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Jacobian Factorization of a C5 Joint Parallel Robot. Analysis of Singular Configurations

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Abstract: This paper presents an approach for parallel robot's Jacobian matrix factorization. Besides its computational efficiency, the proposed approach, which is based on an extension of global formalism developed by Fijany, does'nt require the inversion of the inverse Jacobian matrix. This new factorization is based on a mathematical simplification of the parallel kinematic architecture and by introducing passive and active joint concepts. A singularity study starting from the Jacobian matrix factorization is also presented for the case of a spatial C5 joint parallel robot.

Key–Words: Parallel robot, Jacobian Matrix, Singular Configurations

1 Introduction

The parallel robot concept [1] [2] [3], characterized by a mobile and a fixed part linked by kinematic independent chains, can be considered as an alternative way to avoid the shortcomings of the anthropomorphous classical manipulator structures. One of the drawbacks of a serial manipulator is its lack of accuracy in the positioning of the terminal tool, because of error accumulations in each link. By comparison with serial architectures, parallel architectures present also a greater compactness and stiffness, along with an interesting ratio between the moved load and the robot's mass. However, the increase in stiffness is obtained to the detriment of the workspace, limiting the parallel robotic applications to tasks requiring weak amplitude displacements. The field of simulators [4] [5] [6] which attempts to apply accelerations with weak amplitude displacements and assembly robotics, where the parallel robot is used as an end-effector [7], constitute some of the many applications of parallel robots.
The determination of the inverse geometric model remains, in the case of parallel robots, relatively easy. Some general methods have been developed [8] [9]. Due to non-linearities in the equations, the direct geometric model is difficult to obtain. Some numeric methods, like Newton-Raphson method, have been used successfully [2].
In parallel robot research area, the problem of Jacobian matrix determination is an open and interesting problem. Indeed, the inverse Jacobian matrix computation is currently known and mastered [2] [10] [11], but its analytical expression remains relatively complex. Thus analytical formulation of the Jacobian matrix, by symbolic inversion or even by using some formal computing tools, is difficult [2]. Its expression is generally obtained by a numerical method using a classic algorithm of matrix inversion or by a method based on an iterative scheme. Moreover, in order to limit the computation time, a solution consists of expressing the Jacobian matrix in a nominal position, and to consider it as constant. This corresponds to the hypothesis of using the parallel manipulator around a nominal situation [2] [3].
In the present paper, a new Jacobian matrix factorization is presented. This approach is a generalization of the approach proposed by Fijany et al. [12] [13] [14] for serial robot. Thus we consider the parallel robot as a multi-robot system with k serial robots (the segments) moving a common load (the mobile platform) [15] [16]. The basic idea is to compute the Jacobian matrix associated with each robot link considered as a serial robot and then to compute the Jacobian matrix of the parallel robot by considering the kinematic chain closing constraint. This paper is organized as follows. Some preliminaries and notations are given in section 2. The Jacobian matrix factorization approach is detailed in section 3, before presenting the application of this approach to C5 parallel robot designed at LIIA-Lab [7]. In the section 4, a singularity study starting from the Jacobian matrix factorization, is presented. We conclude with some remarks and perspectives.

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2 Preliminaries

In this section, we define the required notation, and preliminaries are presented. The parallel robot is considered as a multi-robot system with k serial robots (segments) moving a common load (mobile platform). The figure (1) shows the links, the frames and position vectors for the segment i ($i = 1, \dots, k$).

