# Weakness of Shim's New ID-based Tripartite Multiple-key Agreement 

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Abstract - In this article we show that Shim's new ID-based tripartite multiple-key agreement protocol still suffers from the impersonation attack, a malicious user can launch an impersonation attack on their protocol.

Keyword - ID-based, Weil-paring, Impersonation attack, Tripartite authenticated key agreement

## 1. Introduction

The first one-round tripartite Diffiee-Hellman key agreement protocol [1] was proposed by Joux in 2000. However, Joux's protocol does not authenticate the three communicating entities, and is vulnerable to the man-in-the-middle attack. Recently Liu et al. proposed an ID-based one round authenticated tripartite key agreement protocol with pairing[2,4-12] (LZC protocol) which results in eight session keys in the agreement. However, their scheme could not prevent the "unknown key share" attack proposed by Shim et al. in 2005[3]. In [3], they suggest a method to resist the unknown key share attack. This article will show that their protocol is still vulnerable to the impersonation attack.

## 2. The Background

In this section, we will first briefly review the basic concept and some properties of bilinear pairing then review the Shim's protocol.

### 2.1. Bilinear pairing

Let $\mathbb{G}_{1}$ be a cyclic group generated by $P$, whose order is a prime $q$ and $\mathbb{G}_{2}$ be a cyclic multiplicative group of the same order $q$. We assume that the discrete logarithm problem (DLP) in both $\mathbb{G}_{1}$ and $\mathbb{G}_{1}$ are hard. Let $e: \mathbb{G}_{1} \times \mathbb{G}_{1} \rightarrow \mathbb{G}_{2}$ be a pairing which satisfies the following conditions:
(1) Bilinear: $e(a P, b Q)=e(P, Q)^{a b}$, for any $a, b \in \mathbb{Z}$ and $P, Q \in \mathbb{G}_{1}$.
(2) Non-degenerate: there exists $P \in \mathbb{G}_{1}$ and
$Q \in \mathbb{G}_{1}$ such that $e(P, Q) \neq 1$.
(3) Computability: there is an efficient algorithm to compute $e(P, Q)$ for all $P, Q \in \mathbb{G}_{1}$

### 2.2 Shim's protocol

(1) Setup: Key generation center (KGC) chooses a random $s \in \mathbb{Z}_{q}^{*}$ and set $P_{p u b}=s P$. The KGC publishes the system parameters $\left\langle\mathbb{G}_{1}, \mathbb{G}_{2}, q, e, P, P_{p u b}, H, H_{1}\right\rangle$ and keep $s$ as a secret master key, which is known only by itself.
(2) Private key extraction: A user submits his identity information ID to KGC. KGC computes the user's public key as $Q_{I D}=H_{1}(I D)$ and returns $S_{I D}=s Q_{I D}$ to the user as his private key.
(3) Scheme: Assume that there are three entities $A$, $B, C$. Each chooses two random numbers then computers their corresponding parameters. For examples, $A$ chooses random numbers $a$ and $a^{\prime}$, and computes $P_{A}=a P, P_{A}^{\prime}=a^{\prime} P, T_{A}=S_{A}+a^{2} P+a^{\prime} P_{p u b} . B$ chooses random numbers $b$ and $b^{\prime}$, and computes $P_{B}=b P, P_{B}^{\prime}=b^{\prime} P, T_{B}=S_{B}+b^{2} P+b^{\prime} P_{p u b} . C$ chooses random numbers $C$ and $C^{\prime}$, and computes $P_{C}=c P, P_{B}^{\prime}=c^{\prime} P, T_{C}=S_{C}+c^{2} P+c^{\prime} P_{p u b}$. After the computing, they broadcast their values $\left(P_{A}, P_{A}^{\prime}, T_{A}\right),\left(P_{B}, P_{B}^{\prime}, T_{B}\right)$ and $\left(P_{C}, P_{C}^{\prime}, T_{C}\right)$ to all the other parties.
When receiving the other party's communicational parameters, each party performs his/her own verifying equation. For example, $A$ checks whether the following equation holds.

$$
\begin{aligned}
e\left(T_{B}+T_{C}, P\right) & =e\left(S_{B}+b^{2} P+b^{\prime} P_{p u b}+S_{C}+c^{2} P+c^{\prime} P_{p u b}, P\right) \\
& =e\left(s P_{B}+b^{\prime} s P+s P_{C}+c^{\prime} s P, P\right) e\left(b^{2}, P\right) e\left(c^{2}, P\right) \\
& ? \\
& =e\left(Q_{B}+Q_{C}+P_{B}^{\prime}+P_{C}^{\prime}, P_{p u b}\right) e\left(P_{B}, P_{B}\right) e\left(P_{C}, P_{C}\right)
\end{aligned}
$$

