

Assembly Variation Analysis for Deformable Sheet Metal

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Abstract: Assembly variation must be calculated in the process of product design based on part tolerance, and it is assumed that the part is rigid in the process of assembly. However, the conventional addition theorem of variance is no longer valid for deformable sheet metal assemblies. The Monte-Carlo method is the simplest and the most popular method in this situation, but its calculation amount is huge. In this paper, some numbers based on Number Theoretic method are used to calculate the assembly variation, and it can account for the effect of elastic and plastic deformation. Consequently, it is indicated that this method is simple and practical as the Monte Carlo method, and the calculation efficiency can be decreased remarkably also.

Key-Words: sheet metal, variation, number theoretic method

1 Introduction

Sheet metal assembly is widely used in industries such as auto-body, aerospace, electronic and furniture-making. The overall quality of the product is reflected directly on the three dimensional variation of product assembly, such as auto-body assembly. The assembly variation is a key factor for diagnosing and controlling the overall quality of the product, as a result, it is attached to more attention in industry as a key subject to control the quality of the product^[1].

The assembly variation of the product can be obtained based on part tolerances according to certain method, which is called variation analysis. The most commonly used variation analysis methods are Root Sum Squares (RRS) method and Monte-Carlo method. Compared to Worst Case, RRS method is applicable to linearity assembly function and Monte-Carlo Simulation is applicable to nonlinearity assembly function^[2,3].

In these methods, the individual parts are considered as rigid bodies, and the individual deformation in the assembly process are ignored. However, for assemblies that consist of deformable parts, such as auto-body assembly, the

deformation of parts must be considered in the assembly process. In this paper, for accounting the deformation of sheet metals, the assembly variation is calculated using finite element method and NT-net numbers which are produced by number theoretic method. And the calculation result of number theoretic method is compared to the result of Monte-Carlo Simulation and the Method of Influence Coefficient.

2 Tolerance analysis

2.1 The methods of tolerance analysis

Tolerance analysis is a process obtaining assembly variance from the tolerance of parts. The most commonly used methods of tolerance analysis are Worst Case, Root Sum Squares (RSS) method and Monte-Carlo Simulation.

Worst Case is a simple method, it evaluates the assembly under the assumption that all parts are built to their extreme value. However, it generally requires very tight and unrealistic tolerance for the parts in order for the final assembly tolerance to meet the design specifications, which results in the increasing cost of the product. In Root Sum Squares method, the

tolerances of parts are specified as normal distribution, the relationship between the variation of assembly and the tolerances of parts is linear. Compared to Worst Case, RRS method yields a more realistic estimate and looser parts tolerances. However, both the Worst Case and RRS methods are generally applied to one dimensional assembly, while Monte-Carlo Simulation can be applied to complex two dimensional and three dimensional assemblies.

Monte-Carlo Simulation method is the simplest and the most popular method for nonlinear statistical tolerance analysis. Random values for each component are generated according to the component distributions, and the value of response function is computed for each set of component values. A sample of response function value is thus generated, and the moments of this population are computed using the standard statistical formulate. Monte-Carlo Simulation lends itself well to the case where the component parameters have distributions other than normal, because only the random number generator needs to be modified to represent any other kind of distribution. It is the most perfect method in theory. However, the main drawback of this method is that, to get accurate estimates of the moments, it is necessary to generate very large samples, and this is computationally expensive and time consuming. On the other hand, the results may be quite inaccurate if the Monte-Carlo runs without enough samples.

2.2 Background

In most methods, the individual parts are considered as rigid bodies, and the aggregate behaviors are determined by geometric relations. However, for assemblies that consist of deformable parts, such as deformable sheet metals, the part variation dose not stack-up as any of this models predicted because of possible part deformation. Takezawa applied linear regression models to predict auto body panel assembly variation using real production data^[4]. He concluded that “the conventional addition theorem

of variance is no longer valid for deformable sheet metal assemblies. The assembly variance has decreased (compared to part variances), and is closer to the variance of the stiffer part.” Liu summed up the methods of the sheet metal assemblies with one dimensional in practice, and analyzed the stack-up variances of assemblies using the finite element method^[5]. They proposed the Method of Influence Coefficients, in which finite element modeling is used to calculate the variances of the sheet metal assemblies^[6]. Compared to the Monte-Carlo Simulation method, this method improves computational efficiency. In this method, two finite element runs are sufficient to derive the sensitivity matrix needed for the variation simulation. It is assumed that the deformation of the parts is elastic in the process of assembly. And the same kind of method was used by Hu^[7]. By using Finite element method and Monte-Carlo Simulation, the effects of jointing contact on variation transformation were analyzed for the assembly of sheet metal parts^[8]. However, in practice, the plastic deformation can not be avoided because of clamp forces, assembly forces, welding heat and interaction. The Method of Influence Coefficient which based on linear finite element method is no longer applied to this case.