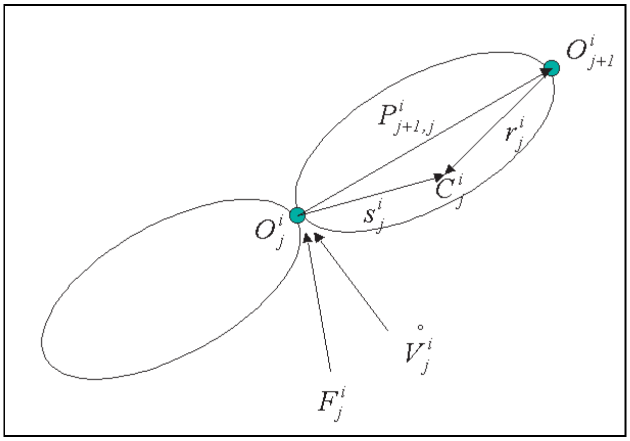

Figure 1: Links, frames and position vectors for the segment i

2.1 Nomenclature

2.1.1 Joint and link parameters

- $\boldsymbol{{}^iP_{j+1,j}}$: position vector from ${}^iO_j$ to ${}^iO_{j+1}$
- k: number of segments
  ${}^iM$: dof number of segment i
- ${}^iN$: joint number of segment i
- S: active joint number by link
- $\theta^a_i, \dot{\theta}^a_i$: position and velocity of active joint of the segment i
- ${}^i\theta^p_j$, ${}^i\dot{\theta}^p_j$: position and velocity of passive joint j of the segment i
- ${}^i\omega_j, {}^iv_j \in \Re^3$: angular and linear velocity of link j for the segment i

2.1.2 Spatial quantities

- ${}^iH_j$: spatial-axis (map matrix) of joint j for the segment i. For instance, for a joint with 2 dof (rotation about Z axis and translation about X axis), the matrix ${}^iH_j \in \Re^{6\times 2}$ is given by:

$$
{}^iH_j = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\begin{array}{l} \text{X-axis rotation} \\ \text{Y-axis rotation} \\ \text{Z-axis rotation} \\ \text{X-axis translation} \\ \text{Y-axis translation} \\ \text{Z-axis translation} \end{array}
$$

- $\boldsymbol{{}^iV_j} = \begin{bmatrix} {}^i\omega_j \\ {}^iv_j \end{bmatrix} \in \Re^6$: spatial velocity of the link j for the segment i
- $\boldsymbol{V_{N+1}} \in \Re^6$: spatial velocity of the end effector

2.1.3 Global quantities

The following global quantities are defined for $j = {}^iN$ to 1 or $j = {}^iM$ to 1 and $i = k$ to 1

- $\boldsymbol{\dot{Q}_i} = Col\left({}^i\dot{\theta}_j \right) \in \Re^{{}^iM}$: global vector of articular coordinate velocity of the segment i, taking into account passive and active joints
- $\boldsymbol{\dot{Q}} = Col\left(\dot{\theta}^a_i \right) \in \Re^k$: vector of generalized coordinate velocity of the system
- $\boldsymbol{\mathcal{V}_i} = Col\left({}^iV_j \right) \in \Re^{6\,{}^iN}$: global vector of spatial velocities for the segment i
- $\mathcal{H}_i = Diag\left({}^iH_j \right) \in \Re^{6\,{}^iN\times {}^iM}$: global matrix of spatial axis for the leg i

2.2 General notations

With any vector $\boldsymbol{V} = \begin{bmatrix} V_x & V_y & V_z \end{bmatrix}^t$, a tensor $\tilde{V}$ can be associated whose representation in any frame is a skew symmetric matrix:

$$
\tilde{V} = \begin{bmatrix} 0 & -V_z & V_y \\ V_z & 0 & -V_x \\ -V_y & V_x & 0 \end{bmatrix}
$$

A matrix $\hat{V}$ associated to the vector $\boldsymbol{V}$ is defined as:

$$
\hat{V} = \begin{bmatrix} U & \tilde{V} \\ 0 & U \end{bmatrix}
$$

where U and 0 stand for unit and zero matrices of appropriate size.
In our derivation, we also make use of global matrices and vectors which lead to a compact representation

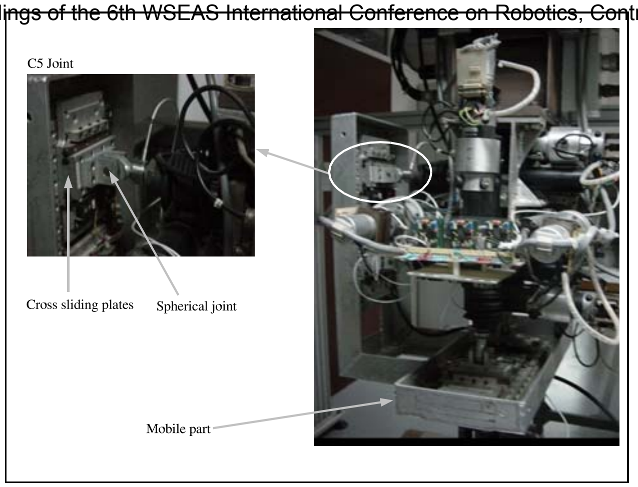

Figure 2: C5 joint parallel robot

of various factorizations. A bidiagonal block matrix $\mathcal{P}_i \in \Re^{6\,^iN \times 6\,^iN}$ is defined as:

$$\mathcal{P}_i = \begin{bmatrix} U & & & & \\ -^i\hat{P}_{N-1} & U & 0 & & \\ 0 & -^i\hat{P}_{N-2} & U & & \\ 0 & 0 & & & \\ \vdots & \vdots & & & \\ 0 & 0 & 0 & -^i\hat{P}_1 & U \end{bmatrix}$$

