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A Distance Protection Algorithm Based on Recursive Minimum Mean-Square Estimation

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Abstrac:- The paper explores an alternative algorithm for line distance protection based on the combined use of Kalman filter and recursive minimum mean-square estimation. The main advantages of the proposed method consist of great robustness and ultra-high-speed in the fault localization in both 50 Hz and 60 Hz systems. The involved parameters, i.e. the resistance and inductance of the portion of the power line between the relay and fault location, is calculated by processing the acquired sampled signals received at the terminations of the relay as given from the voltage and current measurement transformers. A description on how to use the proposed techniques to estimate the fault parameters is properly described.

Key-Words: - Distance protection; Transmission and distribution systems; Computer relaying

1 Introduction

On the basis of well known consolidated approaches [1], [2], [3], [4], [5], [7], a new procedure to evaluate distance protection in sub-transmission lines is investigated. In this context, the identification theory was usefully applied to estimate the line parameters of a power line during a fault condition. As a matter of fact, these procedures can supply quantitative information from measured data coming directly from the observed dynamic system. In general, acquired data can be used to either improve the knowledge of the mathematical model or reconstruct signals not directly measurable but useful to describe the system dynamics. The main two problems that can be solved with the identification theory are parameter identification and state estimation, both performed by processing experimental data coming from the monitored system. The parameter identification can be either deterministic or stochastic. The former is used when the measurement errors can be neglected, which means the experiments can be considered ideal. The latter is adopted when the acquired information is very noisy. Depending on the type of identification (deterministc or stochastic), the approaches towards state reconstruction are called state observation and filtering theory, respectively.

2 System modeling

A power line model usually used in distance relaying is shown in Fig. 1, [3], [10].

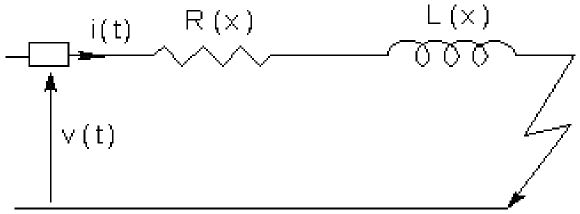

Fig. 1. Simplified model used to schematize a power line.

With reference to Fig. 1, the phenomenon under study is described by the following first-order differential equation:

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$$v(t) = R \cdot i(t) + L \cdot \frac{di(t)}{dt} \quad (1)$$

while the input and state of the system can be respectively defined as:

$$u(t) = v(t)$$
$$x(t) = i(t)$$

The dynamic of the system is described by the following equation:

$$\dot{x}(t) = -\frac{R}{L} \cdot x(t) + \cdot \frac{1}{L} \cdot u(t) \quad (2)$$

By assuming as the system *z* output the current *i(t)*, which is sampled at *Δ* time-intervals and affected by the w_k measurement noise, the following relations can be written:

$$\begin{cases} \dot{x}(t) = -\frac{R}{L} \cdot x(t) + \cdot \frac{1}{L} \cdot u(t) \\ z(k\Delta) = x(k\Delta) + w_k \end{cases} \quad (3)$$

Once all the involved quantities are discretized, the solution of the system (3) can be obtained by applying the recursive minimum mean-square method. The discretization procedure can be performed as explained in the following. The *v(t)* input signal, which is the system forcing function, is sampled at equal *Δ* time-intervals:

$$\{v_k\} = \{v(0) \quad v(1) \quad . \quad . \quad . \quad v(k)\}$$

As shown in Fig. 2, the *v(t)* input signal is linearized between two consecutive time instants.

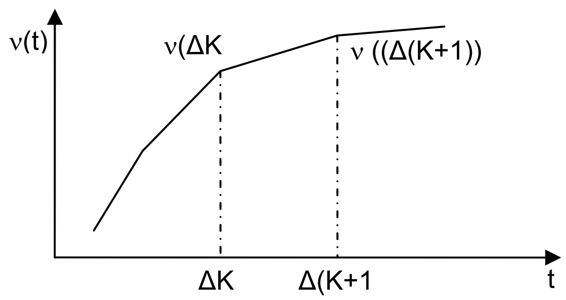

Fig. 2. Feature of the v(t) input signal.

