# Mechanism Design for 3-, 4-, 5- and 6-DOF PMWPSs 

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#### Abstract

Mechanism innovation design is an important topic in the field of robot. Many kinds of limbs with specific degrees of freedom (DOFs) are proposed, in which a new joint, $\mathrm{P}^{+}$-joint is introduced. In order to obtain new mechanisms of parallel manipulators ( PMs ), a principle for mechanism design of PMs is adopted, which is based on the special Plücker coordinates for describing a limb. By the principle, many new mechanisms of 3-, 4-, 5- and 6-DOF parallel manipulators with perpendicular structures (PMWPSs) are put forward, and structure figures are given to denote their configurations. The new mechanism for PMWPSs we design can be used in many areas.


Key-Words: - Parallel manipulator, Perpendicular structures, $\mathrm{P}^{+}$-joint, Mechanism design, PMWPSs

## 1 Introduction

Earlier in 1949, Gough designed a machine adopting a parallel mechanism and introduced it into tire detecting device. Subsequently, Stewart used the parallel mechanism in a flight simulator and named it as Stewart mechanism [1]. From then on, parallel mechanisms have been used in many areas of heavy load platforms, good kinematic and dynamic performance machines, fine motion and precise positioning manipulators etc.
In recent years, theoretical researches in parallel mechanisms have been made increasingly in a variety of areas, and many new mechanisms of parallel manipulators (PMs) are proposed and studied. Pierrot and Reynaud [2] developed a simple and efficient parallel robot, Delta; Gosselin and Angeles studied a planar 3-DOF parallel manipulator [3] that possesses 8 -bar linkages with 2 ternary links connected through three in-parallel legs; Pernette et al. [4] presented a 3-DOF parallel translating manipulator with U-P-U joints kinematic chains; Tsai[5] introduced multi-DOF mechanisms for machine tools; Appleberry [6] studied a truly new translational parallel manipulator with 3-UPU limbs, and so on.
However, most of these mechanisms or manipulators have structures that their limbs connecting the up-platform and fixed frame are not perpendicular. Generally, they possess complicated kinematics models. We will design mechanisms of PMs that their limbs dispose in three vertical directions, so-called PMWPSs.

### 2.1 Joints

PMs consist of links and joints. The traditional types of joints are prismatic joint (P-joint), revolute joint (R-joint), spherical joint (S-joint), and universal joint (U), as shown in Fig.1-a to Fig.1-d. In order to design mechanisms for PMWPSs, a new joint, $\mathrm{P}^{+}$-joint is introduced. $\mathrm{P}^{+}$-joint, shown in Fig.1-e, is composed of two P-joints that dispose vertically.


Fig. 1 Kinematic joints

### 2.2 Classification of limbs

For investigation of the mechanism design of PMs, the key issue is to find the limbs with known kinematics characteristics, specified degrees of freedom (DOFs). Therefore, we have to discuss the classification of the limbs for PMWPSs. The limbs are the sub-chains connected between the up-platform and the fixed platform of a PMs.
In PMs, the sub-chains are limbs composed of the R-joints, P -joints, U-joints, S -joints or the $\mathrm{P}^{+}$-joints.
Table 1 shows the classification of the limbs for PMWPSs, in which the first letter expresses the joint
connected with the fixed platform, and the last letter represents the joint connected with the up-platform. For instance, the limb $\mathrm{PP}^{+} \mathrm{S}$ means that the limb is connected with the fixed platform by a P-joint and linked with the up-platform by an S-joint. By using the joints shown in Fig.1, 3-, 4-, 5- and 6-DOF limbs shown in Fig. 2 to 5 are proposed.

Table 1 Classification of limbs

| DOF | Joint | Type of limbs |
| :---: | :--- | :--- |
| 3 | $\mathrm{P}-, \mathrm{P}^{+}-$ | $\mathrm{PP}^{+}$ |
|  | $\mathrm{P}-, \mathrm{P}-, \mathrm{R}-$ | $\mathrm{PPR}, \mathrm{PRP}$ |
|  | $\mathrm{R}-, \mathrm{P}^{+}-$ | $\mathrm{RP}^{+}$ |
| 4 | $\mathrm{R}-, \mathrm{R}-, \mathrm{P}^{+}-$ | $\mathrm{RRP}^{+}$ |
|  | $\mathrm{P}-, \mathrm{P}^{+}, \mathrm{R}-$ | $\mathrm{PP}^{+} \mathrm{R}, \mathrm{PRP}^{+}$ |
| 5 | $\mathrm{P}-, \mathrm{P}^{+}-, \mathrm{U}-$ | $\mathrm{PP}^{+} \mathrm{U}, \mathrm{PUP}^{+}$ |
|  | $\mathrm{R}-, \mathrm{P}^{+}-, \mathrm{U}-$ | $\mathrm{RP}^{+} \mathrm{U}, \mathrm{RUP}^{+}$ |
|  | $\mathrm{R}-, \mathrm{R}-, \mathrm{P}^{+}-, \mathrm{R}-$ | $\mathrm{RRP}^{+} \mathrm{R}$ |
| 6 | $\mathrm{P}-, \mathrm{P}^{+}-, \mathrm{S}-$, | $\mathrm{PP}^{+} \mathrm{S}, \mathrm{PSP}^{+}$ |
|  | $\mathrm{R}-, \mathrm{P}^{+}-, \mathrm{S}-$, | $\mathrm{RP}^{+} \mathrm{S}, \mathrm{RSP}^{+}$ |


