - 1 - \sum_{1 \leq j \leq i-1} n_j).$$

Thus

$$\begin{aligned} w_{G'} &= \sum_{1 \leq i \leq g} n_i \text{cost}_{t_i} \\ &\leq w_{T_{OPT}} \sum_{1 \leq i \leq g} n_i / (n - 1 - \sum_{1 \leq j \leq i-1} n_j) \\ &= w_{T_{OPT}} \left( \frac{n_1}{n-1} + \frac{n_2}{n-n_1-1} + \dots + \frac{n_g}{n - \sum_{1 \leq j \leq g-1} n_j - 1} \right) \end{aligned}$$

$$\leq w_{T_{OPT}} \left( \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{1} \right) w_{T_{OPT}} H_{n-1},$$

which means that the weight of  $G'$  delivered by Algorithm 1 is at most  $H_{n-1}$  times of the optimal solution of MEBP.  $\square$

After Algorithm 1,  $G'$  is a weakly connected subgraph of  $G$ . Now we compute an arborescence  $T$  rooted at  $s$  and spanning  $V$  from  $G'$ . First a weakly connected spanning tree  $T'$  is generated in  $G'$  arbitrarily. Obviously the weight of  $T'$  is no more than  $G'$ , i.e.,  $w_{T'} \leq w_{G'}$ . Then we convert  $T'$  to an arborescence  $T$  rooted at  $s$  by changing the direction of some edges in  $T'$ .

**Lemma 2**  $w_T \leq 2w_{T'}$

*Proof.* Let  $\langle u, v \rangle \in E(T')$ . By changing the direction of  $\langle u, v \rangle$ , we get a new tree  $T''$ . From the definition of the weight on nodes, there are

$$w_{T''}(u) \leq w_{T'}(u) \tag{4}$$

and

$$\begin{aligned} w_{T''}(v) &= \max\{w_{T'}(v), w(u,v)\} \\ &\leq \max\{w_{T'}(v), w_{T'}(u)\} \end{aligned} \tag{5}$$

Define  $children_T(v)$  be the set of children of  $v$  in  $T$ . Since  $T$  is an arborescence, the direction of every edge is away from  $s$ . So from the equations (4) and (5), we get

$$\begin{aligned} w_T(v) &\leq \max\{w_{T'}(u) : u \in children_T(v) \cup \{u\}\} \\ &\leq \sum_{u \in children_T(v) \cup \{v\}} w_{T'}(u) \end{aligned}$$

For any  $u \neq v$ ,  $children_T(u)$  and  $children_T(v)$  are disjoint, which means any node appears in at most one of the children sets. So

$$\begin{aligned} w_T &= \sum_{v \in V} w_T(v) \\ &\leq \sum_{v \in V} \left( \sum_{u \in children_T(v) \cup \{v\}} w_{T'}(u) \right) \\ &= \sum_{v \in V} (w_{T'}(v) + \sum_{u \in children_T(v)} w_{T'}(u)) \\ &= \sum_{v \in V} w_{T'}(v) + \sum_{v \in V} \left( \sum_{u \in children_T(v)} w_{T'}(u) \right) \\ &\leq 2 \sum_{v \in V} w_{T'}(v) \\ &= 2w_{T'} \end{aligned}$$

The lemma is yielded.  $\square$

From Lemma 1, Lemma 2 and  $w_{T'} \leq w_{G'}$ , we get the following theorem.

**Theorem 3** *Given any MEBP instance with  $n$  nodes, the above algorithm is  $2H_{n-1}$ -approximation.*

Since  $H_n \leq \ln n + 1$  for  $n \geq 1$ , so we can get the following corollary:

**Corollary 1** *Given any MEBP instance with  $n$  nodes, the above algorithm is  $2\ln(n-1) + 2$ -approximation.*

Note that in the second stage of the algorithm, the construction of  $T$  can be done through a depth search, which can be carried out in  $O(n)$  time. And To get the arborescence  $T$  from  $T'$ , at most  $n$  edges will be changed direction. So the time of the second stage is  $O(n)$ . The time complexity of the algorithm is  $O(mn\alpha(m, n)) + O(n) = O(mn\alpha(m, n))$ .