The *v(t)* input can be also written as:

$$\begin{aligned} v(t) &= v(k\Delta) + ((v(k+1)\Delta) - v(k\Delta)) \cdot \frac{t - k\Delta}{\Delta} \\ &= a + b(t - k\Delta) \end{aligned}$$

where:

$$\begin{cases} a = v(k\Delta) \\ b = \dfrac{v((k+1)\Delta) - v(k\Delta)}{\Delta} \end{cases} \quad (4)$$

The general solution of the equation (2), which describes the dynamics of the system, is:

$$x(t) = e^{-\frac{R}{L}(t-t_0)} \cdot x(t_0) + \int_{t_0}^{t} e^{-\frac{R}{L}(t-\tau)} \cdot \frac{1}{L} \cdot u(\tau) d\tau$$

By imposing $t_0 = k\Delta$, $t = (k+1)\Delta$, the following relation can be obtained:

$$x((k+1)\Delta) = e^{-\frac{R}{L}\Delta} \cdot x(k\Delta) + \int_{k\Delta}^{(k+1)\Delta} e^{-\frac{R}{L}((k+1)\Delta-\tau)} \cdot \frac{1}{L} \cdot u(\tau) d\tau \quad (5)$$

Relation (5) can be rewritten in simpler form as:

$$x((k+1)\Delta) = e^{-\frac{R}{L}\Delta} \cdot x(k\Delta) + \int_{0}^{\Delta} e^{-\frac{R}{L}(\Delta-\vartheta)} \cdot \frac{1}{L} \cdot (a + b\vartheta) \cdot d\vartheta \quad (6)$$

where it is assumed $\vartheta = \tau - k\Delta$.

By solving the integral equation (6) and writing the $x(k\Delta)$ state at the $k\Delta$ instant simply as $x(k)$, the following time discrete system can be obtained:

$$\begin{cases} x(k+1) = \alpha \cdot x(k) + \beta \cdot u(k) + \gamma \cdot u(k+1) + \varepsilon_k \\ z(k) = x(k) + w_k \end{cases} \quad (7)$$

where ε_k takes the signal noise into account. The coefficients α, β, and γ are better defined in Section 4.

3. Recursive Minimum Mean-Square Estimation applied through the Kalman filter

Let's suppose a ϑ vector (deterministic and unknown) must be assessed starting from *m* independent measurements. Each y_i measurement is supposedly affected by an added n_i noise, representable with a Gaussian distribution with zero mean and known σ_{ni}^2 variance [11]:

$$\begin{aligned} & y_i = a_i \cdot \vartheta + n_i \\ & E\{n_i\} = 0 \qquad i = 1,\dots,m \\ & E\{n_i^2\} = \sigma_{ni}^2 \end{aligned} \tag{8}$$

If the y_i measurements and n_i noises are reported in two different vectors, respectively named Y and N, equation (8) can be written in a matrix form as follows:

$Y = A \cdot \vartheta + N$ where:

- A is the coefficient matrix.
- N is a random Gussian vector with zero mean and σ_N^2 variance.
- ϑ is a deterministic and unknown vector.
- Y is a random Gussian vector with zero mean and $m_Y = A \cdot \vartheta$ variance.

With regards to the above described notations (in particular to relation 7), the observed X random variable can be written as follows:

$X = A \cdot \vartheta + N$ where:

- $A = [x(k) \quad u(k) \quad u(k+1)]$ is the coefficient matrix, with $x(k)$, $u(k)$ and $u(k+1)$ known quantities at the k instant.
- N is a random Gussian vector with zero mean.
- $\vartheta = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$.

The problem to be solved involves an assessment of the maximum verisimilitude for ϑ parameters starting from the x measured values.

If the known quantities of the X vector are acquired at different, subsequent instants of time, the following dynamic formulation can be given:

$$X(k) = A(k) \cdot \vartheta(k) + N(k)$$

where:

$$X(k) = \begin{bmatrix} X(k-1) \\ --- \\ x(k) \end{bmatrix} = \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ . \\ . \\ x(k) \end{bmatrix}$$

$$A(k) = \begin{bmatrix} A(k-1) \\ --- \\ a(k) \end{bmatrix} = \begin{bmatrix} x(0) & u(0) & u(1) \\ x(1) & u(1) & u(2) \\ x(2) & u(2) & u(3) \\ . & . & . \\ . & . & . \\ x(k) & u(k) & u(k+1) \end{bmatrix}$$

$$N(k) = \begin{bmatrix} N(k-1) \\ --- \\ n(k) \end{bmatrix} = \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_k \end{bmatrix}$$

