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# Onset based optimization of multi-level mixed polarity Reed-Muller functions

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*Abstract:* - A novel onset based optimization method is proposed to convert a Fixed Polarity Reed-Muller (FPRM) function to a compact Multi-level Mixed Polarity Reed-Muller (MMPRM) forms. Onset table is introduced to represent an FPRM function. By operating on the onset table to extract common variables, its compact MMPRM is obtained. The spatial complexity of the proposed algorithm is O(M×n) (M is onset size and n is the number of input variables) with reasonable CPU time. The experimental results show that a significant area improvement is obtained compared with the published results.

*Key-Words:* - Reed-Muller, Logic Synthesis, Fixed Polarity, Mixed Polarity

## 1 Introduction

Reed-Muller (RM) logic has drawn increasing attention because the AND/XOR realization of the circuits require less layout area than their AND/OR counterparts in many applications [1]. Furthermore, AND/XOR PLAs require fewer product terms than AND/OR PLAs [2]. However, XOR gate has the disadvantage of low speed. As the FPGA technology has made significant progress in recent years, XOR gates can be implemented into Look-up Tables as fast as other gates, which changes the situation.

Conversion algorithms between standard Boolean and RM forms have been investigated [3-4]. Furthermore, many optimization techniques for two-level Fixed Polarity RM (FPRM) and mixed polarity RM were proposed in terms of area minimization and/or power minimisation [5-7]. Then, the method in [6] proposed to search all the possible fixed polarities to find the best polarity. For the fixed polarity RM forms, there are $2^n$ polarities for an $n$ variable function. The method may take significant CPU time for large functions. It would not be practical to use the method to optimize mixed polarity RM forms since the number of polarities could be up to $2^{n\cdot 2^{n-1}}$. Recently, truth vector based method for Multi-level Mixed Polarity RM (MMPRM) optimization was proposed in [7]. This method uses a truth vector with length $2^n$ to represent an $n$-variable FPRM expansion. By elimination and decomposition of the truth vector, the compact representation of MMPRM form can be obtained. The main disadvantage of this method is the rapid increase in memory for large functions. In this paper, a novel onset based approach is presented to obtain MMPRM, which requires less memory.

The rest of paper is organized as follows. Section 2 gives the properties of onset table. In section 3, extraction of common variables from on-set truth table is given. The proposed algorithm for the optimization of MMPRM is detailed in section 4. The improved results are presented in section 5. Conclusions are given in section 6.

## 2 Properties of Onset Table

Any $n$-variable Boolean function can be expressed as a two-level fixed polarity RM expansion as

$$f(\dot{x}_{n-1}\cdots\dot{x}_1\dot{x}_0)=\oplus\sum_{i=0}^{2^n-1}b_i\pi_i \tag{1}$$

where $\oplus\sum$ is the EXOR operator, $i$ is in binary form as $i=(i_{n-1}\cdots i_1 i_0)$ and $\pi_i=(\dot{x}_{n-1}\cdots\dot{x}_1\dot{x}_0)$. In the FPRM, $\dot{x}$ can only take either $x$ or its complementary but not both.

$$\dot{x}_j=\begin{cases}1 & i_j=0\\ \dot{x}_j & i_j=1\end{cases} \tag{2}$$

where $j\in\{0,1,\cdots,n-1\}$.

Definition 1: Any $n$-variable RM expansion can be expressed with Set O, which is composed of the decimal equivalent of the coefficients of $\pi$ terms. Set O is also called onset.

Example 1: Given a 3-variable function $f(\dot{x}_2\dot{x}_1\dot{x}_0)=\dot{x}_0\oplus\dot{x}_1\dot{x}_0\oplus\dot{x}_2\dot{x}_0\oplus\dot{x}_2\dot{x}_1\dot{x}_0$ under any polarity, it could be represented by $\pi$-terms as in $f(\dot{x}_2\dot{x}_1\dot{x}_0)=\pi_1\oplus\pi_3\oplus\pi_5\oplus\pi_7$ . It also can be expressed by an $O$ set as $O=\{1,3,5,7\}$, in which the decimal is the sub index of $b_i$.