For general cases, from Theorem 1, we know MEBP cannot be approximated better than  $O(\log n)$ . So our algorithm is almost optimal. In [21] Liang investigated a special but practical case of broadcasting in wireless ad hoc networks, where each node is equipped with the same type of battery. As indicated in [21], in this case, the network has the symmetric property, i.e., the case is a special case of symmetric networks studied above. For the case [21] proposed an approximation algorithm which is  $O(\log^3 n)$ -approximation. Undoubtedly, our algorithm can be applied in the case and improve the known result.

### 3.2 Nearly Symmetric Network

The above algorithm can be generalized to solve the minimum-energy broadcasting problem in asymmetric but nearly symmetric wireless ad hoc networks. In an asymmetric network, define  $w \langle u, v \rangle$  be the energy consumption of  $u$  for unit transmission from  $u$  to  $v$ . For any  $u, v \in V$ , if  $w \langle u, v \rangle / w \langle v, u \rangle$  is bounded by a constant  $K$ , the network is referred to as a nearly symmetric network with constant  $K$ . Our algorithm can be applied to this case without any modification. But the performance analysis is a little different. For the first stage, Algorithm 1, Lemma 1 still holds.

But for the second stage, when we compute  $T$  from  $T'$ , Lemma 2 should be modified as follows.

**Lemma 3**  $w_T \leq (K + 1)w_{T'}$ .

*Proof:* The proof is similar to in Lemma 2. The equation (5) is replaced by the following equation.

$$w_{T'}(v) = \max\{w_{T'}(v), w_{\langle v, u \rangle}\} \leq \max\{w_{T'}(v), Kw_{T'}(u)\} \quad (6)$$

So we have

$$w_T(v) \leq \max\{Kw_{T'}(u) : u \in \text{children}_{T'}(v)\} \cup \{w_{T'}(v)\} \leq w_{T'}(v) + \sum_{u \in \text{children}_{T'}(v)} Kw_{T'}(u)$$

and

$$\begin{aligned} w_T &= \sum_{v \in V} w_T(v) \\ &\leq \sum_{v \in V} (w_{T'}(v) + \sum_{u \in \text{children}_{T'}(v)} Kw_{T'}(u)) \\ &= \sum_{v \in V} w_{T'}(v) + \sum_{v \in V} (\sum_{u \in \text{children}_{T'}(v)} Kw_{T'}(u)) \\ &\leq (K + 1) \sum_{v \in V} w_{T'}(v) \\ &= (K + 1)w_{T'}. \end{aligned}$$

□

So we get the following theorem.

**Theorem 4** Given any nearly symmetric network with constant  $K$  and  $n$  nodes, the minimum-energy broadcasting problem is  $(K + 1)H_{n-1}$ -approximated.

### 4 Special Case with Constant Performance Guarantee

In practice, as pointed out in many literature, the transmission energy at each node cannot be adjusted infinitely. There exist a maximum energy and a minimum energy. Denote the maximum edge weight in a symmetric network by  $w_{\max}$  and the minimum by  $w_{\min}$ . Let  $w_{\max} / w_{\min} = C$  and the degree of the network be  $\Delta$ . If  $C$  and  $\Delta$  are constant, we propose an algorithm with constant performance guarantee.

For this special case, a simple algorithm is as follows:

**step 1:** Ignoring the edge weight, compute a connected dominating set of  $G$ .

**step 2:** Based on the connected dominating set, construct an aborescence  $T$ . First, a spanning tree is constructed, in which all the interior nodes are in the dominating set. Then the edges in the tree are directed away from  $s$ .

Using the approximation algorithms of [24] in step 1, we get that the number of interior nodes in  $T$  is at most  $O(H_\Delta)$  times of the optimal tree. So the weight of  $T$  is at most  $O(CH_\Delta)$  times of the optimal tree. In the following, we devise another algorithm, which is a little similar with Algorithm I of [24]. The performance of the algorithm is better than the above algorithm.

The idea is to grow a tree  $T$ , starting from the source node  $s$  by an iterative procedure. At each step a node  $u$  in  $T$  is picked and scanned. By scanning a node, some new nodes and edges are inserted into  $T$ . Finally we will get an arborescence  $T$  rooted at  $s$  and spanning all the other nodes.