(b) PPR

(c) $\mathrm{RP}^{+}$

(d) PRP

Fig. 2 3-DOF limbs

(a) $\mathrm{RRP}^{+}$

(b) $\mathrm{PP}^{+} \mathrm{R}$

(c) $\mathrm{PP}^{+} \mathrm{R}$

(d) $\mathrm{PRP}^{+}$

Fig. 3 4-DOF limbs

(a) $\mathrm{PP}^{+} \mathrm{U}$

(b) $\mathrm{RP}^{+} \mathrm{U}$

(c) $\mathrm{PUP}^{+}$

(d) $\mathrm{RUP}^{+}$

(e) $\mathrm{RRP}^{+} \mathrm{R}$

(f) $\mathrm{RUP}^{+}$

Fig. 4 5-DOF limbs

(a) $\mathrm{PP}+\mathrm{S}$

(b) $\mathrm{PSP}^{+}$

(c) $\mathrm{RSP}^{+}$

(d) $\mathrm{RP}^{+} \mathrm{S}$

Fig. 5 6-DOF limbs

## 3 Mechanism design for PMWPSs

### 3.1 Principle for mechanism design of PMs

Though some structures of PMs have existed, only a few types of mechanisms are used. Therefore, to find a principle for mechanism design and innovation of PMs is a very important problem. Here, we use a method proposed by F. Gao [7]. For mechanism design of PMs with specific kinematics characteristics, we discuss the limbs with specific kinematics characteristics. For convenience, we let $\$$ be the special Plücker coordinates for describing the displacement of the output link of a limb for a parallel mechanism, which is

$$
\$_{j}=\left(\begin{array}{llllll}
v_{x j} & v_{y j} & v_{z j}, & \omega_{x j} & \omega_{y j} & \omega_{z j} \tag{1}
\end{array}\right)
$$

where $\left(\begin{array}{lll}v_{x j} & v_{y j} & v_{z j}\end{array}\right)$ expresses the translation of the output link of the $j^{\text {th }}$ limb, and $\left(\begin{array}{lll}\omega_{x j} & \omega_{y j} & \omega_{z j}\end{array}\right)$ denotes the rotation of the output link of the $j^{\text {th }}$ limb. The special Plücker coordinates $v_{x j}, v_{y j}, v_{z j}, \omega_{x j}$, $\omega_{y j}$ and $\omega_{z j}$ can be taken as 1 or 0 . When taking 1 , it means that the $j^{\text {th }}$ limb has that DOF; when taking 0 , it means that the $j^{\text {th }} \operatorname{limb}$ has no that DOF. From Eq.(1), we can obtain 3-, 4-, 5- and 6-DOF limbs with specific kinematics characteristics, as shown in table 2 , in which all joints in one limb are in same line denoted in Fig 2 to Fig.5. More, the three axes directions in coordinates system of one limb are shown in Fig.2-a, so are the other limbs.

Table 2 Plücker coordinates $\$_{j}$ of all the kinematic limbs

| DOF | Kinematic Limb | Plücker coordinates $\$_{j}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $v_{x j}$ | $\nu_{y j}$ | $v_{z j}$ | $w_{x j}$ | $w_{y j}$ | $w_{z j}$ |
| 3 | $\mathrm{PP}^{+}$ | 1 | 1 | 1 | 0 | 0 | 0 |
|  | PPR,PRP | 1 | 0 | 1 | 1 | 0 | 0 |
|  | RP ${ }^{+}$ | 1 | 0 | 1 | 0 | 1 | 0 |
| 4 | RRP ${ }^{+}$ | 1 | 1 | 1 | 1 | 0 | 0 |
|  | $\mathrm{PP}^{+} \mathrm{R}, \mathrm{PRP}^{+}$ | 1 | 1 | 1 | 0 | 1 | 0 |
| 5 | $\mathrm{PP}^{+}$U,PUP ${ }^{+}$ | 1 | 1 | 1 | 1 | 0 | 1 |
|  | $\mathrm{RP}^{+} \mathrm{U}, \mathrm{RUP}^{+}$ | 1 | 0 | 1 | 1 | 1 | 1 |
|  | RRP ${ }^{+}$R | 1 | 1 | 1 | 0 | 1 | 1 |
| 6 | $\mathrm{PP}^{+} \mathrm{S}, \mathrm{PSP}^{+}$ | 1 | 1 | 1 | 1 | 1 | 1 |
|  | $\mathrm{RP}^{+} \mathrm{S}, \mathrm{RSP}^{+}$ | 1 | 1 | 1 | 1 | 1 | 1 |