Note according to our notation, $\boldsymbol{^iP_{j+1,j}} = \boldsymbol{^iP_j}$.

2.3 C5 joint parallel robot description

The C5 joint parallel robot [7] consists of a static and a mobile part connected together by six actuated segments (Fig 2 and 3). Each segment is embedded to the static part at point A_i and linked to the mobile part through a C5 passive joint (3 degrees of freedom (DOF) in rotation and 2 DOF in translation) at point B_i. Each C5 joint consists of a spherical joint tied to two crossed sliding plates (Fig 4). Each segment is equipped with a ball and a screw linear actuator driven by a DC motor.

The used notation to describe the parallel robot is defined in the following.

- R_b is the absolute frame, tied to the fixed base: $R_b = (0, x, y, z)$.
- R_p is the mobile frame, tied to the mobile part: $R_p = (C, x_p, y_p, z_p)$.
- Let O be the origin of the absolute coordinate system.

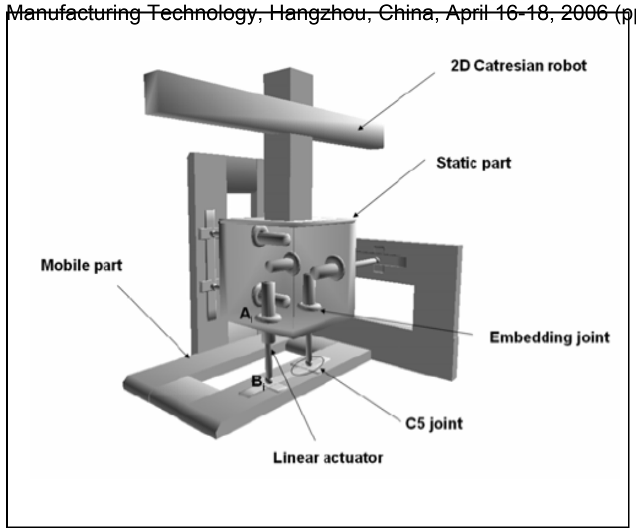

Figure 3: C5 parallel robot representation.

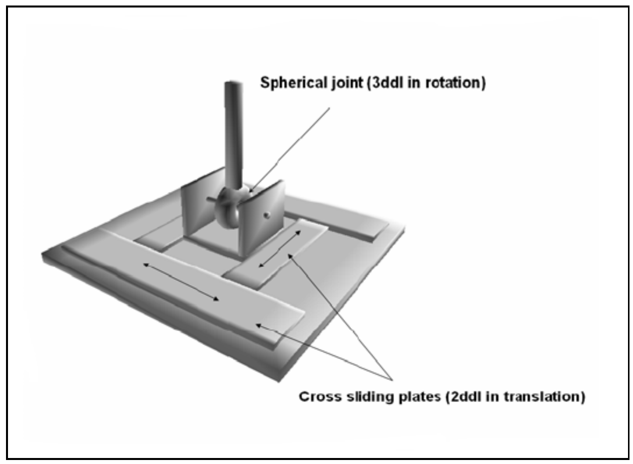

Figure 4: Detail of the C5 joint.