3 Number theoretic method

Number theoretic method (or quasi Monte-Carlo method) is integrated with Number theoretic and approximately analysis. The essence of Number Theory Method is to find uniformly distributed points in a domain of C^s . This sequence can be used to replace the random numbers based on Monte Carlo Simulation, and the uniformity can be illuminated by discrepancy magnitude. The uniformly distributed points which are produced by number theoretic method are called pseudo random numbers or NT-net.

3.1 *glp* aggregate and it's discrepancy of

Number Theoretic Method

Definition 1 Let m be a positive integer, if the residual number that integer a and integer b are divided by m are same, we call integer a and integer b congruence and note it as $a \equiv b(\text{mod } m)$.

definition2 Let $(n; h_1, \dots, h_s)$ be an integral vector, where $1 \leq h_i < n, h_i \neq h_j (i \neq j), s < n$ and the greatest common divisor $(n, h_i) = 1, i=1, \dots, s$. Let

$$\begin{cases} q_{ki} \equiv kh_i(\text{mod } n), \\ x_{ki} = (2q_{ki} - 1)/2n, \end{cases} k = 1, \dots, n, \quad i = 1, \dots, s \quad (1)$$

Where $1 \leq q_{ki} \leq n$. Set

$$P_n = \{ \mathbf{x}_k = (x_{k1}, \dots, x_{ks}), k = 1, \dots, n \} \quad (2)$$

Then $\{P_n\}$ is a set of points in C^s with lower discrepancy if $(n; h_1, \dots, h_s)$ are carefully selected.

Definition 3 For a set of points $P = \{ \mathbf{x}_k, k = 1, \dots, n \}$ in C^s , where $\gamma \in C^s$, let $N(\gamma, P)$ be the number of P satisfying $\mathbf{x}_k \leq \gamma$, and let

$$D(n, p) = \sup_{\gamma \in C^s} \left| \frac{N(\gamma, P)}{n} - v(|\theta, \gamma|) \right| \quad (3)$$

$D(n, P)$ is called the discrepancy of P , $v(|\theta, \gamma|) = \gamma_1 \cdots \gamma_s$ is denoted the volume of a rectangle $|\theta, \gamma|$.

The discrepancy is described as follows: The

absolute value of $\left| \frac{N(\gamma, P)}{n} - v(|\theta, \gamma|) \right|$ is smaller, the NT-net distributions is more uniformity, where n represents the number of all NT-net, $N(\gamma, P)$ represents the number of NT-net in rectangle $|\theta, \gamma|$, and $v(|\theta, \gamma|)$ represents the volume of rectangle $|\theta, \gamma|$. And this is the direct reason that we use discrepancy as the criterion to check up the uniformity of NT-net. Theorem1: For any prime number p , there is always an integral vector $\mathbf{h}_p = (h_1, \dots, h_s)$ to make the Number Theoretic net which is generated by $(p; h_1, \dots, h_s)$ has discrepancy,

$$D(n, p) < c(s)p^{-1}(\log p)^s \quad (4)$$

For a defined vector $(n; h_1, \dots, h_s)$, the Number Theoretic net in C^s is not always uniformity distributes. For a defined prime number p , we can select $\{h_i\}$ to make the vector $\{p; h_1, \dots, h_s\}$ uniformity distributes in C^s .

The vector (h_1, \dots, h_s) can be denoted as follows

$$(h_1, h_2, \dots, h_s) \equiv (1, a, a^2, \dots, a^{s-1}) \pmod{p} \quad (5)$$

Where $1 < a < p$, if it satisfying $a^i \neq a^j \pmod{p}$ and $1 \leq i < j < p$, then a is a primitive root of p .