Definition 2: Onset table, also called $T$ Table (briefly $T$), is to describe the existence of each variable in each $\pi$-term of a RM function. The structure of the $T$ is as follows: the decimal corresponding to the binary digit in each row of the $T$ represents each element of Set $O$ ; each column of the $T$ represents an input variable of the RM function; $k_{ij}$ , where $k_{ij} \in \{0,1\}$ , is a binary digit on the $i$th row and $j$th column of the $T$ , and $k_{ij}=1$ means that the variable $\dot{x}_j$ on the $j$th column appears in $\pi_i$ term while $k_{ij}=0$ means it does not.

Example 2: Given $O=\{1,3,5,7\}$ under any polarity and the corresponding $T$ is shown in Fig. 1.

| T | $\dot{x}_2$ | $\dot{x}_1$ | $\dot{x}_0$ |
|---|---|---|---|
| $\pi_1$ | 0 | 0 | 1 |
| $\pi_3$ | 0 | 1 | 1 |
| $\pi_5$ | 1 | 0 | 1 |
| $\pi_7$ | 1 | 1 | 1 |

Fig. 1 An example of onset table

From Definition 2 on the T table, it is clear that the logic relationship between rows is exclusive-OR while the relationship between columns is logic "AND".

Relationship 1: $f(\dot{x}_{n-1},\cdots,\dot{x}_0)=1$ corresponds to the $T_1$ in Fig.2(a).

Relationship 2: $f(\dot{x})=\bar{x}$ corresponds to the $T_{\bar{x}}$ in Fig.2(b).

| $T_1$ | $\dot{x}_{n-1}$ | $\cdots$ | $\dot{x}_0$ |
|---|---|---|---|
| $\pi_0$ | 0 | $\cdots$ | 0 |

(a)

| $T_{\bar{x}}$ | $\dot{x}$ |
|---|---|
| $\pi_0$ | 0 |
| $\pi_1$ | 1 |

(b)

Fig. 2 $T_1$ and $T_{\bar{x}}$

Corollary 1: Suppose $T \rightarrow f$ is a mapping from $T$ to the RM expression. If any two rows or two columns of $T$ are swapped mutually, a new $T'$ is generated. Let $T \rightarrow f$ and $T' \rightarrow f'$. If $T \rightarrow f$ and $T' \rightarrow f'$, then $f=f'$.

Example 3: Given the 3-variable function shown in Example 1, $T'$ is generated after swapping the third row and fourth row, as shown inFig.3.. The function remains the same as shown in (3).

$$
\begin{aligned}
& f'(\dot{x}_2\dot{x}_1\dot{x}_0) \\
&= \pi_1' \oplus \pi_3' \oplus \pi_5' \oplus \pi_7' \\
&= \pi_1 \oplus \pi_3 \oplus \pi_7 \oplus \pi_5 \\
&= \dot{x}_0 \oplus \dot{x}_1\dot{x}_0 \oplus \dot{x}_3\dot{x}_2\dot{x}_1 \oplus \dot{x}_2\dot{x}_0 \\
&= \dot{x}_0 \oplus \dot{x}_1\dot{x}_0 \oplus \dot{x}_2\dot{x}_0 \oplus \dot{x}_3\dot{x}_2\dot{x}_1 = f(\dot{x}_2\dot{x}_1\dot{x}_0)
\end{aligned} \tag{3}
$$

| T | $\dot{x}_2$ | $\dot{x}_1$ | $\dot{x}_0$ |
|---|---|---|---|
| $\pi_1'$ | 0 | 0 | 1 |
| $\pi_3'$ | 0 | 1 | 1 |
| $\pi_5'$ | 1 | 1 | 1 |
| $\pi_7'$ | 1 | 0 | 1 |

Fig. 3 $T'$ after $\pi_5$ and $\pi_7$ are swapped

The same conclusion can be obtained if the columns are swapped.

Definition 3: $T$ can be divided into $u$ sub-tables either in the horizontal or in the vertical direction, where $u$ is an integer and $u \geq 1$ . The sub-table is notated as $ST_i$, where $i \in \{1,2,\cdots,u\}$. Any sub-table $ST_i$ that covers some of the $\pi$-terms of the $T$ in the horizontal direction is notated as $PST_i$ . Any sub-table $ST_i$ that covers some of the variables of $T$ in the vertical direction is notated as $VST_i$ .