- Let C (or O_{N+1}) be the origin of the mobile coordinate system, whose coordinates are in the absolute frame:

$$\boldsymbol{OC}_{/R_b} = \begin{bmatrix} x_c & y_c & z_c \end{bmatrix}^t$$

- A_i (or $^iO_1$) is the center of the joint between the segment i and the fixed base:

$$\boldsymbol{OA_i}_{/R_b} = \begin{bmatrix} a_i^x & a_i^y & a_i^z \end{bmatrix}^t$$

- B_i (or ^iO_N) is the center of the joint between the segment i and the mobile part:

$$\boldsymbol{CB_i}_{/R_p} = \begin{bmatrix} b_i^x & b_i^y & b_i^z \end{bmatrix}^t$$

- $[R]$ is the rotation matrix of r_{ij} elements (in the RPY formalism), expressing the orientation of the R_p coordinate system with respect to the R_b coordinate system. The expression for this matrix is given by:

$$[R] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \qquad (1)$$

where:
$r_{11} = \cos\beta\cos\gamma$
$r_{12} = -\cos\beta\sin\gamma$
$r_{13} = \sin\beta$
$r_{21} = \sin\gamma\cos\alpha + \cos\gamma\sin\beta\sin\alpha$
$r_{22} = \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma$
$r_{23} = -\cos\beta\sin\alpha$
$r_{31} = \sin\gamma\sin\alpha - \cos\gamma\sin\beta\cos\alpha$
$r_{32} = \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma$
$r_{33} = \cos\beta\cos\alpha$

- α, β and γ are the Bryan angles [17], describing the rotation of the mobile platform with respect to the fixed part.

- $\boldsymbol{X}$ is the task coordinate vector.

$$\boldsymbol{X} = \begin{bmatrix} \alpha & \beta & \gamma & x_c & y_c & z_c \end{bmatrix}^t$$

- R_{b_i} is the frame tied to the segment i: $R_{b_i} = (A_i, x_{b_i}, y_{b_i}, z_{b_i})$.

- α_{b_i}, β_{b_i}, γ_{b_i} are the angles, in the RPY formalism, describing frame R_{b_i} rotation with respect to the absolute frame R_b.

- α_{p_i}, β_{p_i}, γ_{p_i} are the angles, in the RPY formalism, describing the mobile platform rotation with respect to the frame R_{b_i}.

3 Factorized expression of the Jacobian matrix

3.1 General approach

The differential kinematic model of a manipulator can be defined by the relationship between the spatial velocity of the end effector and the vector of generalized coordinate velocities of the robot: $\boldsymbol{V_{N+1}} = \mathcal{J}\dot{\boldsymbol{Q}}$, where $\mathcal{J}$ is the Jacobian matrix.
In the proposed approach, the parallel robot is considered as a multi-robot system, composed of serial robots (the segments) moving a common load (the

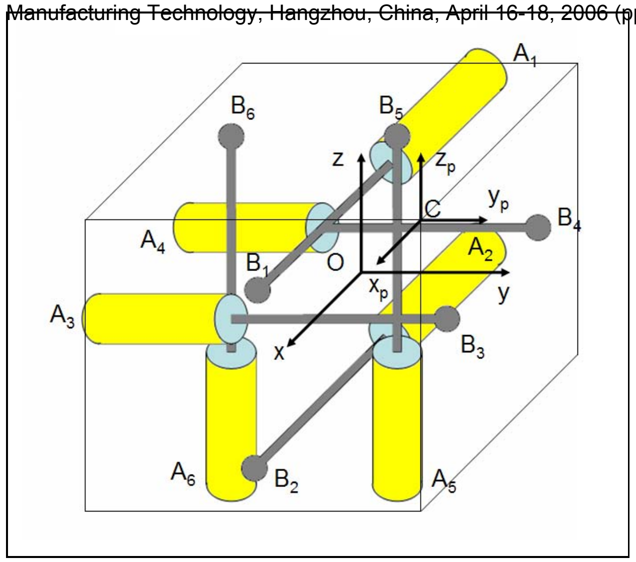

Figure 5: Scheme of the spatial arrangement of the C5 joint parallel robot segments.

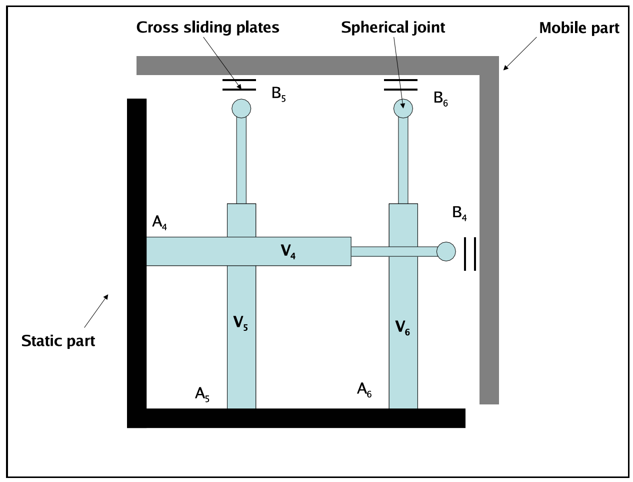

Figure 6: Projection of Fig. 4 onto YOZ plane.

