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# False Alarm Rates for the Shewhart Control Chart with Interpretation Rules

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*Abstract:* -Shewhart $\overline{X}$ chart usually supplemented by interpretation rules is common in practice. These rules are designed to show an earlier detection of un-natural patterns in the process mean. Although these rules are valuable in detecting the "true" problems, they also increase the probability of false alarm. In this paper the false alarm rates of most used seven interpretation rules were obtained through by developing a spread sheet for normal and non-normal distributions. The results show that the number of false alarm can be fairly high for particular rules when the underlying data from a gamma distribution rather than normal distribution.

*Key-Words:* - Interpretation Rules, Shewhart Control Chart, False Alarm Rate, Normal Distribution, Gamma Distribution

## 1 Introduction

Shewhart $\overline{X}$ chart with interpretation rules has been widely applied on industrial practice for quality control and very popular among practitioners. The objective of these rules is to improve the effectiveness of the Shewhart chart to detect the un-natural patterns in the process mean, trends or mixtures. E.S.Page (Page, 1955) suggested rules of the following type of Shewhart chart: "*Choose k, n, N. Take samples of size N. Take action if any (sample average or sample range) falls outside the action line or if any k of the last n points fall outside the warning lines*".

Page's concept of warning lines is more commonly referred to as zone boundaries in the Shewhart charts. They are placed ±1σ, ±2σ and center line and the control limits, divide the Shewhart chart in six zones are shown in figure 1.

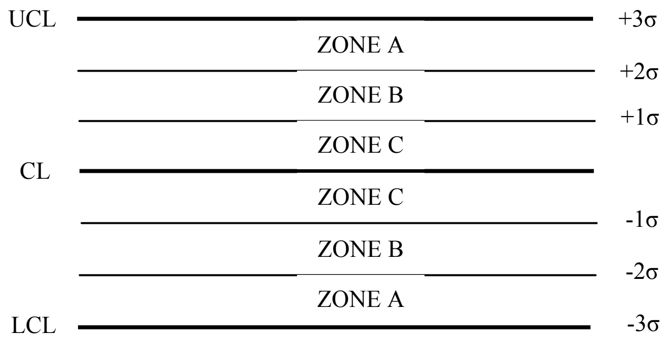


Fig 1. $\overline{X}$ Chart with Different Zones

There are number of sources including Bell Laboratories and Juran have compiled a list of rules for interpreting Shewhart control charts. The most common seven interpretation rules for Shewhart charts are as under

***Rule1***. Any point falls outside the control limits (μ ± 3σ)
***Rule2***. 2/3 consecutive points fall between μ-2σ and μ-3σ or between μ+2σ and μ+3σ
***Rule3.*** 4/5 consecutive points fall between μ-1σ and μ-3σ or between μ+1σ and μ+3σ
***Rule4.*** 8 consecutive points fall between μ and μ-3σ or between μ and μ+3σ
***Rule5.*** 15 consecutive points fall between μ-1σ and μ+1σ
***Rule6.*** 8 consecutive points fall on both sides of center line with none of the points fall between μ ±1σ
***Rule7.*** 7 consecutive points without a change in direction (trend)

Several studies have been done to investigate the false alarm rates of the $\overline{X}$ chart with interpretation rules for example Champ and Woodall (1987), Nelson (1984), Wheeler (1983), Champ and Woodall (1990),Woodall and Davis (1988), Do Sun Bai (2000). The common thing for all these authors were incorporated in their research for investing false alarm rates only first four rules through by simulation studies. Therefore, the object of this research is to further explore the false alarm rates of the $\overline{X}$ chart used most seven rules of interpretation for normal and non-normal distribution. The results were obtained analytically by providing very simple methodology through developing of EXCEL Spread Sheet using statistical functions or from the table of normal curve.

## 2 Methodology

The false alarm rate of Shewhart interpretation rule 1 (the probability of a point exceeding $\mu-3\sigma$ and $\mu+3\sigma$) were obtained, when the area under normal curve less than $\mu-3\sigma$ is 0.00135 or the area under the normal curve greater than $\mu+3\sigma$ is 0.00135. As the normal curve is symmetric around mean. There fore the total area is 0.0027 of the total area under the normal curve. While the false alarm rate for the rule 2 to rule 6 were obtained using the binomial distribution with "SUCCESS" being a sample that falls in the range being examined. The probabilities for "SUCCESS" used in the binomial distribution were obtained from the probability density function of normal distribution.

