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Frequency-Domain Analysis of Non-Linear Circuit Elements

HORIA ANDREI
Faculty of Electrical Engineering
University Valahia Targoviste
18-20 Blv. Unirii, Targoviste, Dambovita
ROMANIA

FANICA SPINEI
Faculty of Electrical Engineering
University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
Email: spinei@elth.pub.ro http://www.elth.pub.ro
COSTIN CEPISCA
Faculty of Electrical Engineering
University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
http://www.elth.pub.ro
VALENTIN DOGARU
Faculty of Electrical Engineering
University Valahia Targoviste
18-20 Blv. Unirii, Targoviste, Dambovita
ROMANIA
Email: dogaru@valahia.ro http://www.valahia.ro
SORIN-DAN GRIGORESCU
Faculty of Electrical Engineering, University Politehnica Bucharest
313 Splaiul Independentei, sector 6, Bucharest
ROMANIA
http://www.elth.pub.ro
NICOLAE JULA
Military Technical Academy
Bucharest
ROMANIA
http://www.mta.ro

Abstract – **This paper presents an alternative mathematical method for determining the spectrum of signal for the nonlinear elements in non sinusoidal steady-state regime. For illustration the time-domain analysis of nonlinear circuit components, are presented two numerical examples.**

1. Introduction

The larger use of the nonlinear elements of the electroenergetic networks, especially during the past decade, has drawn the specialists' and researchers' attention on problems such as to determine the behavior of the nonlinear elements in non-sinusoidal steady-state regime, to find the new spectrum of frequencies and magnitudes in this case.

The analysis of the non-sinusoidal steady-state regime of the circuits is made using the Fourier transform, as in [1], [2], calculating the circuit for each harmonic.

In this paper, is proposed a mathematical method conceived to determine the spectrum and the magnitudes of harmonics for the non-linear circuit elements in the non-sinusoidal steady-state regime.

The alternative method proposes are compared to the classical simulation software methods.

2. The Characterization of the Non-Linear Elements

The non-linearity is a general characteristic of the elements which compose the electroenergetic networks, and generally, it represents a distortion factor for the signals transmission circuits.

The non-linear resistance is described by a non-linear voltage-current $u-i$ characteristic, valid for different functioning regimes. The static characteristic is valid for D.C. regime, the A.C. characteristic is valid for some r.m.s. values, and the dynamic characteristic is valid for instantaneous values for a well defined regime, as in [3], and [4].

We consider a non-linear resistance element, whose characteristic voltage-current, $u(i)$ can be described through a n degree polynomial function, as in [5], and [6]:

$$u(i,t)=\sum_{l=1}^{n}a_l i^l(t) \tag{1}$$

where the coefficients $a_l \in R$ are know.

If the excitation signal $i(t)$ is a non-sinusoidal periodic function with p harmonics

$$i(t)=\sum_{k=1}^{p}I_k \sin(k\omega t+\varphi_k) \tag{2}$$

then the relation (1) can be expressed:

$$u(i,t)=\sum_{l=1}^{n}a_l[\sum_{k=1}^{p}I_k \sin(k\omega t+\varphi_k)]^l \tag{3}$$

The effects of the nonlinear characteristic (1) manifest through the introduction of an other number of voltage harmonics. Using the symbolic method, the sinusoidal current can be expressed by

$$i(t)=I\sin(\omega t+\varphi)=Ie^{j(\omega t+\varphi)}=\frac{1}{2j}(\underline{I}-\underline{I}^*) \tag{4}$$

where:

$\underline{I}=Ie^{j\varphi}e^{j\omega t}$,

$\underline{I}^*=Ie^{-j\varphi}e^{-j\omega t}$,

$j=\sqrt{-1}$.

Then, relations (2) and (3) become

$$i(t)=\sum_{k=1}^{p}\frac{1}{2j}(\underline{I}_k-\underline{I}_k^*) \tag{5}$$

respectively:

$$u(i,t)=\sum_{l=1}^{n}a_l[\sum_{k=1}^{p}\frac{1}{2j}(\underline{I}_k-\underline{I}_k^*)]^l\ . \tag{6}$$

3. Algorithm for Determining the Spectrum and the Magnitudes of Harmonics for the Non-Linear Elements in Non-Sinusoidal Steady State Regime

