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Two-Degree-of-Freedom Controller Design for Takagi-Sugeno Fuzzy Systems

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Abstract: - Controller design strategy for Takagi-Sugeno (T-S) fuzzy systems is considered in the two-degree-of-freedom (TDOF) framework. Firstly, coprime factorization described in the state space formulas for T-S fuzzy systems is introduced based on a common Lyapunov function. Secondly, based on this coprime factorization, TDOF framework for LTI systems is extended to be applied to T-S fuzzy systems. Consequently, good tracking performance and good disturbance rejection (and robustness) are compatibly achieved by a feedforward controller and a feedback controller, respectively. Furthermore, each controller design problem can be formulated with dynamic parallel distributed compensation in terms of linear matrix inequality related to L2 gain performance.

Key-Words: T-S fuzzy system, L2 gain performance, Two-Degree-of-Freedom, Dynamic parallel distributed compensation (DPDC), Nonlinear dynamic control, Linear matrix inequality, Controller design.

1 Introduction

Takagi-Sugeno (T-S) fuzzy systems can be formalized from a large class of nonlinear systems [1,2]. Despite the fact that the global T-S model is nonlinear due to the dependence of the membership functions on the fuzzy variables, it has a special formulation, known as Polytopic Linear Differential Inclusions (PLDI) [3], in which the coefficients are normalized membership functions. That is, local dynamics in different state-space regions are represented by linear models; and the nonlinear system is approximated by the overall fuzzy linear models.

Most of the existing control techniques for T-S model utilized the parallel distributed compensation (PDC) law [4]. With quadratic Lyapunov functions and PDC law, a great deal of attention has been focused on analysis and synthesis of these systems [4-9]. In particular, in [9] sufficient linear matrix inequality (LMI) conditions are provided for the existence of a quadratically stabilizing dynamic compensator or the performance-oriented controller based on the notion of dynamic parallel distributed compensator (DPDC).

On the other hand, two-degree-of-freedom (TDOF) control scheme is fundamental strategy for design of linear time invariant (LTI) systems to deal with both command tracking and disturbance rejection independently [10-13]. It is noted that not only control performance but also controller structures can be independently treated in TDOF framework. To the authors' knowledge, however, general TDOF control scheme including command tracking issue of T-S fuzzy systems has not been discussed explicitly. As for T-S fuzzy systems, it is no use treating transfer functions or eigenvalues of state matrices, thus, the TDOF methodology for LTI systems can not be applied to T-S fuzzy systems straightforwardly.

In the present paper, we extend TDOF control framework for LTI systems to be applied to T-S fuzzy systems. Firstly, coprime factorization for T-S fuzzy systems is described in state space formulas based on a common Lyapunov function. Secondly, TDOF control framework for T-S fuzzy systems is presented based on this coprime factorization. Then two-step controller design approach is proposed. First, a feedforward T-S fuzzy controller that achieves good tracking performance is designed by model matching strategy with L2 gain performance. Second, a feedback controller that rejects disturbances and/or model uncertainties and does not affect tracking performance is designed with another L2 gain performance. Each controller design problem can be formulated with dynamic parallel distributed compensation in terms of linear matrix inequality expressions [14].

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2 Preliminaries

In this section, firstly notations regarding T-S fuzzy systems are introduced. And then, definition of coprime factorization for T-S fuzzy systems is newly introduced . L2 gain performance with L2-norm is also recapped.

Definition 1 [9].

The T-S fuzzy model G consists of a finite set of fuzzy IF-THEN rules. Each rule has the following form:

Dynamic part:

Rule $i = 1, 2, \cdots, r$:

IF $z_1(t)$ is $M_{i1}, \cdots$ and $z_p(t)$ is M_{ip},

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$

Output part:

Rule $i = 1, 2, \cdots, r$:

IF $z_1(t)$ is $M_{i1}, \cdots$ and $z_p(t)$ is M_{ip},

THEN $y(t) = C_i x(t)$

Each variable $z_i(t)$ is a known parameter that is a function of the state variables $x(t)$, external disturbances, and/or time. The symbols M_{ij} represent membership functions for fuzzy sets.

