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ALGORITHMS ON MACHINES

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Abstract: In this Paper we have tried to find an Algorithm which decides whether a Mechanism is deterministic or not. A lot of work has been done in this area,[1],[2],[9] In this we have modeled the problem on Graphs. The rigid bodies in the concerned machine are considered as the vertices of a Graph and the corresponding contacts as the edges of it. Thus the whole problem of determining the indeterminacy of a Plane Mechanism can be converted into a problem of Graph Theory. The Basic Algorithm is concerned with the Adjacency Matrix of the Graph and its corresponding Powers. We have also calculated the Complexity of the Algorithm. The present work is an attempt towards Manufacturing Science and Machine determinism. However in the present work we have not studied the Dynamic aspects of the Machine.

Key Words: Machines, Mechanism, Graph-Theory, Algorithm, Adjacency Matrix of a Graph, Rigid-Bodies.

1. Introduction:

In this article we have attempted to find an Algorithm which can decide upon the determination of a Mechanism. The Theory of Mechanisms and Machines deals with the study of,

1. Geometrical aspects of Motion.
2. Various forces involved in Motion.

The first is called Kinematics and the later is called Dynamics. Dynamics applied to moving bodies is called Kinetics and when it is applied to stationary body is called Statics.For a detailed description cf.[1],[2],[3] and [9]. Let us now give some relevant definitions to be used in sequel.

Definition1.: A Mechanism is a combination of Rigid or Restraining bodies so shaped and connected that they move upon each other with relative motion.

Example1: A simple example is the Slider-Crank Mechanism which is used in Internal Combustion Engine.

Definition 2: A Machine is a Mechanism or a collection of Mechanisms which
transmits force from the source of Power to be overcome, and thus performs useful Mechanical work.

Definition3: If all points of a Mechanism move in parallel Planes then it is called a Plane-Mechanism.

A Space- Mechanism is one in which all points of the Mechanism do not Move in Parallel- planes.

As defined above a Mechanism is defined as a combination of bodies so connected that each move with respect to another. A clue to the behavior of the Mechanism lies in the nature of connections commonly known as Kinematic Pairs.

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Definition 4: The Degree of Freedom of a Kinematic pair is given by the Number of independent co-ordinates required to completely specify the Relative movement. There are three types of Kinematic pairs.

(1) Lower Pair.
(2) Higher Pair.
(3) Wrapping Pair.

For the definitions of the above pairs cf.[9]. We now define the most important element of Mechanism which is called a Link. It appears in the Classification of Kinematic Chains

Link: A material body which is common to two or more kinematic pairs is called a link.

a Kinematic Chain is a series of links connected by kinematic pairs.

Definition 6: A kinematic chain is a series of links connected by kinematic Pairs. A Chain is said to be closed if every link is connected to at least two other links, otherwise it is called an Open- Chain Mechanism. A Robot is an open chain mechanism. A closed chain cannot contain a Singular link. A chain which consists of only binary links is called a simple chain.

It is observed that to form a simple closed chain we need at least three links ,with three kinematic pairs cf.[9]. However if one of these three links are fixed, there cannot be any relative movement and so a closed chain does not form a mechanism. Such an arrangement is referred to as a structure or bridge, which is commonly rigid. Thus a simplest mechanism consists of four binary links each connected by a Kinematic pair known as a four-bar mechanism. Sometimes a synonym of the word mechanism is used, which is called a linkage.

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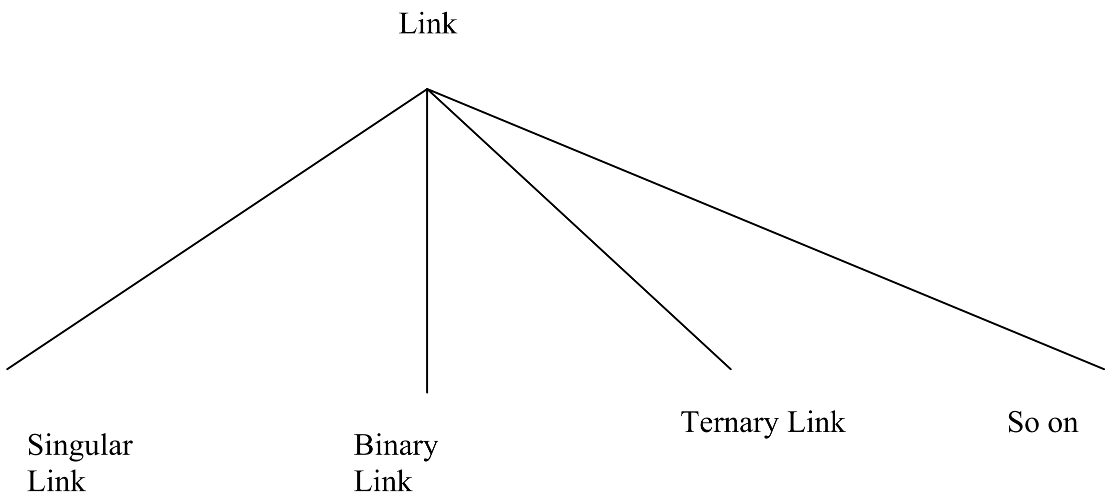

2. Problem Formulation:

Given a collection of rigid bodies and the various contacts amongst each other; the problem is to determine under what condition that will form a mechanism?