mobile platform). A relationship linking the Jacobian matrix of the parallel robot ($\mathcal{J}$) to the Jacobian matrix of each segment ($\mathcal{J}_i$) is presented.
The principle of this approach consists of first computing the Jacobian matrix for each leg considered as an open serial chain. Secondly, the closing constraint is determined, allowing the computation of the parallel robot Jacobian matrix.
The velocity propagation for a serial chain of interconnected bodies is given by the following intrinsic equation [12][13][14]:

$$^{i}\boldsymbol{V_j} - {}^{i}\hat{P}^{t}_{j-1}\,{}^{i}\boldsymbol{V_{j-1}} = {}^{i}H_j\,{}^{i}\dot{\theta}_j \qquad (2)$$

By using the matrix $\mathcal{P}$, equation (2) can be expressed in a global form by:

$$\mathcal{P}_i^t \boldsymbol{\mathcal{V}_i} = \mathcal{H}_i \dot{\boldsymbol{Q}}_{\boldsymbol{i}} \tag{3}$$

thus:

$$\boldsymbol{\mathcal{V}_i} = \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \dot{\boldsymbol{Q}}_{\boldsymbol{i}} \tag{4}$$

The end effector spatial velocity $\boldsymbol{V_{N+1}}$ is obtained by the following relation:

$$\boldsymbol{V_{N+1}} - {}^i\hat{P}_N^t \, {}^{\boldsymbol{i}}\boldsymbol{V_N} = \boldsymbol{0} \tag{5}$$

thus:

$$\boldsymbol{V_{N+1}} = {}^i\hat{P}_N^t \, {}^{\boldsymbol{i}}\boldsymbol{V_N} \tag{6}$$

Let $\beta_i \in \Re^{6\times {}^iN}$ be the matrix defined by $\beta_i = \begin{bmatrix} {}^i\hat{P}_N^t & 0 & \cdots & 0 \end{bmatrix}$, equation (6) becomes:

$$\boldsymbol{V_{N+1}} = \beta_i \boldsymbol{\mathcal{V}_i} \tag{7}$$

Thus, inserting the expression of $\mathcal{V}_i$ from equation (4), we obtain:

$$\boldsymbol{V_{N+1}} = \beta_i \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \dot{\boldsymbol{Q}}_{\boldsymbol{i}} \tag{8}$$

Thus:

$$\mathcal{J}_i = \beta_i \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \tag{9}$$

The Jacobian matrix $\mathcal{J}$ of the parallel robot is obtained by the closing constraint determination of the kinematic chain. This determination can be obtained by expressing the actuated joint velocity $\dot{\boldsymbol{Q}}$ of the parallel robot in function of vectors $\dot{\boldsymbol{Q}}_{\boldsymbol{i}}$ associated to each segment i. Let the matrix Π_i be characterized by:

$$\dot{\boldsymbol{Q}}_{\boldsymbol{i}} = \Pi_i \, \dot{\boldsymbol{Q}} \tag{10}$$

Inserting equation (10) into (8), we obtain:

$$\boldsymbol{V_{N+1}} = \beta_i \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \Pi_i \dot{\boldsymbol{Q}} \tag{11}$$

Therefore, a factorized expression of the parallel robot Jacobian matrix is given by:

$$\mathcal{J} = \beta_i \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \Pi_i \tag{12}$$

The matrices $\mathcal{J}$ and $\mathcal{J}_i$ are linked by the following relationship:

$$\mathcal{J} = \mathcal{J}_i \, \Pi_i \tag{13}$$

The matrix computation of Π_i depends on the parallel robot architecture. In the following sections, we develop the computation of this matrix for the spatial C5 parallel robot.