Using the center of gravity method for defuzzification, we can express the aggregated fuzzy model $G(z)$ as:

$$G(z) \triangleq \left[\begin{array}{c|c} A(z) & B(z) \\ \hline C(z) & 0 \end{array}\right] = \sum_{i=1}^{r} h_i(z) \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & 0 \end{array}\right] \quad (1)$$

Here, z denotes the vector containing all the individual parameters $z_i(t)$. The $h_i(z)$ is the normalized possibility for the ith rule to fire given by

$$h_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)} \quad (2)$$

Where, possibility for the ith rule to fire: $w_i(z)$ is given by the product of all the membership functions associated with the ith rule as:

$$w_i(z) = \prod_{j=1}^{p} M_{ij}(z_j) \quad (3)$$

We will assume that at least one $w_i(z)$ is always nonzero so that $\sum_{i=1}^{r} w_i(z_j) \neq 0$. It is noted that the normalized possibility $h_i(z)$ satisfies conditions $h_i(z) > 0$ and $\sum_{i=1}^{r} h_i(z) = 1$.

Definition 2.

The T-S fuzzy plant $G(z)$ is said to have doubly coprime factorization if there exit right coprime factorization $G(z) = N(z)D(z)^{-1}$ and left coprime factorization $G(z) = \tilde{D}(z)^{-1}\tilde{N}(z)$, where a set of realizations for stable T-S fuzzy systems $N(z)$, $D(z)$, $\tilde{N}(z)$, $\tilde{D}(z)$, $U(z)$, $V(z)$, $\tilde{U}(z)$ and $\tilde{V}(z)$ can be chosen such that

$$\begin{bmatrix} \tilde{V}(z) & \tilde{U}(z) \\ -\tilde{N}(z) & \tilde{D}(z) \end{bmatrix} \begin{bmatrix} D(z) & -U(z) \\ N(z) & V(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (4)$$

A particular set of realizations can be chosen such that

$$\begin{aligned} \begin{bmatrix} D(z) & -U(z) \\ N(z) & V(z) \end{bmatrix} &\triangleq \left[\begin{array}{c|cc} A(z) - B(z)F & B(z) & L \\ \hline -F & I & 0 \\ C(z) & 0 & I \end{array}\right] \\ &= \sum_{i=1}^{r} h_i(z) \left[\begin{array}{c|cc} A_i - B_i F & B_i & L \\ \hline -F & I & 0 \\ C_i & 0 & I \end{array}\right] \end{aligned} \quad (5)$$

$$\begin{aligned} \begin{bmatrix} \tilde{V}(z) & \tilde{U}(z) \\ -\tilde{N}(z) & \tilde{D}(z) \end{bmatrix} &\triangleq \left[\begin{array}{c|cc} A(z) - LC(z) & B(z) & L \\ \hline F & I & 0 \\ -C(z) & 0 & I \end{array}\right] \\ &= \sum_{i=1}^{r} h_i(z) \left[\begin{array}{c|cc} A_i - LC_i & B_i & L \\ \hline F & I & 0 \\ -C_i & 0 & I \end{array}\right] \end{aligned} \quad (6)$$

in which F and L are chosen such that both $A_i - B_i F$ and $A_i - LC_i$ are quadratically stable. That is, $F = -VP_1^{-1}$ and $L = -P_2^{-1}W$ should satisfy the following LMIs, respectively:

$P_1 > 0$;

$$P_1 A_i^T + A_i P_1 + B_i V + V^T B_i^T < 0, \; i = 1, 2, \cdots r \quad (7)$$

and

$P_2 > 0$;

$$A_i^T P_2 + P_2 A_i + W C_i + C_i^T W^T < 0, \; i = 1, 2, \cdots r \quad (8)$$

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Definition 3.
The control system satisfying the property $\sup_{a\neq 0, \|a\|_2<\infty} \frac{\|b\|_2}{\|a\|_2} < \gamma$ is said to have L2 gain performance with the bound γ related to signals a and b, where $\|c\|_2 = \sqrt{\int_0^\infty c^T(t)c(t)dt}$ is the L2-norm of the signal c.

3 TDOF controller design for T-S fuzzy plants

In this section, first, conventional TDOF control structure of LTI system will be extended to T-S fuzzy system using doubly coprime factorization of an T-S fuzzy plant. It is well known that LTI feedback control systems can be constructed as figure 1., where signals r, u, d and y denote reference inputs, control inputs, output disturbances, controlled outputs, respectively. Both K_{ff1} and K_{ff2} are LTI feedforward controllers, K_{fb} denotes an LTI feedback controller and G is a LTI plant.