Example 4: If $T$, as shown in Fig. 1, is divided into 4 sub-tables, as in Fig. 4, $PST_1=\{\pi_1,\pi_3,\pi_5\}$ and $VST_2=\{\dot{x}_0\}$ .

| T | $\dot{x}_2$ | $\dot{x}_1$ | $\dot{x}_0$ |
|---|---|---|---|
| $\pi_1$ | 0 | 0 | 1 |
| $\pi_3$ | 0 $ST_1$ | 1 | 1 $ST_2$ |
| $\pi_5$ | 1 | 0 | 1 |
| $\pi_7$ | 1 $ST_3$ | 1 | 1 $ST_4$ |

Fig. 4 An example of sub-tables of $T$

Corollary 2: Given any two sub-tables, $ST_i$ and $ST_j$ in the $T$ , if $PST_i=PST_j$ , these two sub-tables, $ST_i$ and $ST_j$ can be grouped into a new sub-table $ST_k$ in the vertical direction but without changing the logic functionality of $f$, where $ST_k=\{ST_i,ST_j\}$ . The same is true for $VST_i$ and $VST_j$ .

Definition 4: If $ST_i$ covers all $\pi$-terms of the $T$, it is called $WPST_i$. The variable subset covered by the $WPST_i$ is called $VWPST_i$. If $ST_i$ covers all variables of the $T$, it is called $WVST_i$. The $\pi$- term subset covered by the $WVST_i$ is called $PWVST_i$.

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Corollary 3: Given an $n$-variable function $f$, its corresponding $T$ and $WPST$, where $WPST \subset T$. The variable subset covered by $WPST$ is $VWPST$, where $VWPST \in \{x_m, x_{m-1}, \cdots, x_{q+1}, x_q\}$ and $m>q$. Let $WPST \to s\pi$ and $T^{'} = T - WPST$. If $T \to f$, $T^{'} \to f^{'}$ and each element in the $WPST$ is "1", the $f$ can be rewritten as

$$f = s\pi \bullet f^{'} \tag{4}$$

where $s\pi = \dot{x}_m \bullet \dot{x}_{m-1} \bullet \cdots \bullet \dot{x}_q$.

Example 5: From Fig. 1, $VWPST = \{\dot{x}_0\}$ and hence $s\pi = \dot{x}_0$ can be extracted from the $T$, then, $f^{'}(\dot{x}_2 \dot{x}_1) = 1 \oplus \dot{x}_1 \oplus \dot{x}_2 \oplus \dot{x}_2 \dot{x}_1$ . According to (4), $f(\dot{x}_2 \dot{x}_1 \dot{x}_0) = s\pi \bullet f^{'} = \dot{x}_0 \bullet (1 \oplus \dot{x}_1 \oplus \dot{x}_2 \oplus \dot{x}_2 \dot{x}_1)$.

Definition 5: If two subsets $ST_1$ and $ST_2$ have the same row sizes and satisfy $VST_1 = VST_2$ , and the corresponding locations have the same elements, then $ST_1$ and $ST_2$ are deemed as equal, $ST_1 = ST_2$ .

Corollary 4: Given an $n$-variable function $f$ and its $T$ , $WVST_1$ and $WVST_2$, it holds $T = \{WVST_1, WVST_2\}$, or $T = WVST_1 \oplus WVST_2$. For the subset $SST_i$ where $SST_i \subset WVST_i$, if there is a variable subset $s\pi_i$ so that $WVST_i = s\pi_i \bullet SST_i$ , then we have $T = s\pi_1 \bullet SST_1 \oplus s\pi_2 \bullet SST_2$ . Further, if $SST_1 = SST_2 = SST$ , then $T = (s\pi_1 \oplus s\pi_2) \bullet SST$ . The following equation also holds.

$$f = (s\pi_1 \oplus s\pi_2) \bullet f^{'}$$

where $f^{'} \to SST$ .