Theorem2: Let D be a s dimensional rectangle $[a, b] = [a_1, b_1] \times \cdots \times [a_s, b_s]$, if $D = C^s$, then^[9]

$$DP(P, D) \leq 2\sqrt{s}D(n, P)^{1/s} \quad (6)$$

It has been proved that if P is good uniformity distributes in D , then the sequence discrepancy based on Theorem 2 for any $\varepsilon > 0$ is

$$DP(P, D) = O(n^{-1/s+\varepsilon}) \quad (7)$$

3.2 The random distributions of Monte Carlo Simulation

Generally speaking, Monte-Carlo Simulation needs expansive calculation in its simulation process, and the precision is direct ratio to the root of samples number. In another word, if we want to improve the precision ten times, we have to increase one hundred times of samples. Monte Carlo simulation is low because of the random numbers in $[0, 1]$ is not uniformly distributed^[9].

When function f satisfies $I(f^2) < \infty$, the average rate of convergence of Monte Carlo simulation is $O(1/\sqrt{n})$, and it is not less than

$$O(\sqrt{\ln(\ln(n))/n}) \text{ in any case}^{[9]}.$$

4 Example

For saving time, we use the same example in literature (Liu and Hu, 1997) to illustrate the variation simulation procedures, in which the plastic deformation of parts is ignored. Then we can compare the two results of Monte-Carlo Simulation and Method of Influence Coefficients.

4.1 Problem Definition

The example used is an assembly of two pieces of sheet metal joined by a lap joint. The sizes of the parts used in this example are $100 \times 100 \times 1 \text{mm}^3$ (part1), and $100 \times 100 \times 2 \text{mm}^3$ (part 2). The material of both parts is mild steel with Young modulus $E=20700 \text{N/mm}^2$ and Poisson ratio $\nu = 0.3$. The manner of two sheet metal joint as shown in Fig 1.

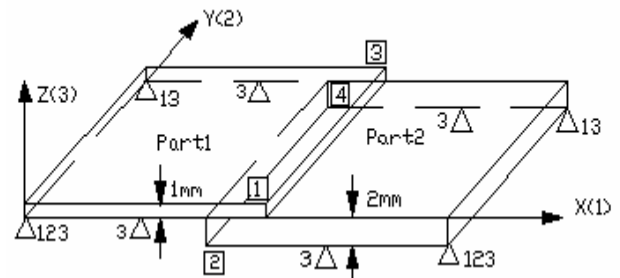


Fig.1 Sheet metal assembly using lap joint

4.2 Finite Element Modeling

The discretization of the problem is shown in Fig.2. 16 elements are generated for each part. Nodes are marked sequentially from 1 to 50 for two parts. After assembly, there are five pairs of nodes which will move “together” in the three dimensional space, they are nodes (21 and 26), (22 and 27), (23 and 28), (24 and 29), (25 and 30). Fixture elements are located at Nodes 21,22,23,24 and 25 on part 1, and nodes 26, 27, 28, 29 and 30 are at part 2.

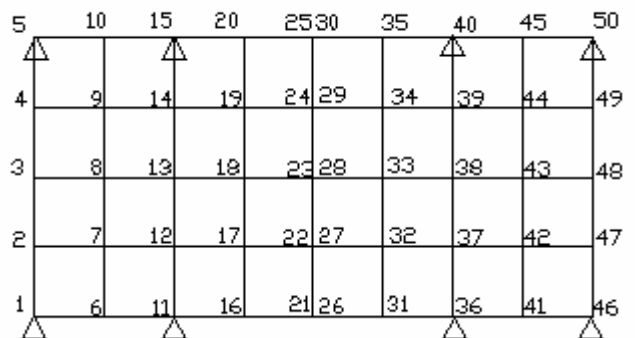


Fig.2 Mesh and nodes of the discretization

The small triangles \triangle with numbers represent the constraints provided by fixture elements, as shown in Fig.1. The numbers represent the directions of constraints provided by these fixtures. For example, a triangle \triangle with a number 123 at a node means that the displacements in the directions of $X(1)$, $Y(2)$ and $Z(3)$ are all constrained at that node.