3.2 Application to the C5 joint parallel robot

3.2.1 Preliminaries

According to our notations, we have :

$$k = 6, \forall i, {}^iM = 6, {}^iN = 2$$

Let ${}^{\boldsymbol{i}}\boldsymbol{P_N} = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^t$,

the propagation vector from B_i to C in the frame tied to the segment i (R_{b_i}):

$$ {}^{\boldsymbol{i}}\boldsymbol{P_N} = \boldsymbol{B_i C}_{/R_{b_i}} = \begin{bmatrix} {}^iR \end{bmatrix}^t \, [R] \; \boldsymbol{B_i C}_{/R_p} \tag{14}$$

${}^iR$ is the rotation matrix from the frame R_{b_i} to the base frame R_b and R the rotation matrix from the frame R_p to the base frame R_b. For the C5 parallel robot, the frames R_{b_i} are chosen parallel to the fixed frameR_b (each segment is embedded). Thus, we have:

$$ {}^iR = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (14) is written as:

$$\begin{cases} x_i = -r_{11}b_i^x - r_{12}b_i^y - r_{13}b_i^z \\ y_i = -r_{21}b_i^x - r_{22}b_i^y - r_{23}b_i^z \\ z_i = -r_{31}b_i^x - r_{32}b_i^y - r_{33}b_i^z \end{cases} \tag{15}$$

The spatial arrangement of the segments (see Fig. 5 and 6) is as follow:

- The segments 1 and 2 are in the direction of the x-axis ($Y_i = Z_i = 0$ for $i = 1, 2$).
- The segments 3 and 4 are in the direction of the y-axis ($X_i = Z_i = 0$ for $i = 3, 4$).
- The segments 5 and 6 are in the direction of the z-axis ($X_i = Y_i = 0$ for $i = 5, 6$).

Thus, we deduce the following relations:

$$\begin{matrix} y_1 = y_2 = y_c \\ z_3 = z_4 = z_c \\ x_5 = x_6 = x_c \end{matrix} \tag{16}$$

The global vector of articular coordinate velocity of the leg i is given by:

$$\dot{\boldsymbol{Q}}_{\boldsymbol{i}} = \begin{bmatrix} \dot{w}_{p_i} & \dot{u}_{p_i} & \dot{\gamma}_{p_i} & \dot{\beta}_{p_i} & \dot{\alpha}_{p_i} & \dot{\theta}_i^a \end{bmatrix}^t \tag{17}$$

where $\dot{u}_{p_i}$ and $\dot{w}_{p_i}$ are translation velocities due to the crossed sliding plates.
The vector of generalized coordinate velocity is given by:

$$\dot{\boldsymbol{Q}} = \begin{bmatrix} \dot{\theta}_6^a & \dot{\theta}_5^a & \dot{\theta}_4^a & \dot{\theta}_3^a & \dot{\theta}_2^a & \dot{\theta}_1^a \end{bmatrix}^t \tag{18}$$

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3.2.2 Matrix Π_i determination

The aim of this part is to determine the matrix Π_i given in equation (10).
We have:

$$\begin{bmatrix} \dot{w}_{p_i} \\ \dot{u}_{p_i} \\ \dot{\gamma}_{p_i} \\ \dot{\beta}_{p_i} \\ \dot{\alpha}_{p_i} \\ \dot{\theta}_i^a \end{bmatrix} = \Pi_i \begin{bmatrix} \dot{\theta}_6^a \\ \dot{\theta}_5^a \\ \dot{\theta}_4^a \\ \dot{\theta}_3^a \\ \dot{\theta}_2^a \\ \dot{\theta}_1^a \end{bmatrix} \quad (19)$$

The elements ${}^i\pi_{jk}$ of the matrix Π_i are computed by equation (8). This equation is true for $i = 1$ to 6, thus:

$$\beta_i \left(\mathcal{P}_i^t\right)^{-1} \mathcal{H}_i \dot{\boldsymbol{Q}}_{\boldsymbol{i}} = \beta_j \left(\mathcal{P}_j^t\right)^{-1} \mathcal{H}_j \dot{\boldsymbol{Q}}_{\boldsymbol{j}} \quad (20)$$

for i and $j = 1$ to 6.
From equation (20), we can show that for all $i, j = 1, \ldots, 6$ we obtain the following relations:

$$\begin{cases} \dot{\alpha}_{p_i} = \dot{\alpha}_{p_j} \\ \dot{\beta}_{p_i} = \dot{\beta}_{p_j} \\ \dot{\gamma}_{p_i} = \dot{\gamma}_{p_j} \end{cases} \quad (21)$$