In a parallel manipulator, if it has specific DOFs, the first, second, . . . and $n^{\text {th }} \operatorname{limb}$ by which the
up-platform is connected with the fixed platform have to satisfy the following condition

$$
\begin{equation*}
\$=\$_{1} \cap \$_{2} \cap \ldots \cap \$_{j} \ldots \cap \$_{n} \tag{2}
\end{equation*}
$$

In Eq.(2), $\$$ denotes the motion of the up-platform of a parallel manipulator, and

$$
\$=\left(\begin{array}{lll}
v_{x} & v_{y} & v_{z}, \omega_{x} \tag{3}
\end{array} \omega_{y} \omega_{z}\right)
$$

Analogously, the coordinates $v_{x}, v_{y}, v_{z}, \omega_{x}, \omega_{y}$, $\omega_{z}$ can be taken as 1 or 0 . When taking 1 , it means that the up-platform has that DOF, when taking 0 , it means that up-platform has no that DOF.

### 3.2 Mechanism design for PMWPSs

By aid of the principle for mechanism design of PMs, we design many types of 3 -, $4-, 5$-, and 6 -DOF PMWPSs, whose kinematics characteristics of the up-platform are listed in Table 3. All the mechanisms or manipulators possess characteristics that their limbs are divided into three groups and distributed in three perpendicular directions. We name it perpendicular structures. Fig. 6 to Fig. 9 denote configurations of all the mechanisms for $3-, 4-$ - 5 -, and 6-DOF PMWPSs. More, in Fig.6-a the three axes in fix coordinates system are parallel to the tree groups of limbs in three perpendicular directions respectively, so are the axes in the other PMWPSs. In the sign denoting one manipulator, the numeral shows limb number distributed in three perpendicular directions. For example, 6- $\mathrm{PP}^{+}$S structure in Fig.9-a denotes that the limb number distributed in three perpendicular directions are all 2, while 1-PPR\&3-PP ${ }^{+}$R in Fig.4-a denotes the lime number are 1,1 and 2 respectively.

Table 3 Kinematics characteristics of 3-, 4-, 5and 6-DOF PMWPSs

| DOF | Name of the mechanism | Plücker coordinates $\$$ of the up-platform |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $v_{x}$ | $v_{y}$ | $v_{z}$ | $w_{x}$ | $w_{y}$ | $w_{z}$ |
| 3 | $1-\mathrm{RP}^{+}$\& 2-PRP | 1 | 0 | 1 | 0 | 1 | 0 |
|  | $\begin{gathered} \text { 3--PP } \\ \text { 3-RRP } \end{gathered}$ | 1 | 1 | 1 | 0 | 0 | 0 |
|  | $3-\mathrm{RP}+\mathrm{U}$ | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | $\begin{gathered} \text { 1-PPR\&3-PP }{ }^{+} \text {R } \\ 4-R U P P^{+} \end{gathered}$ | 1 | 1 | 1 | 1 | 0 | 0 |
| 5 | $\begin{aligned} & \text { 1-PP+U\&4-PP }{ }^{+} \text {S } \\ & \text { 1-RUP+\&4-RP } \end{aligned}$ | 1 | 1 | 1 | 1 | 0 | 1 |
| 6 | $\begin{aligned} & \hline \text { 6- }-\mathrm{PP}^{+} \mathrm{S} \\ & \text { 6-RP } \\ & \text { 6- } \mathrm{PSP}^{+} \\ & \text {6-RSP } \end{aligned}$ | 1 | 1 | 1 | 1 | 1 | 1 |


(a) $1-\mathrm{RP}^{+} \& 2-\mathrm{PRP}$

(b) $3-\mathrm{PP}^{+}$

(c) $3-\mathrm{RRP}^{+}$

(d) $3-\mathrm{RP}^{+} \mathrm{U}$

Fig. 6 Mechanisms for 3-DOF PMWPSs

(a) 1-PPR\&1-PP ${ }^{+}$\& $2-\mathrm{PP}^{+} \mathrm{R}$

(b) 4 -RUP ${ }^{+}$

Fig. 7 Mechanisms for 4-DOF PMWPSs

(a) $1-\mathrm{PP}^{+} \mathrm{U} \& 4-\mathrm{PP}^{+} \mathrm{S}$

(b) $1-\mathrm{RUP}^{+} \& 4-\mathrm{RP}^{+} \mathrm{S}$

Fig. 8 Mechanisms for 5-DOF PMWPSs

(a) $6-\mathrm{PP}^{+} \mathrm{S}$

(b) $6-\mathrm{RP}^{+} \mathrm{S}$

(c) $6-\mathrm{PSP}^{+}$

(d) $6-\mathrm{RSP}^{+}$

Fig. 9 Mechanisms for 6-DOF PMWPSs

## 4 Conclusion

Mechanism innovation design is an important topic in the field of robot. In this paper, a new joint, $\mathrm{P}^{+}$-joint is introduced and many kinds of limbs with specific degrees of freedom (DOFs) are proposed. By a principle for mechanism design of PMs, which is based on the special Plücker coordinates for describing a limb, many new mechanisms of 3-, 4-, 5and 6-DOF PMWPSs are put forward, and structure figures are given to denote their configurations. Generally, the PMWPSs possess simply kinematics. The new mechanisms we design can be used in many areas.

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