For the sake of simplicity, only four sources of variation are considered. Four sources of variation at the free corners of the two flat sheet metal parts marked by small squares \square with numbers within

them. Number 1 and 3 in squares □ represent the sources of variation on part 1, number 2 and 4 represent the sources of variation on part 2.

4.3 simulation process

Suppose the mean deviation is 1mm in the Z-direction at each source of variation:

$$\{\mu_p\} = [1 \ 1 \ 1 \ 1]^T$$

(8)

And the standard deviation at each source of variation is also 1mm:

$$\{\sigma_p\} = [1 \ 1 \ 1 \ 1]^T$$

(9)

For the sake of simplicity, we assume that all the sources of variation are independent.

Monte-Carlo Simulation is a computer simulation of the sheet metal assembly process. The task is to calculate assembly variation (output) based on part variation. A sheet metal assembly parts process typically consists of the following steps:

(1) Loading sheet metal part to work-holding fixtures. Given the part variation V . If more than one source of variation are considered, parts variation will be expressed in a form of a vector $\{V_u\}$;

(2) Clamping parts to nominal positions. Clamp forces can be calculated by using an equation

$$\{F_u\} = [K_u]\{V_u\} \quad (10)$$

The components of vector $\{F_u\}$ represent forces provided by clamps, the vector $[K_u]$ represent the stiffness matrix; items with subscript u represent quantities of the unwelded structures.

(3) Joining the sheet metal parts. The deformations caused by welding heat and pressure are ignored. The vector $[K_w]$

represents the welded stiffness matrix, the vector $\{F_w\}$ represents clamp forces, and

here the vector $\{F_w\}$ is identical to the vector $\{F_u\}$.

(4) Releasing the clamps and /or fixture. The vector $\{V_w\}$ is the spring-back of the welded structures at each node, and we can calculate the variation of welded structure as follows

$$\{V_w\} = [K_w]^{-1}\{F_w\} \quad (11)$$

4.3.1 Monte-Carlo Simulation

Monte-Carlo Simulation method is used to simulate the change of variations. A random number generator is used to generate the distributions with a unit standard deviation ($\sigma = 1mm$) and a unit mean deviation ($\mu = 1mm$) for each source of variation. For simplicity, we suppose that all sources of variations are independent. The number of assemblies (sample size) simulated is selected as 1000.

4.3.2 Number theoretic method

Selecting $n=61$, $s=4$, and $a=3$ in the method of NTM, we can obtain the Number Theoretic net which is uniformly distributes in C^4 . Then we transform the uniformly distributes to the normal distributes with a unit standard deviation ($\sigma = 1mm$) and a unit mean deviation ($\mu = 1mm$) for each source of variation. We can simulate the assembly process and calculate the variation of each node after assembly by using software MSC/PATRAN. Among all nodes the maximum deviation are Node 21 (or 26), where $\mu = 0.4868$ mm, $\sigma_p = 0.5569$ mm. It can be concluded that the variations of assemblies are less than the variations of parts for the sheet metal assemblies.

4.4 Results comparison

A comparison of the results among Monte-Carlo Simulation, Method of Influence Coefficients and Number Theoretic Method is shown in Table 1. As shown in Table 1, the mean deviation from Monte-Carlo Simulation is smaller than others. On the other hand, the number of assemblies (sample size) simulated is 1000, while Number Theoretic Method only needs 61 (sample size). It can be seen from the comparison that NTM needs less calculation and shorter time than others in simulation process.

Table 1 Comparison of three methods

Methods	Mean deviation	Standard deviation	No. of FEM Runs
Monte-Carlo Simulation*	0.44	0.55	1000
Influence Coefficients*	0.47	0.54	2
Number Theoretic	0.47	0.55	61

*: The data comes from literature [6]

5Conclusions

In this paper, for accounting for the effect of deformation of sheet metals, the assembly variation is calculated using finite element analysis and pseudo random numbers which are produced by number theoretic method. This method can account for the effect of elastic and plastic deformation, and the calculation amount can be decreased remarkably comparing with others. To obtain the same magnitude results, Monte-Carlo Simulation needs 1000 samples which are 15 times of Number Theoretic Method (61 samples). The computational efficiency improves remarkably. However, pseudo random numbers can influence the calculation results, so further research is needed on how to confirm the best pseudo random number of simulation

process.

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