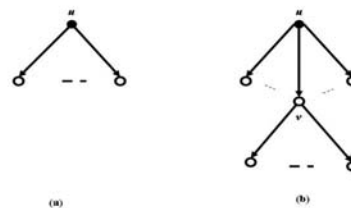


Fig. 1: Two ways to scan nodes

There are two ways to scan a node, as shown in Figure 1. In the first way (Figure 1(a)), a node  $u$  in  $T$  is scanned. Some neighboring nodes of  $u$  are inserted into  $T$ . In the second way (Figure 1(b)), a node  $u$  in  $T$  and a neighboring node  $v$  of  $u$  are scanned. Node  $v$  and some neighboring nodes of  $u$  or  $v$  are inserted into  $T$ . Similarly to Section 3, define the cost of such an operation as the ratio of the increased weight of  $T$  to the number of nodes inserted into  $T$ . For any node  $u$  in  $T$ , there exists a minimal cost, denoted as  $\text{cost}_u$ . The rule in the algorithm is to pick the node with the global minimum cost and scan it according to the cost.

Initially, node  $s$  is black, while all the other nodes are white. At each iteration, for all black nodes, its

minimal cost is computed. Then the black node with global minimum cost is picked. Some white nodes are inserted into  $T$  and colored black. Those white nodes may be neighbors of or neighbors of a neighbor of the black node. The algorithm ends after there is no white node, i.e., all white nodes have been inserted into  $T$ . A formal description of the algorithm is given in Algorithm 2.

---

**Algorithm 2**

---

**Input:** A symmetric network  $G(V,E)$  with Constants  $C$  and  $\Delta$  and source  $s \in V$   
**Output:** An arborescence  $T$  rooted as  $s$   
**Begin**  
 1. Let  $T = (\{s\}, \emptyset)$ .  
 2. **while**  $T$  does not span  $V$  **do**  
 3. **for** every node  $v \in V(T)$  **do**  
 4. compute the minimal cost  $c_v$  of  $v$   
 5. **end for**  
 6. Select the node with global minimum cost  $c_{\min} = \min\{c_v : v \in V(T)\}$  from  $V(T)$ .  
 7. Insert the nodes and directed edges corresponding to  $c_{\min}$  to  $T$ .  
 8. **end while**  
**End**

---

**Theorem 5** Given any symmetric wireless ad hoc network with constants  $C$  and  $\Delta$ , Algorithm 2 yields a broadcasting tree whose energy consumption is at most  $2(C + H_{\Delta-1})$  times of the minimum energy broadcasting tree.

*Proof:* Let  $T_{OPT}$  be the optimal tree. For any node  $v \in V$ , let  $children(v)$  be the set of children of  $v$  in  $T_{OPT}$ . For  $v$  being a leaf,  $children(v) = \emptyset$ . Every node except for  $s$  belongs to one and only one of the children sets.

The proof is based on a charging scheme. Each time we scan a black node; we insert some white nodes into  $T$  and color them black. We charge each of these inserted white nodes in this step. Since each node except for  $s$  is colored and inserted exactly once, it is charged exactly once. Node  $s$  will not be charged. For any node  $v \in V \setminus \{s\}$ , denote  $\delta_v$  as its charge. If in the iteration  $v$  is colored and inserted, the global minimum cost is  $c_{\min}$ , then charge each newly colored node  $c_{\min}$ , i.e.,  $\delta_v = c_{\min}$ . Assume

there are  $k$  iterations in the algorithm and  $V_i$  is the set of newly colored nodes in the  $i^{th}$  iteration. There is the following equation:

$$\begin{aligned} w_T &= \sum_{i=1}^k \sum_{v \in V_i} \delta_v \\ &= \sum_{v \in V \setminus \{s\}} \delta_v \\ &= \sum_{v \in V \setminus \{s\}} \sum_{u \in children(v)} \delta_u \end{aligned} \tag{7}$$

Now we prove the upper bound on the total charges to nodes belonging to a children set  $children(i)$  (for any  $i \in V \setminus \{s\}$ ). Denote  $u_0 = |children(i)|$  and  $u_j$  be the number of white nodes in  $children(i)$  after the  $j^{th}$  iteration. Without loss of generality, assume that at each iteration some nodes of  $children(i)$  are colored, so the number of white nodes in  $children(i)$  decreases at each iteration.