Further more the false alarm rate for rule 3 having following additional probabilities,

1. The probability of 4/4 points falling in the range $\mu-1\sigma$ to $\mu-3\sigma$
2. The probability of 4/5 points falling in the range $\mu-1\sigma$ to $\mu-3\sigma$ with one of the first four points not falling in the range.
3. The probability of 4/4 points falling in the range $\mu+1\sigma$ to $\mu+3\sigma$.
4. The probability of 4/5 points falling in the range $\mu+1\sigma$ to $\mu+3\sigma$ with one of the first four points not falling in the range

As in the 7$^{th}$ rule, the points do not lie in range being examined but continuously increasing or decreasing. So the false alarm rate of this rule can not calculated as previous mention rules but rather calculated using the number of "SUCCESS" of the rule occurring dividing by the number of possible outcomes. There fore, the false alarm rates of this rule can be calculated by taking the number of successes (2$\rightarrow$ 1 increasing and 1 decreasing) by the number of possible outcomes (7!=5040)

$$P\text{ (All Ascending)} = 1/5040 = 0.000198$$
$$P\text{ (All Descending)} = 1/5040 = 0.000198$$
$$\alpha = P\text{ (All Ascending)} + P\text{ (All Descending)}$$
$$\alpha = 0.000198+0.000198=0.000397$$

The values of all rules for false alarm rates are shown in table 1 for normal distribution.

Table1: False Alarm Rates of Shewhart $\overline{X}$ Control Charts with Interpretation Rules (Normal Distribution)

| Rule | Test | p(s) | Binomial | Probability | False Alarm Rates |
|---|---|---|---|---|---|
| 1 | P(x>3σ) | 0.001350 | 1 of 1 | 0.001350 | 0.002700 |
| | P(x<-3σ) | 0.001350 | 1 of 1 | 0.001350 | |
| 2 | P(-3σ < x < -2σ) | 0.021400 | 2 of 2 | 0.000458 | 0.002704 |
| | | | 2 of 3 | 0.000894 | |
| | P(2σ < x < 3σ) | 0.021400 | 2 of 2 | 0.000458 | |
| | | | 2 of 3 | 0.000894 | |
| 3 | P(-3σ < x < -1σ) | 0.157305 | 4 of 4 | 0.000612 | 0.005339 |
| | | | 4 of 5 | 0.002057 | |
| | P(1σ < x < 3σ) | 0.157305 | 4 of 4 | 0.000612 | |
| | | | 4 of 5 | 0.002057 | |
| 4 | P(x < μ) | 0.500000 | 8 of 8 | 0.003906 | 0.007813 |
| | P(x > μ) | 0.500000 | 8 of 8 | 0.003906 | |
| 5 | P(-1σ < x < 1σ) | 0.682689 | 15 of 15 | 0.003261 | 0.003261 |
| 6 | P(-1σ < x < 1σ) | 0.682689 | | | 0.000103 |
| | 1-P(-1σ < x < 1σ) | 0.317311 | 8 of 8 | 0.000103 | |
| 7 | Combination of 7 items=5040 | | All Ascending | 0.000198 | 0.000397 |

The above calculations were carried out for Shewhart individual rules of normal distribution. The above model or methodology can also be applied for non-normal distribution. Therefore above EXCEL spread sheet model was extended for gamma distribution.

The PDF of Gamma Distribution is

$$f(x\,|\,a,b) = \frac{1}{b^{a}\Gamma(a)} * x^{a-1} * e^{-\frac{x}{b}} \qquad x,a,b > 0$$

Where $\Gamma(\bullet)$ =gamma function, a = Shape parameter and b = Scale parameter.

$$E(x) = \mu = a * b = \text{Scale} * \text{Shape}$$

$$\text{var}(x) = a * b^{2} = \text{Scale}^{2} * \text{Shape}$$

We consider the shape parameter (a) = 0.5 and scale parameter (b) = 1. From these parameters a mean and standard deviation were calculated. After calculation of mean and standard deviation then we have adopted above same methodology as the results shown in below table 2.

