Optimal Layout for Branched Networks using Harmony Search

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Abstract: - A recently-developed meta-heuristic algorithm, harmony search was introduced and applied to layout optimization of branched networks. The algorithm mimics each music player's behavior in improvisation performance to find best engineering harmony, that is, global optimum. The computing results showed that the harmony search algorithm found global optimum, while other algorithms such as genetic algorithm and evolutionary algorithm only found near-optima.

Key-Words: - harmony search, genetic algorithm, optimization, layout, design, branched network, tree network, transportation

1 Introduction

Networks, which deliver water, oil, gas, electricity, or electronic information, play an important role in our daily lives. Out of various networks, this study focuses on the layout geometry of tree-like fluid transportation network.

Since last couple decades, the layout optimization of fluid network has performed using meta-heuristic algorithms: Walters and Lohbeck [1] used genetic algorithm (GA) for the layout problem of a branched network from a directed base graph while Walters and Smith [2] used evolutionary algorithm (EA) for the selection of a branched network from a non-directed base graph; Davidson and Goulter [3] applied EA to layout design of rectilinear branched networks and Davidson [4] applied EA to that of rectilinear looped networks.

In this study, recently-developed meta-heuristic algorithm, harmony search (HS) is introduced and applied to the layout optimization and compared with existing algorithms.

2 Harmony Search Algorithm

The harmony search has the following algorithm steps [5], which has been applied to various engineering optimization problems such as water network design, truss structure design, school bus routing, water pump operation, and hydrologic parameter calibration [6-10].

2.1 Problem and Algorithm Initialization

First, objective function is formulated.

Minimize
$$f(\mathbf{x})$$
 s.t. $x_i \in \mathbf{X}_i, i = 1, 2, \dots, N$ (1)

where $f(\mathbf{x})$ is an objective function; and \mathbf{X}_i is the set of possible candidate values for each decision variable x_i , that is, $\mathbf{X}_i = \{x_i(1), x_i(2), \dots, x_i(K_i)\}$.

The objective function in this study is the cost function for a branched network. The cost function can have the following form if water is delivered through the network [2].

$$f(\mathbf{x}) = \sum_{j=1}^{NL} L_j Q_j^{0.5}$$
(2)

where L_j is the length for the arc j; and Q_j is the flowrate along the arc j.

The candidate value set \mathbf{X}_i for decision variable x_i is the set of candidate adjacent node numbers for current node *i* in this study.

Next, harmony memory (HM) in the form of matrix is filled with as many random vectors as harmony memory size (HMS).

$$\begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{N-1}^1 & x_N^1 & f(\mathbf{x}^1) \\ x_1^2 & x_2^2 & \cdots & x_{N-1}^2 & x_N^2 & f(\mathbf{x}^2) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \cdots & x_{N-1}^{HMS-1} & x_N^{HMS-1} & f(\mathbf{x}^{HMS-1}) \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_{N-1}^{HMS} & x_N^{HMS} & f(\mathbf{x}^{HMS}) \end{bmatrix}$$
(2)

2.2 New Harmony Improvisation

In this step, a new harmony vector, $\mathbf{x}' = (x'_1, x'_2, ..., x'_N)$ is improvised with three rules (1) random selection, (2) HM consideration; and (3) pitch adjustment.

The value for the decision variable x'_i in the new harmony vector can be randomly chosen from entire value range, $\{x_i(1), x_i(2), \dots, x_i(K_i)\}$ in random selection process. Or, the value can be chosen from value specified any in the HM range, $\{x_i^1, x_i^2, \cdots, x_i^{HMS-1}, x_i^{HMS}\}$ in HM consideration process with the probability of harmony memory considering rate (HMCR; $0 \leq$ HMCR \leq 1). The variable, whose value was chosen from HM consideration, has additional chance to be pitch-adjusted with the probability of pitch adjusting rate (PAR; $0 \le PAR \le 1$). Thus, the new value for each decision variable can be chosen as follows:

 $x'_i \leftarrow$

	$x'_i \in \{x_i(1), \dots, x_i(K_i)\}$	<i>w.p</i> .	(1 - HMCR)	
ł	$x'_i, x'_i \in \{x^1_i,, x^{HMS}_i\}$	<i>w.p</i> .	$HMCR \times (1 - PAR)$ (3))
	$x'_{i} + \Delta, x'_{i} \in \{x^{1}_{i}, \dots, x^{HMS}_{i}\}$	<i>w.p</i> .	$HMCR \times PAR$	

For this layout problem, the HS chooses a successive node out of any adjacent nodes (random selection), out of nodes in HM (HM consideration). However, the pitch adjustment process is not used in this study because adjacent nodes do not have any order in magnitude.

2.3 Harmony Memory Update

If the new harmony vector improvised in previous step is better than the worst vector in the HM in terms of the objective function, the new vector is included in the HM and the existing worst vector is excluded from the HM. In order to prevent premature harmony vectors, the number of identical vectors in HM is limited.

After update, if the iteration reached termination criterion (maximum improvisations; MaxImp), the

computation is terminated. Otherwise, the previous step is repeated.

3 Applications

In order to demonstrate the ability of HS, the algorithm is applied to two test networks.

3.1 9-Node Test Network

The HS algorithm for optimal layout is applied to the test network as shown in Figure 1.



Fig. 1 Base Graph of 9-Node Network

The network has 9 nodes and 12 arcs, where the number of total combinations is $2^{12} = 4,096$, and the number of possible networks, obtained by total enumerations, is 192.