The first step of the algorithm, as in [7], is the determination of the number m of voltage harmonics which are generated at the nonlinear resistance terminals, like a consequence of the nonlinearity. It is easy to demonstrate that:

$$m=n\cdot p \tag{7}$$

In the second step we determine the magnitude of each $q=0,...,m$ harmonic of voltage. This can be calculated using (6) and the formula of the multinom. We obtain:

$$u_q=\sum_{l=1}^{n}a_l[\sum_{k=1}^{p}\frac{1}{2j}(\underline{I}_k-\underline{I}_k^*)]^l\Bigg|_q=$$
$$=A_q\sum_{\substack{n_1,..,n_p\geq 0\\ 1\cdot n_1+..+p\cdot n_p=q\\ n_1+..+n_p=n}}\frac{n!}{n_1!...n_p!}\times \tag{8}$$
$$\times(\frac{1}{2j})^{n_1+..n_p}\left[(\underline{I}_1-\underline{I}_1^*)\right]^{n_1}...\left[(\underline{I}_p-\underline{I}_p^*)\right]^{n_p}$$

The determination of integer numbers $n_1,...,n_p$ represents the third step of the algorithm. Generally, the integer numbers n and p are known. If q is variable, resolving in the N set the equations system

$$n_1,...,n_p \geq 0$$
$$1 \cdot n_1 + ... + p \cdot n_p = q \qquad (9)$$
$$n_1 + ... + n_p = n$$

we get the integer numbers $n_1,...,n_p$.

In this way we obtain all the magnitudes of voltage harmonics, and we known the contribution of each current harmonic on the each voltage harmonic. In table 1 are illustrated the conclusions of this algorithm, and we marked with a black point the influence of each current harmonic on the harmonics of voltage.

We established that, generally, the p order current harmonic generates and influences all the voltage harmonics starting with the first order and finishing with the $p + p(n-1) = p \cdot n$ order.

u \ i	u_0	u_1	..	$u_{1+p(n-1)}$	$u_{2+p(n-1)}$	.	$u_m = u_{n \cdot p}$
i_1	•	•	. . .	•			
i_2	•	•	. . .	•	•		
...							
i_p	•	•		•	•	•	•

Table1. The current and voltage magnitudes harmonics and their influences.

So, the supplementary number of voltage harmonics generated by the current harmonic of p order $\Delta_p = p(n-1)$.

4. Example

We get by measurements a non-linear resistive $u-i$ characteristic shown in figure 2. Practically, this characteristic represents a lamp with the nominal power of 100W.

Using, for example, the small square values method, as in [8], we obtain the $n = 3$ degree polynomial function which approximates the best the set of measured values:

$$u(i,t) = 9.03 i(t) + 2390 i^3(t) .$$

In the first example we consider a sinusoidal current generator

$$i(t) = 0.5 \sin(100\pi t) A$$

which supplied the lamp and contain only the fundamental, $p = 1$.

The curve of voltage obtained by LabView measurements are shown in figure 3, and the magnitude of the voltage harmonics are presented in table 4, third column.

For implementing the algorithm described in section 3, we calculate first, (7), the number m of voltage harmonics which are generated at the nonlinear lamp terminals, $m = n \cdot p = 3$.

The magnitudes of the voltage harmonics are calculated using relation (8), where $q = 0,1,2,3.$ These values are shown in fourteen column of table 4. It is important to observe that the second harmonic of voltage is null.

In table 4, we observe the coincidence between the values of voltage magnitudes obtained through the two methods: by LabView measurements and by the algorithm described in section (3).

This particular example put in evidence an only one supplementary harmonic, the third, of voltage generated by the non-linearity of lamp and the current generator, shown in table 5. The third value of voltage harmonic it is very important value and makes an important disturbation of voltage characteristic.

u \ i	u_0	u_1	u_2	u_3
i_1		•		•

Table5. The current and voltage magnitudes harmonics and their influences.

The second example considers the same nonlinear element supplied by a non sinusoidal steady-state current source

$$i(t) = 0.75 \sin 100\pi t + 0.1 \sin(200\pi t - \frac{\pi}{8}) +$$
$$+ 0.2 \sin(300\pi t + \frac{\pi}{3}) A$$

which contain $p = 3$ harmonics. The curve of voltage obtained by LabView measurements are shown in figure 6, and the magnitude of voltage harmonics are presented in table 7, third column.

For implementing the algorithm described in section 3, we calculate first, using relation (7), the number m of voltage harmonics which are

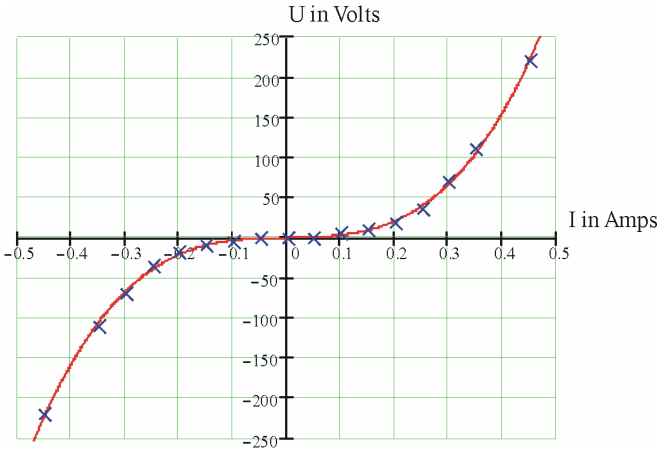

Figure 2. The non-linear characteristic of a lamp.