After some manipulations on relation (20), we obtain:

- For $i = 1$ and $j = 2$

$$\dot{\theta}_1^a = (z_2 - z_1)\,\dot{\beta}_{p_i} + (y_1 - y_2)\,\dot{\gamma}_{p_i} + \dot{\theta}_2^a \quad (22)$$

- For $i = 3$ and $j = 4$:

$$\dot{\theta}_3^a = (z_3 - z_4)\,\dot{\alpha}_{p_i} + (x_4 - x_3)\,\dot{\gamma}_{p_i} + \dot{\theta}_4^a \quad (23)$$

- For $i = 5$ and $j = 6$:

$$\dot{\theta}_5^a = (y_6 - y_5)\,\dot{\alpha}_{p_i} + (x_5 - x_6)\,\dot{\beta}_{p_i} + \dot{\theta}_6^a \quad (24)$$

- For $i = 1$ and $j = 3$:

$$\dot{u}_{p_1} = (z_1 - z_3)\,\dot{\alpha}_{p_i} + (x_3 - x_1)\,\dot{\gamma}_{p_i} + \dot{\theta}_3^a \quad (25)$$

- For $i = 1$ and $j = 5$:

$$\dot{w}_{p_1} = (y_5 - y_1)\,\dot{\alpha}_{p_i} + (x_1 - x_5)\,\dot{\beta}_{p_i} + \dot{\theta}_5^a \quad (26)$$

From equation (16), we have $y_1 = y_2, z_3 = z_4$ and $x_5 = x_6$. Thus, the equations (22, 23, 24) can be written as follow:

$$\begin{aligned} \dot{\theta}_1^a &= (z_2 - z_1)\,\dot{\beta}_{p_i} + \dot{\theta}_2^a \\ \dot{\theta}_3^a &= (x_4 - x_3)\,\dot{\gamma}_{p_i} + \dot{\theta}_4^a \\ \dot{\theta}_5^a &= (y_6 - y_5)\,\dot{\alpha}_{p_i} + \dot{\theta}_6^a \end{aligned} \quad (27)$$

In the following, the matrix Π_1 is computed with equations (19), (25), (26) and (27). We obtain:

$$\Pi_1 = \begin{bmatrix} \frac{z_1-z_3}{y_5-y_6} & \frac{z_1-z_3}{y_6-y_5} & \frac{x_3-x_1}{x_3-x_4} & \frac{x_4-x_1}{x_4-x_3} & 0 & 0 \\ \frac{y_5-y_1}{y_5-y_6} & \frac{y_6-y_1}{y_6-y_5} & 0 & 0 & \frac{x_1-x_5}{z_1-z_2} & \frac{x_1-x_5}{z_2-z_1} \\ 0 & 0 & \frac{1}{x_3-x_4} & \frac{1}{x_4-x_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{z_1-z_2} & \frac{1}{z_2-z_1} \\ \frac{1}{y_5-y_6} & \frac{1}{y_6-y_5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

4 Singularity study

Singularities are particular configurations where the robot becomes uncontrollable. The singular configurations are determined by the analysis of the $\mathcal{J}^{-1}$ rank. There are two kinds of singularities [7] [20]:

1. Singularities of first type, which occur when the determinant of the matrix $\mathcal{J}^{-1}$ becomes infinite. In their configurations we can find nonzero vectors $\dot{\boldsymbol{Q}}$ for which $\boldsymbol{V_{N+1}}$ will be equal to zero.

2. Singularities of second type, which occur when the determinant of the matrix $\mathcal{J}^{-1}$ is equal to zero. They correspond to an uncontrollable displacement of the mobile part when all the active links are locked.