The algorithm of the recursive minimum mean-square estimation allows a computation of the optimum assessment of $\hat{\vartheta}(k+1)$ using the knowledge of the $\hat{\vartheta}(k)$ assessment of the previous time instant.
In order to use the Kalman filter formulation [6], [8], the problem must be rewritten in a dynamic form. Since the ϑ vector parameters are constant, the state equation can be simply written as:

$$\begin{cases} \vartheta(k+1) = \vartheta(k) \\ x(k) = a(k) \cdot \vartheta(k) + \varepsilon_k \end{cases} \tag{9}$$

As concerns the dynamic problem (9), it can be useful to neglect the farther and emphasize the more recent observations. This aim can be reached by introducing the following f forgetting factor:

$$f = \frac{1}{\lambda} \quad \text{where} \quad \lambda \in (0,1) .$$

This parameter is responsible for the algorithm rapidity in forgetting past observations. More specifically, the closer λ is to 1, the smaller the difference is between the weight of old and newly received data.

The algorithm of the recursive minimum mean-square estimation is the following:

1. Initial conditions:
   - $S(0) = S_0$
   - $\hat{\vartheta}(0) = \hat{\vartheta}_0$
   - $k = 0$
2. Computation of the $K(k+1)$ gain matrix:

$$K(k+1) = S(k) \cdot a^T(k) \cdot (f \cdot a(k) \cdot S(k) \cdot a^T(k) + \sigma_w^2)^{-1} \cdot f$$

3. Computation of the $\hat{\vartheta}(k+1)$ assessment:

$$\hat{\vartheta}(k+1) = \hat{\vartheta}(k) + K(k+1) \cdot (x(k+1) - a(k) \cdot \hat{\vartheta}(k))$$

4. Computation of the $S(k+1)$ matrix:

$$S(k+1) = (I - K(k+1) \cdot a(k)) \cdot S(k) \cdot f$$

   Increase of the k step:

$$k = k+1$$

5. GO TO 2.

As in all recursive algorithms, also in this case the initialisation problem must be solved. Because the ϑ vector is not a random variable, there no expectation value and covariance exist to start the algorithm.
In these conditions, an initial $\overline{\vartheta}$, usually available from prior information, is established. As a consequence, $S(0)$ is not a covariance matrix, even if in the algorithm the $S(k)$ quantity plays a role similar to a covariance matrix of the estimation error in the Kalman filter. This observation suggests to choose a $S(0)$ sufficiently great to model the uncertainty of the estimated starting value of $\overline{\vartheta}$.

4 The fault parameters

In order to perform a computation of the line parameters during a fault condition, i. e. the value of the resistance and inductance "seen" by a distance relay, it is necessary to know the coefficients α, β and γ that are reported in the dynamic equation of the system (7):

$$x(k+1) = e^{-\frac{R}{L}\Delta} \cdot x(k) + \int_0^\Delta e^{-\frac{R}{L}(\Delta-\vartheta)} \cdot \frac{1}{L} \cdot (a + b\vartheta) \cdot d\vartheta$$

$$= e^{-\frac{R}{L}\Delta} \cdot x(k) + \frac{a}{L} \cdot e^{-\frac{R}{L}\Delta} \cdot \int_0^\Delta e^{\frac{R}{L}\vartheta} d\vartheta + \frac{b}{L} \cdot e^{-\frac{R}{L}\Delta} \cdot \int_0^\Delta \vartheta \cdot e^{\frac{R}{L}\vartheta} d\vartheta$$

$$= e^{-\frac{R}{L}\Delta} \cdot x(k) + \frac{a}{L} \cdot e^{-\frac{R}{L}\Delta} \cdot \left[\frac{L}{R} \cdot e^{-\frac{R}{L}\vartheta} \right]_0^\Delta + \frac{b}{L} \cdot e^{-\frac{R}{L}\Delta} \cdot \left[\vartheta \cdot \frac{L}{R} \cdot e^{-\frac{R}{L}\vartheta} - \frac{L^2}{R^2} \cdot e^{\frac{R}{L}\vartheta} \right]_0^\Delta$$

$$x(k+1) = e^{-\frac{R}{L}\Delta} \cdot x(k) + \frac{\left(1 - e^{-\frac{R}{L}\Delta}\right)}{R} \cdot a + \frac{\left[\Delta - \frac{L}{R} \cdot \left(1 - e^{-\frac{R}{L}\Delta}\right)\right]}{R} \cdot b \quad (10)$$