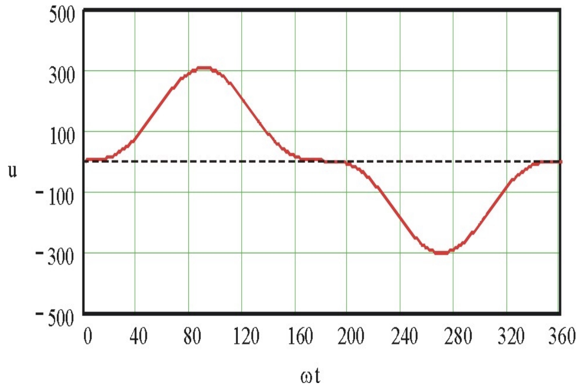

Figure 3. The measured curve of voltage at the terminals of lamp supplied with a sinusoidal current generator.

Voltage Harmonics U_m	Frequency (Hz)	Magnitudes obtain by measuremnts (V)	Magnitudes obtain by algorithm (V)
0	0	-	-
1	50	228.5775	228.57
2	100	-	-
3	150	-896.2570	896.25

Table 4. The magnitudes of voltage harmonics for a lamp supplied with a sinusoidal current generator obtain by measurements and by the proposed algorithm.

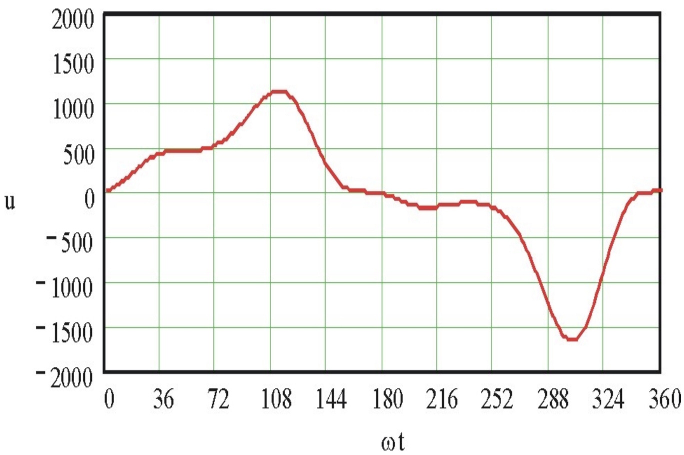

Figure 6. The non sinusoidal voltage at the terminals of lamp.

Voltage Harmonics U_q	Frequency (Hz)	Magnitudes obtain by measuremnts (V)	Magnitudes obtain by algorithm (V)
0	0	-14.7297	15
1	50	439.7816	440
2	100	-35.3468	35,35
3	150	834.8563	834.85
4	200	129.1337	129.13
5	250	-255.7831	255.8
6	300	-74.5562	74.55
7	350	24.0671	24
8	400	6.1235	6
9	450	4.7835	4.78

Table 7. The voltage harmonics of a lamp supplied with a non sinusoidal current generator obtain by measurements and by the proposed algorithm.

generated at the nonlinear lamp terminals, $m = n \cdot p = 9$, and it also appears a d.c. component or $q = 0$. The magnitudes of the voltage harmonics are calculated using relation (8), where $q = 0,1,..,9$.

u \ i	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
i_1	•	•	•	•	•	•	•	•		
i_2	•	•	•	•	•	•	•	•	•	
i_3	•	•	•	•	•	•	•	•	•	•

Table 8. The current and voltage magnitudes harmonics and their influences.

These values are shown in fourteen column of table 7. Looking at the columns three and four, in table 7, we observe the coincidence between the values of voltage magnitudes obtained through the two methods: by LabView measurements and by the algorithm described in section (3).

This example put in evidence a great number of supplementary harmonics voltage generated by the non-linearity of the lamp and the non sinusoidal current generator, shown in table 8.

5. Conclusions

The paper proposes a new, efficient and easy to use method for analysis of the nonlinear elements behaviour in non sinusoidal steady-state regime.

The results of this algorithm are comparables with the results obtained, for example, through a classical LabView measurement method or PSPICE simulation methods, presented in [9].

Thus, we know the generated effects by each nonlinear element in the electroenergetical system and the type of electromagnetic pollution introduced by these.

Using this algorithm the analysis of the nonlinear elements in non sinusoidal steady-state regime can be made through the equivalent coupled circuits for each harmonic, which will contain current or voltage controlled sources. This last idea will be developed in another future paper.

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