Example 6: Given a 4-variable function $f(x_3 x_2 x_1 x_0) = x_2 x_1 \oplus x_2 x_1 x_0 \oplus x_3 x_2 \oplus x_3 x_2 x_0$ under polarity 0, Set O is $O = \{6, 7, 12, 13\}$. $T$ can be obtained as shown in Fig. 5. After changing the order of variables in $T$ , $T^{'}$ can be obtained as shown in Fig. 6. $T^{'}$ can be divided into two sub-tables, $WVST_1$ and $WVST_2$, where $WVST_1 = \{\pi_6^{'}, \pi_7^{'}\}$ and $WVST_2 = \{\pi_{12}^{'}, \pi_{13}^{'}\}$.

For $WVST_1$, there are $s\pi_1 = x_1$ and sub-table $SST_1$ such as $WVST_1 = x_1 \bullet SST_1$ . Also for $WVST_2$ , it has $WVST_2 = x_3 \bullet SST_2$. Since $SST_1 = SST_2 = SST$ and both of them cover variables $\{x_2, x_0\}$, $T = (x_1 \oplus x_3) \bullet SST$ . To obtain the RM expansion corresponding to $T$, the logic expansion corresponding to $SST$ is required. Based on Definition 2 and two mapping relationships, we have $SST \to f^{'} = x_2 \overline{x_0}$ .

Then, $f = (x_1 \oplus x_3) \bullet f^{''} = (x_1 \oplus x_3) \bullet x_2 \overline{x_0}$ .

| | $x_3$ | $x_2$ | $x_1$ | $x_0$ |
|---|---|---|---|---|
| $\pi_6$ | 0 | 1 | 1 | 0 |
| $\pi_7$ | 0 | 1 | 1 | 1 |
| $\pi_{12}$ | 1 | 1 | 0 | 0 |
| $\pi_{13}$ | 1 | 1 | 0 | 1 |

Fig. 5 $T$ of a 4-variable function

| | $x_1$ | $x_3$ | $x_2$ | $x_0$ | | |
|---|---|---|---|---|---|---|
| $\pi_6^{'}$ | 1 | 0 | 1 | 0 | $SST_1$ | $WVST_1$ |
| $\pi_7^{'}$ | 1 | 0 | 1 | 1 | | |
| $\pi_{12}^{'}$ | 0 | 1 | 1 | 0 | $SST_2$ | $WVST_2$ |
| $\pi_{13}^{'}$ | 0 | 1 | 1 | 1 | | |

Fig. 6 $T^{'}$ after changing the order of variables

# 3 Extraction of Common Variables

In the process of the simplification of RM expansion, the important method is the extraction of common variables. There are two kinds of common variables called global and local common variables, where "global" means the variable exists in all $\pi$ terms while "local" means the variable exists in some of $\pi$ -terms.

According to Corollary 3, the global common variables can be easily obtained. Unfortunately, in reality, global common variables are not always available. Therefore, extraction of common variables usually is for local ones.

However, it is not straightforward to extract local common variables in a given $T$. The following procedure will help to achieve this given a $T$ with $R$ rows and $C$ columns.

Procedure 1: Extraction of local common variables

1. Generate a sub-table $WPST_{xi}$ from the $T$ table for each variable, where $i \in \{0, 1, \cdots, C-1\}$ .
2. Generate the sub-table $ST_{\dot{x}_i}$ by deleting $WPST_{xi}$ from $T$ , where $ST_{xi}$ has $R$ rows with $C$-1 columns.
3. Categorise the $\pi$ -terms into different classes based on the same elements in a row. Count the 1s, $n_{\dot{x}_i}$ , for each class and record $\max(n_{\dot{x}_i})$ .
4. Obtain $C$ number of $max(n_{x_i})$ for all $ST_{\dot{x}_i}$ by repeating steps 1 and 3.
5. Divide the $T$ into sub-tables again. Delete $k(k = [c/2], k > 0)$ $WPST_{\dot{x}_i}$ , which have larger $n_{\dot{x}_i}$ , from the $T$ and record corresponding columns $\{C_k\}, k \in \{1, 2, \cdots, C\}$ .

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6. The new generated $T'$ has $R$ rows and $(C-k)$ columns. Repeat Step 3 for $T'$. Obtain the classes corresponding to $max(n_{x_i})$ and record their rows $\{R_j\}, j \in \{1,2,\cdots,R\}$;

7. Reorganize the $T$. Carry out row exchange so that the rows of $\{R_j\}$ from Step 5 are gathered as a $WVST_1$ and the rest is a $WVST_2$, and have $T=\{WVST_1, WVST_2\}$; Carry out column exchange so that the columns $\{C_k\}$ are gathered. Then divide the $WVST_1$ into two sub-tables $ST_{11}$ and $ST_{12}$ along a column so that all of the elements in one of the sub-table are 1s. Then, local common variables can be extracted.