The number of colored nodes in  $children(i)$  after the first iteration is  $u_0 - u_1$ . There are two cases:

**case 1:** Algorithm 2 scans node in the first way (Figure 1(a)). Then each newly colored node gets a charge of at most  $\frac{Cw_{T_{OPT}}(i)}{u_0 - u_1}$ ;

**case 2:** Algorithm 2 scans node in the second way (Figure 1(b)). Since there are two nodes being scanned, so each newly colored node gets a charge of at most  $\frac{2Cw_{T_{OPT}}(i)}{u_0 - u_1}$ .

Once any node in  $children(i)$  is colored black, node  $i$  becomes a candidate node to be scanned as a part of the two nodes in the second way, since it is adjacent to a black node. In the  $j^{th}$  iteration, the number of nodes of  $children(i)$  that are colored is  $u_{j-1} - u_j$ . The charge to each newly colored node in

$children(i)$  is at most  $\frac{2Cw_{T_{OPT}}(i)}{u_j - 1}$ , since a scan of  $i$

in the second way is eligible and the network is symmetric. We get

$$\begin{aligned} \sum_{v \in children(i)} \delta_v &\leq \frac{2Cw_{T_{OPT}}(i)}{u_0 - u_1} (u_0 - u_1) + \\ &\sum_{j=2}^k \frac{2w_{T_{OPT}}(i)}{u_j - 1} (u_{j-1} - u_j) \end{aligned} \tag{8}$$

$$\begin{aligned}
 &= 2w_{T_{OPT}}(i)(C + \sum_{j=2}^k \frac{u_{j-1} - u_j}{u_{j-1}}) \\
 &= 2w_{T_{OPT}}(i)(C + \frac{u_1 - u_2}{u_1} + \\
 &\quad \frac{u_2 - u_3}{u_2} + \dots + \frac{u_{k-1}}{u_{k-1}}) (\because u_k = 0) \\
 &\leq 2w_{T_{OPT}}(i)(C + H_{u_i}) \\
 &\leq 2w_{T_{OPT}}(i)(C + H_{\Delta-1}) \\
 &\quad (\because u_1 < u_0 \leq \Delta)
 \end{aligned}$$

From (7) and (8), there is

$$\begin{aligned}
 w_T &\leq \sum_{v \in V \setminus \{s\}} 2w_{T_{OPT}}(v)(C + H_{\Delta-1}) \\
 &= 2(C + H_{\Delta-1})w_{T_{OPT}}.
 \end{aligned}$$

□

Since  $C$  and  $\Delta$  are constant, so the above theorem means that MEBP in the case can be constant approximated.

## 5 Summary and Conclusions

In this paper we have studied the minimum-energy broadcast problem on symmetric wireless ad hoc networks. An almost optimal approximation algorithm has been proposed, which also improves the known result for a special case where each node is equipped with the same type of battery. In addition the algorithm has been generalized to some nearly symmetric network with constant  $K$ . Finally a constant approximation algorithm has been devised for a special case.

In the future we intend to implement simulations of the algorithms proposed in this paper, explore how to implement our algorithm in distributed environment efficiently and study how to cope with the mobility of the nodes.

### References:

[1] Estrin D., Govindan R., Heidemann J., and Kumar S. Next century challenges: Scalable coordination in sensor networks. in *Proceedings of ACM MobiCom*, Seattle, Washington, USA, August, 1999, 263-270.

[2] Hubaux J.P., Gross T., Boudec J.Y.L., and Vetterli M., Towards self-organized mobile ad hoc networks: terminodes project. *IEEE Communications Magazine*, 2001. 39(1).

[3] Pottie G. and Kaiser W., Wireless intergrated network sensors, *Communications of the ACM*, 2000, 43(5): p. 51--58.

[4] Rabaey J., Ammer M., da Silva Jnr J., Patel D., and Roundy S., Picoradio supports ad hoc ultra-low power wireless networking, *IEEE Computer Magazine*, 2000, 33(7): p. 42--48.

[5] Chang J. and Tassiulas L. Energy conserving routing in wireless ad hoc networks. in *Proceedings of IEEE InfoCom 2000*, Tel Aviv, Israel, 2000.

