Using Optimization Costs Modeling to Maintain Performance Constraints for the Failure Mode Identification

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Abstract: - Failure identification procedures are currently being used for quality control and for the detection of potential failure modes (includes failure location) during the design stage or product producing. Although many methods have given, such as failure modes and effects analysis (FMEA), fault tree analysis (FTA), failure modes, effects and criticality analysis (FMECA), they do not mention the optimization cost and maintain performance constraints for the production. This paper aims to give the optimization algorithmic modeling for typical censoring structures to identification failure and analysis the reason. Application to typical example of failure censored data of the motor will be presented and failure identification procedures will be made based on some preliminary research.

Key-Words: - Failure modes; Failure modes and effects analysis; Failure-free design; Quality; Performance.

1 Introduction

Business reality is that products must come to market quickly, be more reliable and cost less. Recently manufacturers have worked very hard to improve their quality of products and services in order to remain their product competitive. For a manufacturer, it is very important to meet customer satisfaction in order to stay on the globally competitive market. Product reliability is a critical factor affecting the customers perception of product quality and ultimately impacts market share. So product design has to be improved by focusing on the weakness of the product.

To detection the Failure mode is very important during the design stage or product producing, many methods have given, such as failure modes and effects analysis(FMEA)[1], fault tree analysis (FTA) and failure modes, effects and criticality analysis (FMECA), as well as prior knowledge and experience, to determine potential failure modes. These procedures require designers to have both a broad knowledge of commonly occurring failure modes and an understanding of any connections or causality between failures in order for the design to be successful.

The traditional FMEA, when performed rigorously, contains valuable information about failures of various components, but has two fundamental weaknesses: (1) methodological guideline to conduct the FMEA; and (2) the lack of natural, repeatable language to record the information of the cost. Current industrial FMEA practice is severely restricted, representing and reasoning with system failure knowledge. Thus, the standardization of the failure mode vocabulary would make the procedure more useful and repeatable.

In early, the product quality control is only focus on the reliability, the cost of the production often not been taken into the mind. But product development in today’s highly competitive market, the key to success is to get product to
the customer in the shortest possible time whilst ensuring both maximum performance and safety. The issue is whether this can be accomplished without a substantial increase in the cost of product development. The way that using optimization costs modeling to maintain performance constraints for the failure mode identification may be a solution.

2 Model of Censoring Structure

Statistical tools have been employed for some time now in quality control and reliability measurement. A structural approach based on probability theory for the design and safety analysis of aircraft began in the early 1960s [3]. The use of numerical probabilities may not be a prerequisite for performing system safety analyses, but it provides valuable guidance to the designer in determining the architecture required and assessing its failure tolerance and then improve the product’s construct and parameters. And then, the above construct and parameters change will lead the cost of production increasing.

The prediction of system failure probabilities is not a precise science; however, the process does provide an extremely good framework on which to hang engineering experience. This approach constitutes a mathematically sound method for representing and reasoning with joint probability distributions in an internally consistent manner. Traditional FMEA ignores these connections and implicitly assumes that all failure states and events, together with their cause and effects, are probabilistically independent. Standardization of a product function vocabulary to enable archival and retrieval of product design knowledge has been a primary research area for many years[4]-[7].

The work presented here employs the function-failure design method; a functional approach to guide the determination of the potential failure modes that a product may be subjected to once it is placed in its operation environment. The methodology involves the formation of a function-failure matrix that can be used as a knowledge base to identify and analyze potential failure for new designs and redesign. The overall procedure to create the knowledge base id outline in Fig.1.

How to analyze and make certain the occurring probability effectively can be one of a goal of FMEA, which emphasize on the quantification analysis of indefinite structure and the probability which will be able to occur. So we’ll obtain the associated information, such as the reliability, optimization and the cost. Thus, design parameters can be adjusted with that. During the process, some character allows to be neglected such as the geometric complexity, the materials and the sensitivity. The current product is designed mainly aiming at a given function performance. Collins et al. has included such conclusion based on the 23 types of electric machine: the

![Fig.1 The relationship of the product function and failure mode](image-url)
failure mode generally appears the deficiency of partial function. And the identification and amendment of failure mode mainly aims at the positions which have occurred failure, then it will result in the deficiency of partial function (or global deficiency). We have adopted the function failure mode in this article to carry out such technical development of optimization analysis. Above all, the algorithm based on the function-failure-matrix analyzes the potential failure with the correlative knowledge which will occur in the process of product design or redesign.

Function-component-EC is constituted of multi-array and multi-row that denotes components and functions respectively. CF Component-failure is a matrix which is composed of multi-row of components and multi-array of failure modes. Information analysis received from FMEA by using the information of the two matrices, this “function-failure matrix analysis” is one of common designing methods in reliability projects. The components in these matrices should tie up the functions of product and may become ineffective, it requires high reliability for this components (the components whose safe system outruns the life expectation consumedly are not to be included in this matrix). Similarly, CF bases on the importance of affect on the inefficiency by the components, which are in BOM table of products. Meanwhile, EF (the latent failure modes effecting function of product) calculated by the two matrix above.