$B$ and $C$ also do their corresponding verification to check if the equations hold.
If each equation holds, then $A, B$ and $C$ compute the eight session keys respectively, as in the LZC protocol, as follows.
A computes:
$K_{A}^{(1)}=e\left(P_{B}, P_{C}\right)^{a}, K_{A}^{(2)}=e\left(P_{B}, P_{C}^{\prime}\right)^{a}, K_{A}^{(3)}=e\left(P_{B}^{\prime}, P_{C}\right)^{a}$, $K_{A}^{(4)}=e\left(P_{B}^{\prime}, P_{C}^{\prime}\right)^{a} K_{A}^{(5)}=e\left(P_{B}, P_{C}\right)^{a^{\prime}}, K_{A}^{(6)}=e\left(P_{B}, P_{C}^{\prime}\right)^{a^{\prime}}$, $K_{A}^{(7)}=e\left(P_{B}^{\prime}, P_{C}\right)^{a^{\prime}}, K_{A}^{(8)}=e\left(P_{B}^{\prime}, P_{C}^{\prime}\right)^{a^{\prime}}$
$B$ computes:
$K_{B}^{(1)}=e\left(P_{A}, P_{C}\right)^{b}, K_{B}^{(2)}=e\left(P_{A}, P_{C}^{\prime}\right)^{b}, K_{B}^{(3)}=e\left(P_{A}, P_{C}\right)^{b^{\prime}}$, $K_{B}^{(4)}=e\left(P_{A}, P_{C}^{\prime}\right)^{b^{\prime}} K_{B}^{(5)}=e\left(P_{A}^{\prime}, P_{C}\right)^{b}, K_{B}^{(6)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{b}$, $K_{B}^{(7)}=e\left(P_{A}^{\prime}, P_{C}\right)^{b^{\prime}}, K_{B}^{(8)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{b^{\prime}}$
$C$ computers:
$K_{C}^{(1)}=e\left(P_{A}, P_{B}\right)^{c}, K_{C}^{(2)}=e\left(P_{A}, P_{B}\right)^{c^{\prime}}, K_{C}^{(3)}=e\left(P_{A}, P_{B}^{\prime}\right)^{c}$, $K_{C}^{(4)}=e\left(P_{A}, P_{B}^{\prime}\right)^{c^{\prime}} K_{C}^{(5)}=e\left(P_{A}^{\prime}, P_{B}\right)^{c}, K_{C}^{(6)}=e\left(P_{A}^{\prime}, P_{B}\right)^{c^{\prime}}$,
$K_{C}^{(7)}=e\left(P_{A}^{\prime}, P_{B}^{\prime}\right)^{c}, K_{C}^{(8)}=e\left(P_{A}^{\prime}, P_{B}^{\prime}\right)^{c^{\prime}}$
We can find that
$K_{A}^{(1)}=K_{B}^{(1)}=K_{C}^{(1)}=e(P, P)^{a b c}=K^{(1)}$. Similarly,
we also have
$K_{A}^{(i)}=K_{B}^{(i)}=K_{C}^{(i)}=K^{(i)}$, for $i=2,3, \ldots, 8$.Each entity then takes the eight computed values $K^{(i)}$
$(i=1,2, \ldots, 8)$ as the final session keys, where
$K^{(1)}=e(P, P)^{a b c}, K^{(2)}=e(P, P)^{a b c^{\prime}}, K^{(3)}=e(P, P)^{a b^{\prime} c}$, $K^{(4)}=e(P, P)^{a b^{\prime} c^{\prime}}, K^{(5)}=e(P, P)^{a^{\prime} b c}, K^{(6)}=e(P, P)^{a^{\prime} b c^{\prime}}$, $K^{(7)}=e(P, P)^{a^{\prime} b^{\prime} c}, K^{(8)}=e(P, P)^{a^{\prime} b^{\prime} c^{\prime}}$

## 3. Our Attack

In this section, we show that how the Shim's protocol is insecure against the impersonation attack.

Assume that there is an adversary $X$, who wants to impersonate $B$ to communicate with $A$ and $C$ shown as follows:

## Step1: $X$ computes

 $P_{X}=x P, P_{X}^{\prime}=x^{\prime} P-Q_{B}, T_{X}=\quad x^{\prime} P_{p u b}+x^{2} P \quad$ and broadcast them to $A$ and $C$. After receiving the broadcast parameters sent by $X$ and $C, A$ verify the equation and we will find that the equation would be hold show below:$e\left(T_{X}+T_{C}, P\right)=e\left(x^{\prime} P_{p u b}+x^{2} P+S_{C}+c^{2} P+c^{\prime} P_{p u b}, P\right)$
$=e\left(x^{\prime} P+Q_{C}+c^{\prime} P, P_{p u b}\right) e\left(x^{2} P+c^{2} P, P\right)$
$=e\left(x^{\prime} P-Q_{B}+Q_{B}+Q_{C}+c^{\prime} P, P_{p u b}\right) e(x P, x P) e(c P, c P)$
$=e\left(P_{X}^{\prime}+Q_{B}+Q_{C}+c^{\prime} P, P_{p u b}\right) e(x P, x P) e(c P, c P)$
$=e\left(Q_{B}+Q_{C}+P_{X}^{\prime}+P_{C}^{\prime}, P_{p u b}\right) e\left(P_{X}, P_{X}\right) e\left(P_{C}, P_{C}\right)$