[6] Chang J. and Tassiulas L. Fast approximate algorithms for maximum lifetime routing in wireless ad hoc networks. in *IFIP-TC6/European Commission International Conference, Lecture Notes in Computer Science, Vol 1815*, 2000, pp 702-713.

[7] Heinzlman W., Chandrakasan A., and Balakrishnan H. Energy-efficient communication protocol for wireless mirosensor networks. in *Proceedings of the 33rd Hawaii International Conference on System Science*. January 2000, Maui, Hawaii.

[8] Li L., Bahl V., Wang Y., and Wattenhofer R. Distributed topology control for power efficient operation in multihop wireless ad hoc networks. in *Proceedings of IEEE InfoCom 2001*, Anchorage, Alaska, 2001.

[9] Li Q., Aslam J., and Rus D. Online power-aware routing in wireless ad hoc networks. in *Proceedings of ACM MobiCom*. Rome, Italy, July, 2001, pp 97-107.

[10] Rodoplu V. and Meng T.H., Minimum energy mobile wireless networks. *IEEE Journal on selected Areas in Communications*, 1999. 17(8): pp. 1333--1344.

[11] Schurgers C., Tsiatsis V., Ganeriwal S., and Srivastava M.B., Optimizing sensor networks in the energy-latency-density design space. *IEEE Transactions on Mobile Computing*, 2002. 1(1): pp. 70--80.

[12] Singh S., Raghavendra C., and Stepanek J. Power-aware broadcasting in mobile ad hoc networks. in *Proceedings of IEEE PIMRC'99*. Osaka, Japan, September 1999.

[13] Singh S., Woo M., and Raghavendra C. Power-aware routing in mobile ad hoc networks. in *Proceedings of MobiCom'98*. 1998, pp. 181-190.

[14] Xu Y., Heidemann J., and Estrin D. Geography-informed energy conservation for ad hoc routing. in *Proceedings of ACM MobiCom*, 2001, pp. 70-84.

[15] Royer E.M. and Toh C.K., A review of current routing protocols for ad hoc mobile wireless



- networks. *IEEE Personal Communications*, 1999, 6(2): p. 46--55.
- [16]Wieselthier J.E., Nguyen G.D., and Ephremides A. On the construction of energy-efficient broadcast and multicast trees in wireless networks. in *Proceedings of IEEE InfoCom2000*. 2000. Tel Aviv, Israel.
- [17]Cagalj M., Hubaux J.P., and Enz C. Minimum-Energy Broadcast in All-Wireless Networks: NP-Completeness and Distribution issues. in *Proceedings of ACM MobiCom*. Atlanta, USA, 2002, pp. 586-594.
- [18]Clementi A.E.F., Crescenzi P., Penna P., Rossi G., and Vocca P. On the complexity of Computing minimum energy consumption broadcast subgraphs. in *Proceedings of the 18th Annual Symposium on Theoretical Aspects of Computer Science (STACS), LNCS, Volume 2010* Springer Verlag, 2001, pp. 121-131.
- [19]Clementi A.E.F., Di Ianni M., and Silvestri R., The minimum broadcast range assignment problem on linear multi-hop wireless networks. *Theoretical Computer Science*, To appear.
- [20]Egecioglu O. and Gonzalez T. Minimum-energy broadcast in simple graphs with limited node power. in *Proceedings of IASTED International Conference on Parallel and Distributed Computing and Systems (PDCS 2001)*, Anaheim, CA, August, 2001, pp. 334-338.
- [21]Liang W. Constructing Minimum-Energy Broadcast Trees in Wireless Ad Hoc Networks. in *Proceedings of ACM MobiHoc 2002*. Lausanne, Switzerland, June 2002, pp. 112-122.
- [22]Wan P.J., Calinescu G., Li Y.X., and Frieder O. Minimum-energy broadcast routing in static ad hoc networks. in *Proceedings of IEEE InfoCom2001*, 2001, pp.1162-1171.
- [23]Cormen T.H., Leiserson C.E., and Rivest R.L., *Introduction to algorithms*. 1990: MIT Press.
- [24]Guha S. and Khuller S., Approximate algorithms for connected dominating sets. *Algorithmica*, 1998. 20(4): p. 374--387.