The following example shows how it works.

Example 7: Given a 5-variable RM expansion, its $O$ Set is $O=\{1,6,9,21,23,26,31\}$ and its function is

$$f(x_4,x_3,x_2,x_1,x_0)=x_0 \oplus x_2x_1 \oplus x_3x_0$$
$$\oplus x_4x_2x_0 \oplus x_4x_2x_1x_0 \oplus x_4x_3x_1 \oplus x_4x_3x_2x_1x_0 \quad (5)$$

The corresponding $T$ is drawn in Fig. 7. Based on the above algorithm, it can be carried out as follows:

1. Generate sub-table $WPST_{xi}$ from $T$ for each variable.
2. Generate sub-table $ST_{xi}$ by deleting $WPST_{xi}$ from $T$ and regrouping the rest of $T$ into $ST_{xi}$. For example, if $WPST_{x3}$ is generated for variable $x_3$ and $ST_{x3}$ is generated after excluding $WPST_{x3}$ and regrouping the rest of *T*. Fig. 8 shows the resulting sub-table $ST_{x3}$.
3. According to the content in each row of $ST_{x3}$, $ST_{x3}$ can be categorised into 5 classes: $\{s\pi_1, s\pi_9\}$, $\{s\pi_{23}, s\pi_{31}\}$, $\{s\pi_6\}$, $\{s\pi_{21}\}$ and $\{s\pi_{26}\}$. The number of "1"s in each class is 2,8,2,3 and 2 respectively. Hence $max(n_{x3})$=8.
4. Similarly, $max(ST_{x0})$=4, $max(ST_{x1})$=6, $max(ST_{x2})$=4 and $max(ST_{x4})$=4 are obtained.
5. Since $k$=[$C$/2]=[2.5]=2, deleting $WPST_{x1}$ and $WPST_{x3}$ from $T$, we have $T'$ as shown in Fig. 9. Record $\{C\}=\{x_1,x_3\}$.
6. $T'$ can be categorised into 4 classes: $\{\pi_1, \pi_9\}$, $\{\pi_6\}$, $\{\pi_{26}\}$ and $\{\pi_{21}, \pi_{23}, \pi_{31}\}$. $\{\pi_{21}, \pi_{23}, \pi_{31}\}$ has the largest number of 1s. Hence, $\{R_j\}$ = $\{\pi_{21}, \pi_{23}, \pi_{31}\}$.
7. Divide $T$ into $WVST_1$ and $WVST_2$ such as $T=\{WVST_1, WVST_2\}$. $WVST_1$ can be further divided into two sub-tables $ST_{11}$ and $ST_{12}$. Hence $T=\{WVST_1, WVST_2\}=\{\{ST_{11}, ST_{12}\}, WVST_2\}$, as shown in Fig. 10. If $WVST_2 \rightarrow f_2$, $WVST_1 \rightarrow f_1$, $ST_{11} \rightarrow f_{11}$ and $ST_{12} \rightarrow s\pi$, $f = s\pi \bullet f_{11} \oplus f_2$, where $s\pi = x_4x_2x_0$ is local common variables.

| | $x_4$ | $x_3$ | $x_2$ | $x_1$ | $x_0$ |
|---|---|---|---|---|---|
| $\pi_1$ | 0 | 0 | 0 | 0 | 1 |
| $\pi_6$ | 0 | 0 | 1 | 1 | 0 |
| $\pi_9$ | 0 | 1 | 0 | 0 | 1 |
| $\pi_{21}$ | 1 | 0 | 1 | 0 | 1 |
| $\pi_{23}$ | 1 | 0 | 1 | 1 | 1 |
| $\pi_{26}$ | 1 | 1 | 0 | 1 | 0 |
| $\pi_{31}$ | 1 | 1 | 1 | 1 | 1 |