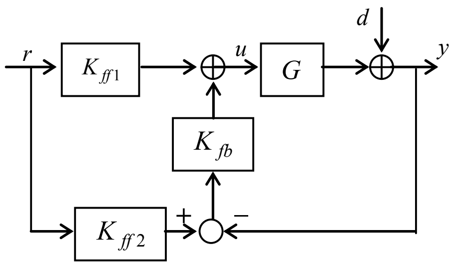

Fig.1. Configuration of general LTI control systems

In this configuration, if K_{ff1} is set to be zero matrix and K_{ff2} is set to be unit matrix, then Fig.1 shows an One-Degree-Of-Freedom (ODOF) control system. Otherwise, Fig.1 represents a TDOF control system; and particularly if K_{ff1} is set to be $D \cdot K_{ff}$ and K_{ff2} is set to be $N \cdot K_{ff}$, where D and N are right and left factor of right coprime factorization of the plant, then feedback and feedforward controllers can be designed independently. This TDOF configuration of LTI system can be easily extended to T-S fuzzy system with doubly coprime factorization of the T-S fuzzy plant introduced in section 2. In Fig.2, controlled output can be obtained as

$$y = N(z)K_{ff}(z)r + (I + G(z)K_{fb}(z))^{-1}d \qquad (9)$$

Thus, tracking performance and disturbance rejection (and also robustness against model uncertainties) are independently achieved by a feedforward controller: $K_{ff}(z)$ and a feedback controller: $K_{fb}(z)$, respectively.

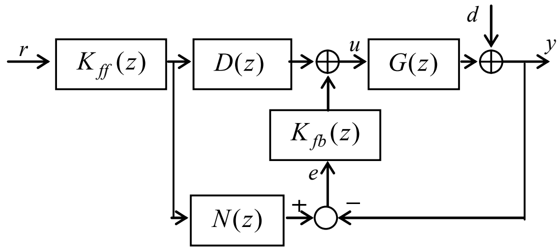

Fig.2. TDOF structure for T-S fuzzy plants

On the other hand, in linear fractional transformation described by Fig.3, it has been proposed to designing T-S fuzzy output-feedback controller: $K(z)$ obtaining internal stability and guaranteeing L2 gain bound for augmented plant: $P(z)$ [9].

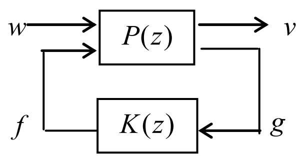

Fig.3. LFT configuration of T-S fuzzy control systems

Consequently, the design of each T-S fuzzy controller $K_{ff}(z)$ and $K_{fb}(z)$ can be reduced to design of each T-S fuzzy controller $K(z)$, by considering respective plant $P(z)$ including fuzzy plant $G(z)$.

In the rest of this section, two-step controller design approach is introduced. First, a feedforward controller: $K_{ff}(z)$ that achieves good tracking performance is designed by model matching strategy with L2 gain performance for augumented plant $P(z)$ related to $K_{ff}(z)$. Second, feedback controller: $K_{fb}(z)$ that rejects disturbances or model uncertainties while does not affect tracking performance is designed with another L2 gain performance and another augumented plant $P(z)$ related to $K_{fb}(z)$. Each controller design problem can be formulated in terms of linear matrix inequality expression. In the following text, we will omit the dependent parameter z for the simplicity.

3.1 Feedforward configuration

As for T-S fuzzy system, it is no use treating transfer functions or eigenvalues to test the stability. Thus, approximation of transfer functions can not be applied to solve command tracking problem. Instead, we treat output error between target signal and controlled output. Accordingly, the augmented plant P_{ff} for tracking problem can be constructed as shown in Fig. 4 with reference model: T and a weighting function: W_r. LTI reference model and weighting function are also available, besides the latter can be omitted.

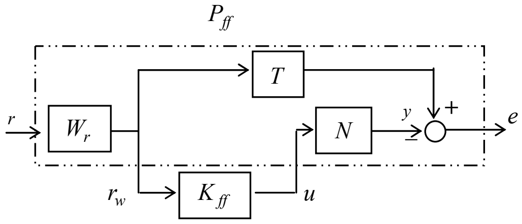

Fig.4. LFT configuration regarding command tracking problem

According to Fig.4, the augmented plant holds the relation as

$$\begin{bmatrix} e \\ r_w \end{bmatrix} = P_{ff} \begin{bmatrix} r \\ u \end{bmatrix} = \begin{bmatrix} TW_r & -N \\ W_r & 0 \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix} \tag{10}$$