3. Problem Solution:

Here we present a Graph-Theoretic Algorithm to investigate indeterminacy of a Plane Mechanism. We consider the rigid bodies in the concerned machine as the vertices of a Graph and the corresponding contacts as the edges of the Graph. Thus the whole problem of determining the indeterminacy of a Plane mechanism can be converted into a problem of Graph-Theory.

Hypothesis: We floor the hypothesis that if there is a 4-cycle in the system, the system is Determinate; otherwise the system is indeterminate. For the relevant definitions of Graph-theory cf.[7].

We provide the solution to the problem in the following steps.

1.Let A be the adjacency matrix of a Graph.

2. Compute A^2.

3.Identify all pairs (i,j),(i≠j) such that $a_{ij}^2 > 1$. (a_{ij}^2 is the $(i,j)^{th}$ entry of A^2).If no such pair exists the system is indeterminate. 3.Identify all pairs (i,j),(i≠j) such that $a_{ij}^2 > 1$. (a_{ij}^2 is the $(i,j)^{th}$ entry of A^2).If no such pair exists the system is indeterminate.

Remark: Existence of such a pair ensures a 4-cycle.

4. Augment the minor of A corresponding to v_i, v_j, v_k, v_l.

5.Does the augmented A become K-I?(here K is the adjacency matrix of the complete graph with the specified number of vertices and I is the Identity Matrix. If yes, the system is determinate.

6. If not the again calculate A^2.

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7.Identify a new pair (m,n) such that a_{mn}^2 is greater than 1,where at least One of m,n should be different from i,j,k,l.If no new pair exists then the System is indeterminate. If a new pair exists, go back to step 3.

8. Continue the process until A is augmented to K-I or no new pair (m,n) is identified for which $a_{mn}^2 > 1$.

We shall illustrate the above Algorithm by means of some Examples.

Example 1: Consider a system G(8,10),it means that the system has 8 rigid bodies along with 10 contacts amongst each other. Now following the above Algorithm we get a Graph whose adjacency matrix is

$$A = \begin{bmatrix} 0\,1\,0\,0\,1\,0\,0\,1 \\ 1\,0\,1\,0\,0\,0\,0\,0 \\ 0\,1\,0\,1\,0\,0\,1\,0 \\ 0\,0\,1\,0\,1\,0\,0\,0 \\ 1\,0\,0\,1\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0\,1\,0 \\ 0\,0\,1\,0\,0\,1\,0\,1 \\ 1\,0\,0\,0\,0\,0\,1\,0 \end{bmatrix}$$

then,

$$A^2 = \begin{bmatrix} 3\,0\,1\,1\,0\,1\,1\,0 \\ 0\,2\,0\,1\,1\,0\,1\,1 \\ 1\,0\,3\,0\,1\,1\,0\,1 \\ 1\,1\,0\,2\,0\,1\,1\,0 \\ 0\,1\,0\,0\,3\,0\,1\,1 \\ 1\,0\,1\,1\,0\,2\,0\,1 \\ 1\,1\,0\,1\,1\,0\,3\,0 \\ 0\,1\,1\,0\,1\,1\,0\,2 \end{bmatrix}$$

Here we observe that except for the diagonals none of the entries of A^2 is greater than 1. Thus there does not exist a 4-Cycle in the system and the System is indeterminate.

Example2:

Now consider another system whose graph is G=(8,10) and the corresponding Adjacency matrix is

$$A = \begin{bmatrix} 0&1&0&0&1&0&0&1 \\ 1&0&1&0&0&1&0&0 \\ 0&1&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0&0 \\ 1&0&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&1&0&1 \\ 1&0&0&0&0&0&1&0 \end{bmatrix}$$

Now,

$$A^2 = \begin{bmatrix} 3&0&1&1&0&2&1&0 \\ 0&3&0&1&2&0&1&1 \\ 1&0&2&0&1&1&0&0 \\ 1&1&0&2&0&1&0&0 \\ 0&2&1&0&3&0&1&1 \\ 2&0&1&1&0&3&0&1 \\ 1&1&0&0&1&0&2&0 \\ 0&1&0&0&1&1&0&2 \end{bmatrix}$$

Here we observe that $a_{16}^{(2)} = 2$ and $a_{25}^{(2)} = 2$,also $a_{15} = a_{65} = a_{12} = a_{62} = 0$. Applying our algorithm we augment the matrix A at a_{16} and at a_{25} , a_{61} *and* a_{52} to get the new matrix as,

$$A = \begin{bmatrix} 0&1&0&0&1&1&0&1 \\ 1&0&1&0&1&1&0&0 \\ 0&1&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0&0 \\ 1&1&0&1&0&1&0&0 \\ 1&1&0&0&1&0&1&0 \\ 0&0&0&0&0&1&0&1 \\ 1&0&0&0&0&0&1&0 \end{bmatrix}$$

We continue our said Algorithm to infer that the above mechanism is determinate.

Example 3: However if we consider a system having the graph G=(12,16) then applying the above Algorithm we obtain that it is a deterministic mechanism ,which can be easily verified.

Complexity

We now calculate the Complexity of the above Algorithm

Total number of times of A has augmentation has complexity = O(mn-(m+n)).

To find A^2 we require $O(m^2n)$ time hence the total time required is equal to $O[\{mn-(m+n)\}(m^2n)\}/4 = O(m^3 n^2)$.

Thus we have the following theorem.

Theorem 1: The Complexity of the above Algorithm is $O(m^3n^2)$,where m and n are respectively the rows and columns of the Adjacency matrix of the graph.

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Examples of Lower Pair Links with associated Degrees of Freedom