Fig. 7 *T* of a 5-variable RM expansion

| | $x_4$ | $x_2$ | $x_1$ | $x_0$ |
|---|---|---|---|---|
| $\pi'_1$ | 0 | 0 | 0 | 1 |
| $\pi'_6$ | 0 | 1 | 1 | 0 |
| $\pi'_9$ | 0 | 0 | 0 | 1 |
| $\pi'_{21}$ | 1 | 1 | 0 | 1 |
| $\pi'_{23}$ | 1 | 1 | 1 | 1 |
| $\pi'_{26}$ | 1 | 0 | 1 | 0 |
| $\pi'_{31}$ | 1 | 1 | 1 | 1 |

Fig. 8 The resulting sub-table $ST_{x3}$ after deleting variable $x_3$ from $T'$

| | $x_4$ | $x_2$ | $x_0$ |
|---|---|---|---|
| $\pi'_1$ | 0 | 0 | 1 |
| $\pi'_6$ | 0 | 1 | 0 |
| $\pi'_9$ | 0 | 0 | 1 |
| $\pi'_{21}$ | 1 | 1 | 1 |
| $\pi'_{23}$ | 1 | 1 | 1 |
| $\pi'_{26}$ | 1 | 0 | 0 |
| $\pi'_{31}$ | 1 | 1 | 1 |

Fig. 9 $T'$ after deleting $WPST_{x1}$ and $WPST_{x3}$

| | $x_4$ | $x_2$ | $x_0$ | $x_3$ | $x_1$ | |
|---|---|---|---|---|---|---|
| $\pi_1'$ | 0 | 0 | 1 | 0 | 0 | |
| $\pi_6'$ | 0 | 1 | 0 | 0 | 1 | |
| $\pi_9'$ | 0 | 0 | 1 | 1 | 0 | $WVST_2$ |
| $\pi_{26}'$ | 1 | 0 | 0 | 1 | 1 | |
| $\pi_{21}'$ | 1 | 1 | 1 | 0 | 0 | |
| $\pi_{23}'$ | 1 $ST_{12}$ | 1 | 1 | 0 $ST_{11}$ | 1 | $WVST_1$ |
| $\pi_{31}'$ | 1 | 1 | 1 | 1 | 1 | |

Fig. 10 The resulting sub-table $T'$

## 4 Onset Table Method for Multi-level Mixed Polarity RM

A given fixed polarity RM expansion can be represented as an onset table. Using the properties of onset table and proposed extraction of common variables in sections 2 and 3, the onset table can be divided into smaller sub-tables by extracting shared common variables, which leads to compact MMRM expansion.

To improve the efficiency of the proposed algorithm, two stop criteria are set as follows.

1. Only one π-term appears in the table.
2 Only one variable appears in each π-term.

Fig. 11 shows the algorithm of the onset table method for MMPRM.

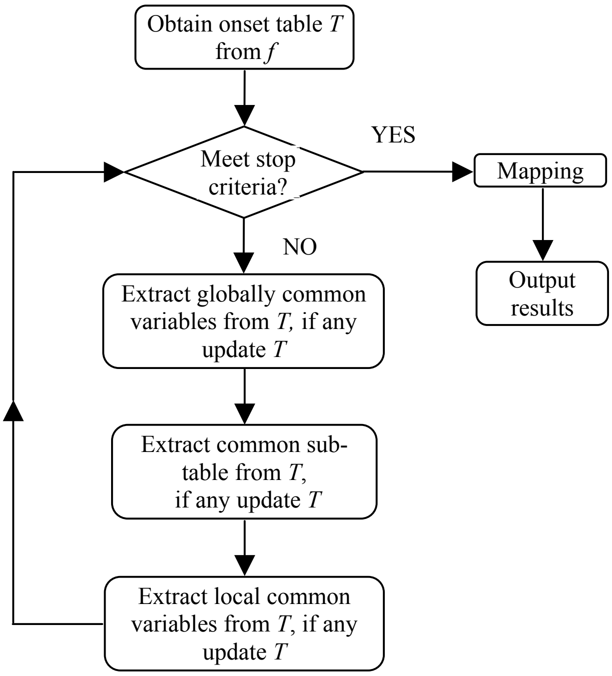


Fig. 11 Onset table method for MMPRM

Take Example 7 in Section 3 for example. We can obtain the expressions of $f_{11}$ and $f_2$ as follows.