and is derived with the state space realization of $W_r \triangleq \left[\begin{array}{c|c} A_{wr} & B_{wr} \\ \hline C_{wr} & D_{wr} \end{array}\right]$ and $T \triangleq \left[\begin{array}{c|c} A_t & B_t \\ \hline C_t & D_t \end{array}\right]$ as

$$P_{ff} \triangleq \sum_{i=1}^{r} h_i \left[\begin{array}{ccc|cc} A_{wr} & B_{wr}C_t & 0 & B_{wr}D_t & 0 \\ 0 & A_t & 0 & B_t & 0 \\ 0 & 0 & A_i - B_iF & 0 & -B_i \\ \hline C_{wr} & D_{wr}C_t & C_i & D_{wr}D_t & 0 \\ C_{wr} & 0 & 0 & D_{wr} & 0 \end{array}\right] \tag{11}$$

Consequently, the design problem of the feedforward controller K_{ff} obtaining L2 gain performance related to r and e is formulated in terms of linear matrix inequality mentioned later in subsection 3.3. It should be noted that K_{ff} itself is a stable controller.

3.2 Feedback configuration

Standard T-S fuzzy control methodology can also be applied to the design of a feedback controller: K_{fb}. In this case, the augmented plant is constructed as shown in Fig. 5 with a weighting function: W_d.

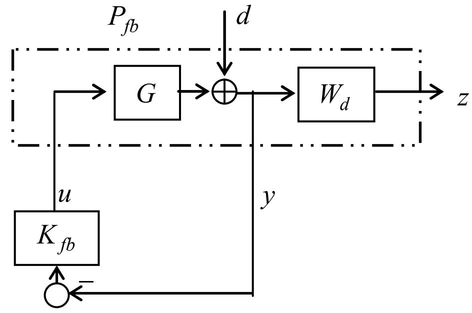

Fig.5. LFT configuration regarding disturbance rejection problem

According to Fig.5, the augmented plant holds the relation as

$$\begin{bmatrix} z \\ y \end{bmatrix} = P_{fb} \begin{bmatrix} d \\ u \end{bmatrix} = \begin{bmatrix} W_d & -W_dG \\ I & -G \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix} \tag{12}$$

and is derived with the state space realization of $W_d \triangleq \left[\begin{array}{c|c} A_{wd} & B_{wd} \\ \hline C_{wd} & D_{wd} \end{array}\right]$ as

$$P_{fb} \triangleq \sum_{i=1}^{r} h_i \left[\begin{array}{cc|cc} A_{wd} & B_{wd}C_i & B_{wd} & 0 \\ 0 & A_i & 0 & -B_i \\ \hline C_{wd} & D_{wd}C_i & D_{wd} & 0 \\ 0 & C_i & I & 0 \end{array}\right] \tag{13}$$

Likewise the design of K_{ff}, the design problem of K_{fb} obtaining L2 gain performance related to d and z can also be formulated in terms of linear matrix inequality.

3.3 Controller construction

Both augmented plants: P_{ff} as shown (11) and P_{fb} as shown (13) have the following formulation:

$$P \triangleq \sum_{i=1}^{r} h_i \left[\begin{array}{c|cc} A_i & B_w^i & B_i \\ \hline C_z^i & D_{zw}^i & D_z^i \\ C_i & D_w^i & 0 \end{array}\right] \tag{14}$$

A unified systematic scheme for designing dynamic feedback controllers for the plant (14) has been presented to assure both internal stability and L2 gain performance according to theorem 8 of [9]. The control laws are in the form of so-called dynamic parallel distributed compensation (DPDC). Moreover, controller design scheme has been formulated by solving the sufficient LMI conditions.

The T-S fuzzy controller K_{ff} and K_{fb} regarding respectively P_{ff} and P_{fb} can be derived as follows.

$$K \triangleq \left[\begin{array}{c|c} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \frac{1}{2}(A_k^{ij} + A_k^{ji}) & \sum_{j=1}^{r} h_i B_k^i \\ \hline \sum_{i=1}^{r} h_i C_k^i & D_k \end{array}\right] \tag{15}$$

where A_k^{ij}, B_k^i, C_k^i and D_k^i can be written as

$$A_k^{ij} = \frac{1}{2} P_{12}^{-1} \left\{ 2\hat{A}_{ij} - P_{12} B_k^i C_j Q_{11} - P_{12} B_k^j C_i Q_{11} \right.$$
$$-P_{11} B_i C_k^j Q_{12}^T - P_{11} B_j C_k^i Q_{12}^T$$
$$\left. -P_{11}(A_i + B_i D_k C_j) Q_{11} - P_{11}(A_j + B_j D_k C_i) Q_{11} \right\} Q_{12}^{-1},$$
$$B_k^i = P_{12}^{-1}(\hat{B}_i - P_{11} B_i D_k),$$
$$C_k^i = (\hat{C}_i - D_k C_i Q_{11}) Q_{12}^{-T},$$
$$D_k = \hat{D}.$$