By substituting relations (4) in equation (10) and after simple mathematical steps the following relations can be obtained:

$$\alpha = e^{-\frac{R}{L}\Delta}$$

$$\beta = \frac{(1 - e^{-\frac{R}{L}\Delta})}{R} - \frac{\Delta^2}{R} + \frac{\Delta L}{R^2} \cdot \left(1 - e^{-\frac{R}{L}\Delta}\right) \quad (11)$$

$$\gamma = \frac{\Delta^2}{R} + \frac{\Delta L}{R^2} \cdot \left(1 - e^{-\frac{R}{L}\Delta}\right)$$

Equations (11) represent a non linear system of three equations and two unknowns, *R* and *L*. The same equations (11) can be rewritten in the following form:

$$\alpha = e^{-\frac{R}{L}\Delta} \quad (12a)$$

$$\beta \cdot R^2 + \left(\Delta^2 + \alpha - 1\right) \cdot R - (1-\alpha)\Delta \cdot L = 0 \quad (12b)$$

$$\gamma \cdot R^2 - \Delta \cdot R + (1-\alpha) \cdot \Delta \cdot L = 0 \quad (12c)$$

After the summation of the equations (12b) and (12c), the following relation can be obtained:

$$R \cdot \left[R \cdot (\beta + \gamma) - (1-\alpha)\right] = 0 \quad (13)$$

One solution of the (13) equation is $R = 0$. In this case equation (13) can be rewritten, without losing generality, as:

$$R \cdot (\beta + \gamma) - (1-\alpha) = 0 \quad (14)$$

Relations (14) and (12a) represent a linear system of two equations that can be solved to compute the *R* and *L* unknowns:

$$\begin{cases} L \cdot \ln\alpha + R \cdot \Delta = 0 \\ R \cdot (\beta + \gamma) = 1 - \alpha \end{cases} \quad (15)$$

System (15) can be written in a matrix form as:

$$\begin{bmatrix} \beta+\gamma & 0 \\ \Delta & \ln\alpha \end{bmatrix} \cdot \begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} 1-\alpha \\ 0 \end{bmatrix}$$

and finally, in a more significant form, as :

$$\begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} \beta+\gamma & 0 \\ \Delta & \ln\alpha \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1-\alpha \\ 0 \end{bmatrix}$$

5 Further considerations about the adopted line model

For a complete analysis of the problem, it is useful to highlight some further considerations about the limits of the series R-L model adopted to represent a single power line. As already said in Section 1, this is a simplified model used to investigate the system during a fault condition. To better symbolize the fault circuit, the model shown in Fig. 3 can be used; where C(x) is the shunt capacitance of the line.
Actually, the model shown in Fig. 4 involves the estimation of four fault parameters, which are: R, L, RC and LC. Of course, the consequence is a more complicate algorithm to be implemented to solve the problem.
In real cases, this approach appears not justified because also the R-L-C model involves a simplified representation and its use do not reduce significantly the error made on the estimation of the line parameters [9].

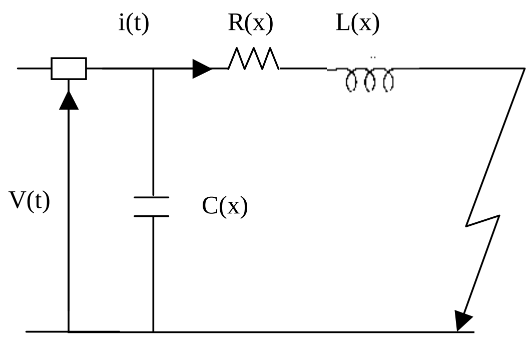

Fig. 3: Single-phase line model with the shunt capacitance.

6 Conclusions

The proposed algorithm represents a valid alternative to other algorithms now used in line distance digital protection. As a matter of fact, the algorithm is characterized by a very small response time, high robustness, and good precision on the fault parameter estimation, which are fundamental requirements for efficient distance protection. The presence of non linear loads does not significantly affect the algorithm behavior.

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