Each node has geographical coordinates and water demand as shown in Table 1.

Table 1. Data of 9-Node Network

Node	х	у	Demand
1	0	0	10
2	10	0	20
3	0	10	10
4	20	0	20
5	10	10	10
6	0	20	20
7	20	10	10
8	10	20	20
9	20	20	-120

When the HS tackled the problem with the algorithm parameter values (HMS = 20; HMCR = 1.0; and MaxImp = 50), it could find 7 solutions out of top 10 solutions within 1.2% of total searching space. Moreover, the HS could reach the global optimum (Cost = 427.8118) only after 3 objective function evaluations as shown in Fig. 2.



Fig. 2 Optimum for 9-Node Network

3.2 64-Node Test Network

The HS was also applied to another larger network as shown in Figure 3.



Fig. 3 Base Graph of 64-Node Network

The network has 64 nodes and 112 arcs, where the number of possible networks is 1.26×10^{26} [2]. The

total enumeration of possible networks is technically impracticable. However, the ever-found optimum using exact solution techniques is 5062.8.

Each node has geographical coordinates and water demand as shown in Table 2.

Table 2. Data of 64-Node Network

Node	Х	у	D	Node	Х	у	D
1	0	0	10	33	30	40	10
2	10	0	20	34	20	50	20
3	0	10	10	35	10	60	10
4	20	0	20	36	0	70	20
5	10	10	10	37	70	10	10
6	0	20	20	38	60	20	20
7	30	0	10	39	50	30	10
8	20	10	20	40	40	40	20
9	10	20	10	41	30	50	10
10	0	30	20	42	20	60	20
11	40	0	10	43	10	70	10
12	30	10	20	44	70	20	20
13	20	20	10	45	60	30	10
14	10	30	20	46	50	40	20
15	0	40	10	47	40	50	10
16	50	0	20	48	30	60	20
17	40	10	10	49	20	70	10
18	30	20	20	50	70	30	20
19	20	30	10	51	60	40	10
20	10	40	20	52	50	50	20
21	0	50	10	53	40	60	10
22	60	0	20	54	30	70	20
23	50	10	10	55	70	40	10
24	40	20	20	56	60	50	20
25	30	30	10	57	50	60	10
26	20	40	20	58	40	70	20
27	10	50	10	59	70	50	10
28	0	60	20	60	60	60	20
29	70	0	10	61	50	70	10
30	60	10	20	62	70	60	20
31	50	20	10	63	60	70	10
32	40	30	20	64	70	70	-940

When the HS tackled the problem with the algorithm parameter values (HMS = 2; HMCR = 0.96; and MaxImp = 1500), it could find the assumed the global optimum (5062.8), as shown in Fig. 4, after 1500 function evaluations (1.19 * 10^{-21} % of total searching space) while EA [2] found approximately 5095.3 and GA [1] found 5218.0 after 3200 iterations.

Moreover, the GA required tedious pre-initialization of flow direction on the base graph. However, HS did not request any assumption of flow direction in an arc.



Fig. 4 Optimum for 64-Node Network

4 Conclusions

The recently-developed HS algorithm was introduced and applied to layout optimization of rectilinear branched networks.

The HS algorithm could find the global optimum on 9-node or 64-node network problem within very small amount of total solution space, while other meta-heuristic algorithms such as genetic algorithm and evolutionary algorithm only reached near-optima.

It is also expected that the HS algorithm can be successfully applied to more complicated and realistic networks in the future.

References:

- G. Walters, and T. Lohbeck, Optimal Layout of Tree Networks using Genetic Algorithms, *Engineering Optimization*, Vol.22, 1993, pp.27-48.
- [2] G. Walters, and D. Smith, Evolutionary Design Algorithm for Optimal Layout of Tree Networks, *Engineering Optimization*, Vol. 24, 1995, pp.261-281.
- [3] J. Davidson, and I. Goulter, Evolution Program for Design of Rectilinear Branched Networks, *Journal*

of Computing in Civil Engineering, ASCE, Vol.13, No.4, 1995, pp.246-253.

- [4] J. Davidson, Evolution Program for Layout Geometry of Rectilinear Looped Networks, *Journal of Computing in Civil Engineering*, ASCE, Vol.13, No.4, 1999, pp.246-253.
- [5] Z. Geem, J. Kim, and G. Loganathan, A New Heuristic Optimization Algorithm: Harmony Search, *Simulation*, Vol.76, No.2, 2001, pp.60-68.
- [6] Z. Geem, Optimal Cost Design of Water Distribution Networks using Harmony Search, *Engineering Optimization*, Vol.38, No.3, 2006.
- [7] K. Lee, and Z. Geem, A New Structural Optimization Method Based on the Harmony Search Algorithm, *Computers and Structures*, Vol.82, No.9-10, 2004, pp.781-798.
- [8] Z. Geem, K. Lee, and Y. Park, Application of Harmony Search to Vehicle Routing, *American Journal of Applied Sciences*, Vol. 2, No. 12, 2005, pp.1552-1557.
- [9] Z. Geem, Harmony Search in Water Pump Switching Problem, *Lecture Notes in Computer Science*, Vol.3612, 2005, pp.751-760.
- [10] J. Kim, Z. Geem, and E. Kim, Parameter Estimation of the Nonlinear Muskingum Model using Harmony Search, *Journal of the American Water Resources Association*, Vol.37, No.5, 2001, 1131-1138.