Carrying out Steps 3 to 7 in Procedure 1 on two sub-tables $ST_2$ and $ST_{11}$ in Example 7, these sub-tables can be divided into some smaller sub-tables, which can be mapped into a MMPRM form based on Definition 2 and some mapping relationships in Section 2.

Finally, we have

$$f(x_4,x_3,x_2,x_1,x_0) = x_0\bar{x}_3 \oplus x_1(x_2 \oplus x_4x_3) \oplus x_4x_2x_0(\bar{x}_1 \oplus x_3x_1) \quad (6)$$

It can be seen that in equation 5, the number of literals is 20 but it only needs 12 literals in equation 6. Hence, the area saving is 40%.

## 5 Experimental Results

The proposed algorithm is implemented in C. The results are obtained by a PC with Intel P4/1.8G under Windows 2000. The proposed algorithm is applied to MCNC benchmark circuits. Performance is measured on 7 MCNC benchmark circuits and three randomly generated circuits. The test circuit size is up to 25 input variables. The number of literals is used to measure the area of the circuit implementation.

The area improvement is defined as

$$imp = \frac{literals - literals\ in\ MMPRM}{literals} \times 100\% \quad (7)$$

Here, the "literals" stands for the results from [6] while the "literals in MMPRM" from the proposed method.

Table 1 shows the comparison results from the RM expansion under polarity 0 to a MMPRM expansion. The first column is circuit name, the second is the number of input variables, the third and the fourth list the number of literals, and the last column shows the literal improvement. It can be seen that the maximum improvement could be up to 80% and the average improvement is 68%.

Table 2 shows the comparison results from the FPRM expansion under the best polarity to a MMPRM expansion. The columns have the same meaning as in Table 1. Although the improvement is less than under polarity 0, it still can achieve 69% at maximum and 55% on average, respectively.

In terms of the spatial complexity, the proposed algorithm only need to store a $T$. Therefore, it can be estimated that the spatial complexity is $O(M \times n)$, where $M$ is the element number of the onset and $n$ is the number of input variables for a specific function.

For the time complexity, the CPU time used to solve a function with 25 input variables is 236 seconds using stated PC. This should not be a major problem with the rapid improvement of today's computer performance.

Table 1 The comparison from a RM expansion under polarity 0 to a MMPRM expansion

| Name | *i* | Literals from [6] | Literals from the proposed | imp(%) |
|---|---|---|---|---|
| 9sym | 9 | 756 | 304 | 60 |
| newill | 8 | 237 | 70 | 70 |
| newtag | 8 | 88 | 27 | 69 |
| life | 9 | 792 | 321 | 59 |
| ryy6 | 16 | 624 | 168 | 73 |
| sym10 | 10 | 1300 | 528 | 59 |
| t481 | 16 | 108 | 55 | 49 |
| test_21 | 21 | 135273 | 27304 | 80 |
| test_22 | 22 | 153654 | 30262 | 80 |
| test_25 | 25 | 90209 | 20399 | 77 |
| Avg. (%) | | | | 68 |

Table 2 The comparison from a FPRM expansion under the best polarity to a MMPRM expansion

| Name | *i* | Literals from [6] | Literals from the proposed | imp(%) |
|---|---|---|---|---|
| 9sym | 9 | 636 | 276 | 57 |
| newill | 8 | 78 | 24 | 69 |
| newtag | 8 | 27 | 15 | 44 |
| life | 9 | 596 | 218 | 63 |
| ryy6 | 16 | 464 | 171 | 63 |
| sym10 | 10 | 1300 | 528 | 59 |
| T481 | 16 | 40 | 28 | 30 |
| Avg. (%) | | | | 55 |

## 6 Conclusions

In this paper, a novel onset table (or *T* table) based method is proposed to obtain a compact MMPRM from an FPRM form. This method takes much less memory than previous method [7]. With the efficient extraction of common variables, the onset table is divided into smaller sub-tables. Using the mapping relationship between the *T* table and the RM expansion, the compact MMPRM form can be obtained. The experimental results show a great improvement of literals can be achieved compared to the published results.

The future work will develop algorithms to obtain the minimum MMPRM form through searching $2^n$ polarities under the FPRM.

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