Variables $Q_{11}, P_{11}, T_{ij}, \hat{A}_{ij}, \hat{B}_i, \hat{C}_i$ and $\hat{D}$ satisfies the following LMI conditions

$$\begin{bmatrix} \bar{E}_{11}^{ij} & \bar{E}_{12}^{ij} & \bar{E}_{13}^{ij} & \bar{E}_{14}^{ij} \\ (\bar{E}_{12}^{ij})^T & \bar{E}_{22}^{ij} & \bar{E}_{23}^{ij} & (\bar{E}_{42}^{ij})^T \\ (\bar{E}_{13}^{ij})^T & (\bar{E}_{23}^{ij})^T & -2\gamma I & (\bar{E}_{43}^{ij})^T \\ (\bar{E}_{14}^{ij})^T & \bar{E}_{42}^{ij} & \bar{E}_{43}^{ij} & -2\gamma I \end{bmatrix} < T_{ij} \quad \forall i \le j, \tag{16}$$

$$T = T^T = \begin{bmatrix} T_{11} & \cdots & T_{1n} \\ \vdots & \ddots & \vdots \\ T_{1n} & \cdots & T_{nn} \end{bmatrix} < 0. \tag{17}$$

where

$$\bar{E}_{11}^{ij} = A_i^T Q_{11}^T + Q_{11} A_i + A_j^T Q_{11}^T + Q_{11} A_j + B_i \hat{C}_j + (B_i \hat{C}_j)^T$$
$$+ B_j \hat{C}_i + (B_j \hat{C}_i)^T$$
$$\bar{E}_{12}^{ij} = A_i + A_j + B_i \hat{D} C_j + B_j \hat{D} C_i + 2\hat{A}_{ij}^T,$$
$$\bar{E}_{13}^{ij} = B_w^i + B_w^j + B_i \hat{D} D_w^j + B_j \hat{D} D_w^i,$$
$$\bar{E}_{14}^{ij} = (C_z^i Q_{11} + C_z^j Q_{11} + D_z^i \hat{C}_j + D_z^j \hat{C}_i)^T,$$
$$\bar{E}_{22}^{ij} = A_i P_{11}^T + P_{11} A_i^T + A_j P_{11}^T + P_{11} A_j^T + \hat{B}_i C_j + \hat{B}_j C_i$$
$$+ (\hat{B}_i C_j)^T + (\hat{B}_j C_i)^T,$$
$$\bar{E}_{23}^{ij} = P_{11} B_w^i + P_{11} B_w^j + \hat{B}_i D_w^j + \hat{B}_j D_w^i,$$
$$\bar{E}_{42}^{ij} = C_z^i + C_z^j + D_z^i \hat{D} C_j + D_z^j \hat{D} C_i,$$
$$\bar{E}_{43}^{ij} = D_{zw}^i + C_{zw}^j + D_z^i \hat{D} D_w^j + D_z^j \hat{D} D_w^i,$$

with constraint $P_{11} Q_{11} + P_{12} Q_{12}^T = I$ and $\begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0$.

4 Conclusions

We have developed two-step nonlinear dynamic controller design strategy for T-S fuzzy plant in TDOF framework. The first step is to design a feedforward controller that achieves good tracking performance by model matching strategy with L2 gain performance. The second step is to design a feedback controller that rejects disturbances while does not affect tracking performance with another L2 gain performance. Each controller design problem is formulated in terms of linear matrix inequality and both problems can be solved by DPDC methodology.

Concerning feedback controllers, it is well known that all proper stabilizing controllers are parameterized in terms of arbitrary $Q \in RH^\infty$ for LTI control system. The configuration of TDOF control system is deeply considered with the Q-parameter and its consequences — sophisticated design framework has been developed. In the present paper, we have newly developed general TDOF control scheme for T-S fuzzy system, however, we just focused on L2 gain performance to design controllers. The *Q*-parameter approach will give us more practical validity to deduce the solution and covers more general control system designs including multi-objective and/or switching system for T-S fuzzy plants. Based on our results, *Q*-parameter approach can be applicable to T-S fuzzy control system and it will be treated in another paper.

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