Several authors [7] [20] [21] [22] have extensively studied the singular configurations of mechanisms. Among them, Merlet [2] has proposed a geometric method based on the Grassman geometry. This method consists of defining the conditions of linear dependency between the Plucker vectors of the lines associated to the segments of the manipulator. This linear dependency leads to a degeneracy of the inverse Jacobian matrix as it is formed of these vectors.
In our case, we determine the singular configurations starting from the factorization Jacobian matrix $\mathcal{J}$. The equation (13) gives the Jacobian expression as (for $i = 1$):

$$\mathcal{J} = \mathcal{J}_1\,\Pi_1 \quad (29)$$

Thus

$$det\left(\mathcal{J}\right) = det\left(\mathcal{J}_1\right)\ det\left(\Pi_1\right) \quad (30)$$

The matrix $\mathcal{J}_1$ is given by the equation (9). We obtain:

$$\mathcal{J}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -y_1 & z_1 & 0 & 1 \\ 0 & 1 & x_1 & 0 & -z_1 & 0 \\ 1 & 0 & 0 & -x_1 & y_1 & 0 \end{bmatrix} \quad (31)$$

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We deduce the determinant of this matrix:

$$det\left(\mathcal{J}_1\right) = 1 \tag{32}$$

The matrix Π_1 is given by the equation (28). We deduce:

$$det\left(\Pi_1\right) = \frac{1}{(x_3 - x_4)\ (y_5 - y_6)\ (z_1 - z_2)} \tag{33}$$

- The singular configurations of first type are then defined when:

$$det\left(\Pi_1\right) = \infty \tag{34}$$

These singularities appear when:

$$x_3 = x_4 \text{ or } y_5 = y_6 \text{ or } z_1 = z_2$$

Considering the equations (1) and (15) and the geometrical considerations, $b_1^y = b_2^y$, $b_3^z = b_4^z$, $b_5^x = b_6^x$, given by Fig (5), we deduce:

1. Configrations for $x_3 = x_4$

$$\begin{array}{l} \cos\beta\cos\gamma\ (b_3^x - b_4^x) = 0 \\ \Rightarrow\ \ \beta = \pm\frac{\pi}{2},\ \ or\ \ \gamma = \pm\frac{\pi}{2} \end{array} \tag{35}$$

2. Configurations for $y_5 = y_6$

$$\begin{array}{l} (\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma)\ (b_5^y - b_6^y) = 0 \\ \Rightarrow \begin{cases} \alpha = \pm\frac{\pi}{2},\ \ and\ \ (\gamma = 0\ \ or\ \ \beta = 0) \\ \gamma = \pm\frac{\pi}{2},\ \ and\ \ (\alpha = 0\ \ or\ \ \beta = 0) \\ \alpha = \frac{\pi}{4}\ \ and\ \ \gamma = \frac{\pi}{4}\ \ and\ \ \beta = \frac{\pi}{2} \\ \alpha = -\frac{\pi}{4}\ \ and\ \ \gamma = -\frac{\pi}{4}\ \ and\ \ \beta = -\frac{\pi}{2} \\ \alpha = -\frac{\pi}{4}\ \ and\ \ \gamma = \frac{\pi}{4}\ \ and\ \ \beta = -\frac{\pi}{2} \\ \alpha = \frac{\pi}{4}\ \ and\ \ \gamma = -\frac{\pi}{4}\ \ and\ \ \beta = -\frac{\pi}{2} \end{cases} \end{array} \tag{36}$$

3. Configurations for $z_1 = z_2$

$$\begin{array}{l} \cos\beta\cos\alpha\ (b_1^z - b_2^z) = 0 \\ \Rightarrow\ \ \beta = \pm\frac{\pi}{2},\ \ or\ \ \alpha = \pm\frac{\pi}{2} \end{array} \tag{37}$$

Finally, first type singular configurations appear for $\alpha = \pm\frac{\pi}{2}$, or $\beta = \pm\frac{\pi}{2}$, or $\gamma = \pm\frac{\pi}{2}$, or for any combination of these values.

- The singular configurations of second type are defined when:

$$det\left(\Pi_1\right) = 0 \tag{38}$$

Considering the equation (33), these singularities appear when:

$$(x_3 - x_4)\ (y_5 - y_6)\ (z_1 - z_2) = \infty$$

These configurations are not geometrically possible for the C5 parallel robot, thus does not have any singularity of the second type.

5 Conclusion

In this paper we have presented a new factorization technique of the Jacobian matrix for parallel robots. This method which gives an analytical expression, without any need of the forward kinematic model, has been tested on the C5 joint parallel robot built in our laboratory.
The Jacobian matrix factorization have been used for singular configuration determination. The interest of our approach is within parallel robot simulation, design and operational space control. The dynamic modeling, based on this formalism is under investigation for the factorization of the inertia matrices (joint and operational spaces) and their inverses, leading to the modeling algebra for robot modeling and control [23].

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