Step2: $\quad C$ can obtain his parameters sent from other parties and also pass his/her verification by the equation
$e\left(T_{A}+T_{X}, P\right)=e\left(Q_{A}+Q_{B}+P_{X}^{\prime}+P_{A}^{\prime}\right) e\left(P_{A}, P_{A}\right) e\left(P_{X}, P_{X}\right)$

Step3: After that, A can compute the session keys as follows.
$K_{A}^{(1)}=e\left(P_{X}, P_{C}\right)^{a}, K_{A}^{(2)}=e\left(P_{X}, P_{C}^{\prime}\right)^{a}, K_{A}^{(3)}=e\left(P_{X}^{\prime}, P_{C}\right)^{a}$, $K_{A}^{(4)}=e\left(P_{X}^{\prime}, P_{C}^{\prime}\right)^{a} K_{A}^{(5)}=e\left(P_{X}, P_{C}\right)^{a^{\prime}}, K_{A}^{(6)}=e\left(P_{X}, P_{C}^{\prime}\right)^{a^{\prime}}$, $K_{A}^{(7)}=e\left(P_{X}^{\prime}, P_{C}\right)^{a^{\prime}}, K_{A}^{(8)}=e\left(P_{X}^{\prime}, P_{C}^{\prime}\right)^{a^{\prime}}$

And $C$ can compute the session keys as follows:
$K_{C}^{(1)}=e\left(P_{A}, P_{X}\right)^{c}, K_{C}^{(2)}=e\left(P_{A}, P_{X}\right)^{c^{\prime}}, K_{C}^{(3)}=e\left(P_{A}, P_{X}^{\prime}\right)^{c}$, $K_{C}^{(4)}=e\left(P_{A}, P_{X}^{\prime}\right)^{c^{\prime}} K_{C}^{(5)}=e\left(P_{A}^{\prime}, P_{X}\right)^{c}, K_{C}^{(6)}=e\left(P_{A}^{\prime}, P_{X}\right)^{c^{\prime}}$, $K_{C}^{(7)}=e\left(P_{A}^{\prime}, P_{X}^{\prime}\right)^{c}, K_{C}^{(8)}=e\left(P_{A}^{\prime}, P_{X}^{\prime}\right)^{c^{\prime}}$

Each entity, $A$ and $C$, then takes the following eight
computed values $K^{(i)}=(i=1, \ldots, 8)$ as their final session keys
$K^{(1)}=e(P, P)^{a x c}, K^{(2)}=e(P, P)^{a x c^{\prime}}, K^{(3)}=e(P, P)^{\alpha \alpha^{\prime} c} e\left(Q_{B}, P\right)^{-a c}$,
$K^{(4)}=e(P, P)^{\alpha \alpha^{\prime} c^{\prime}} e\left(Q_{B}, P\right)^{-a c^{\prime}}, K^{(5)}=e(P, P)^{d_{x c}}, K^{(6)}=e(P, P)^{d^{\prime} x^{\prime}}$,
$K^{(7)}=e(P, P)^{d \alpha^{\prime} c} e\left(Q_{B}, P\right)^{-d c}, K^{(8)}=e(P, P)^{d \times c^{\prime}} e\left(Q_{B}, P\right)^{-d c^{\prime}}$

Finally, the adversary $X$ can also get the same session keys $K^{(1)}, K^{(2)}, K^{(5)}$ and $K^{(6)}$ as $A$ and $C$ by computing:
$K_{X}^{(1)}=e\left(P_{A}, P_{C}\right)^{x}=e(P, P)^{a x c} \equiv K^{(1)}$,
$K_{X}^{(2)}=e\left(P_{A}, P_{C}^{\prime}\right)^{x}=e(P, P)^{a x c} \equiv K^{(2)}$,
$K_{X}^{(5)}=e\left(P_{A}^{\prime}, P_{C}\right)^{x}=e(P, P)^{a^{\prime} x c} \equiv K^{(5)}$,
$K_{X}^{(6)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{x}=e(P, P)^{a^{\prime} x c^{\prime}} \equiv K^{(6)}$.
As a result, $X$ can share these four keys $K^{(1)}$, $K^{(2)}, K^{(5)}, K^{(6)}$ in the eight session keys. Under this situation, $A$ and $C$ think these four session keys are shared with $B$, but indeed, they are shared with $X$. Besides, both $A$ and $C$ come to share the same eight session keys. Thus, the impersonation attack on four of the eight session keys can be successfully mounted. More precisely, the attacker $X$ can use these four session keys to communicate with $A$ and $C$, and he can have one half of the probability to realize what the communication contents are between $A$ and $C$.

## 4. Conclusion

In this article, we show that Shim et al.'s new ID-based tripartite multiple-key agreement protocol in [3] can not resist an impersonation attack. How to design a secure and efficient ID-based authenticated tripartite multiple-key agreement scheme to prevent all kinds of attacks remains an open problem.

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