Table2: False Alarm Rates of Shewhart $\overline{X}$ Control Charts with Interpretation Rules (Gamma Distribution)

| Rule | Test | p(s) | Binomial | Probability | False Alarm Rates |
|---|---|---|---|---|---|
| 1 | P(x>3σ) | 0.014306 | 1 of 1 | 0.014306 | 0.014306 |
| | P(x<-3σ) | 0.000000 | 1 of 1 | 0.00000 | |
| 2 | P(-3σ < x < -2σ) | 0.000000 | 2 of 2 | 0.000000 | 0.001810 |
| | | | 2 of 3 | 0.000000 | |
| | P(2σ < x < 3σ) | 0.031194 | 2 of 2 | 0.000973 | |
| | | | 2 of 3 | 0.000837 | |
| 3 | P(-3σ < x < -1σ) | 0.0000000 | 4 of 4 | 0.000000 | 0.000880 |
| | | | 4 of 5 | 0.000000 | |
| | P(1σ < x < 3σ) | 0.142993 | 4 of 4 | 0.000418 | |
| | | | 4 of 5 | 0.000462 | |
| 4 | P(x < μ) | 0.682689 | 8 of 8 | 0.047183 | 0.047286 |
| | P(x > μ) | 0.317311 | 8 of 8 | 0.000103 | |
| 5 | P(-1σ < x < 1σ) | 0.160012 | 15 of 15 | 0.000000 | 0.000000 |
| 6 | P(-1σ < x < 1σ) | 0.160012 | | | 0.247849 |
| | 1-P(-1σ < x < 1σ) | 0.839988 | 8 of 8 | 0.247849 | |
| 7 | Combination of 7 items=5040 | | All Ascending | 0.000198 | 0.000397 |

## 3 Comparisons of False Alarm Rates

Table 3: False Alarm Rates Comparisions of Shewhart Control chart with Interpretation rules

| Rules | False Alarm Rates | |
|---|---|---|
| | Normal Distribution | Gamma Distribution |
| 1 | 0.002700 | 0.014306 |
| 2 | 0.002704 | 0.001810 |
| 3 | 0.005339 | 0.000880 |
| 4 | 0.007813 | 0.047286 |
| 5 | 0.003261 | 0.000000 |
| 6 | 0.000103 | 0.247849 |
| 7 | 0.000397 | 0.000397 |

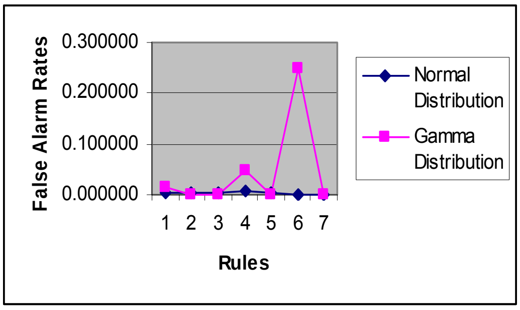


Fig.1: False Alarm Rates Comparisons of Shewhart Control Chart with Interpretation Rules

## 4 Conclusion

Interpretation rules were designed to increase the effectiveness of Shewhart $\overline{X}$ chart to detect the presence of assignable causes. It was found in this study, that the False Alarm Rates for individual Shewhart control chart interpretation rules for the statistical distributions has significant differences. After the analysis of table 3 and figure 1 of comparisons, It was found that the false alarm rates for the rules 1 4, 6 are much larger for the gamma distribution than the normal distribution. While the false alarm rate for rule # 5 of gamma distribution is zero. After the analysis of distributions that the interpretation rules # 6 and # 7 for normal distribution and rule # 3 and 7 for gamma distribution have a very low false alarm rates and very low power to detect the changes in the process mean in terms of standard deviation.

The benefits of interpretation rules are obvious. It is recommended to use them wisely according to distribution of the data .If particular assignable causes are suspected to be present in certain situation, then applying only the specific tests that will mostly likely detect them will greatly reduce the false alarm rates. The interpretation of the rules should be valuable in selecting the appropriate rules to apply for different statistical distributions.

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