Such as the fig, it could be \[ EC \times CF = EF \] 
\[
\begin{bmatrix}
4 & 0 & 7 & 1 & 0 & 2 \\
3 & 0 & 2 & 0 & 5 & 0 \\
2 & 6 & 3 & 0 & 3 & 4 \\
0 & 2 & 4 & 6 & 0 & 6 \\
6 & 0 & 3 & 6 & 3 & 0 \\
3 & 5 & 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 5 & 5 & 1 \\
1 & 9 & 0 & 1 & 5 & 4 \\
\end{bmatrix}
\times
\begin{bmatrix}
0 & 2 & 4 & 0 & 1 & 0 \\
5 & 0 & 0 & 1 & 0 & 5 \\
3 & 1 & 0 & 0 & 3 & 4 \\
0 & 3 & 2 & 4 & 4 & 0 \\
0 & 0 & 1 & 0 & 3 & 0 \\
0 & 0 & 3 & 3 & 0 & 0 \\
5 & 1 & 0 & 0 & 3 & 5 \\
1 & 9 & 0 & 1 & 5 & 4 \\
\end{bmatrix}
= \\
\begin{bmatrix}
4 & 0 & 7 & 4 & 0 & 2 \\
3 & 0 & 2 & 0 & 5 & 0 \\
0 & 2 & 6 & 0 & 0 & 3 & 4 \\
4 & 0 & 2 & 4 & 6 & 0 & 6 \\
6 & 0 & 3 & 6 & 3 & 0 & 3 \\
3 & 5 & 3 & 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 5 & 5 & 1 \\
1 & 9 & 0 & 1 & 5 & 4 & 0 \\
\end{bmatrix}
\times
\begin{bmatrix}
0 & 4 & 0 & 7 & 1 & 0 & 2 \\
2 & 3 & 0 & 2 & 0 & 5 & 0 \\
0 & 2 & 6 & 3 & 0 & 3 & 4 \\
4 & 0 & 2 & 4 & 6 & 0 & 6 \\
3 & 5 & 3 & 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 5 & 5 & 1 \\
1 & 9 & 0 & 1 & 5 & 4 & 0 \\
\end{bmatrix}
\]

Actually, the probability of the failure modes of product is by the manufacturer. Consider products that are various, the usual failure modes of components in some essentially an assembly of major components, such as a computer or a motor. In this case, the manufacturer has to ensure that the system is functional to prevent “dead-on-arrival” and this involves testing. Often these components might have to be in storage for some time and could have got affected during shipment to retailer. Consider also high-value added products; since the reputation of the manufacturer is at stake for the failure modes happened.

2.1 Model for cost of failure modes

As mentioned earlier, produce quality have connect with component failure. Following this, we model the attention of manufacture on product quality. We conclude with game theoretic formulation to determine the optimal actions. And then, we can make the following notation to describe the relationship of the failure modes and cost:

- \( G(x, \theta) \) —— The probability distribution function for the time to the product first failure;
- \( F(x, \theta) \) —— the probability distribution function for the time a component of product first failure;
- \( f(x, \theta) \) —— the probability density function associated with \( F(x, \theta) \)

\[ \theta \] —— The component reliability parameter in the production design

\[ r(\theta) \] —— probability of an component item being defecting \[ \{ \theta < r(\theta) < 1, dr(\theta)/d\theta < 0 \} \]

- \( C_m \) —— Manufacture cost per unit \[ dC_m / d\theta < 0 \];
- \( C_d \) —— cost to rectify each non-conform failure item;
- \( C_r \) —— cost of each rectification attempt;

We model the production of non-conforming failure items as follows. Each failure item manufactured is non-conforming with probability \( r(\theta) \) and conforming with probability \( 1 - r(\theta) \) reflects the quality of conformance and depends on the component quality bigger value corresponding to lower quality. Since smaller \( \theta \) corresponds to a more stringent design for the components probability of an failure occur. For example, better product performance achieved through use of redundancy not only involves more components (due to replication of one or more of the components) but also more assembly operations and this increases the probability of an failure modes items being non-conforming. We model this by relationship \( dr(\theta)/d\theta < 0 \).

A non-conforming failure item is detected through testing and can be rectified to make it conforming. The cost of such rectification is \( C_d \) per failure item and borne
An failure item can happen several times during the time frame L (product life) of its useful life. A failure item can be made operational through a rectification action. We assume that all failed items are repaired minimally. Under a minimal repair, the failure rate after repair is the same as that just before failure. This type of repair characterization is appropriate for multi-component items where the item failure is due to failure of either one or a small number of components are replaced by new ones. Since the remaining unfailed components, constituting the bulk of the item, are unaffected, the failure rate of the repaired item differs only slightly from that just before item failure.

During the time frame L, the product failure have several distribution functions for the components: \( F_1(x, \theta) \), \( F_2(x, \theta) \), \ldots \( F_m(x, \theta) \). The \( x \) denote the random time of component failure, If failure of component happened, then we signed the product have a failure. We have a formula to describe product failure distribution function as following:

\[
F(t, \theta) = \int_0^t r(\theta) d\theta = (\theta_1, \theta_2, \ldots, \theta_m)
\]  

When a failure item is brought for repair, it is subjected to a testing procedure following by appropriate repair action. Assuming that the repair process is imperfect in the sense that it is done properly with probability \( q \) and not with \((1-q)\). In the latter case, the item comes back again for repair. The number of times an item is returned due to imperfect repair before it is repaired properly is a random variable. Note that \( q=1 \) corresponds to perfect repair and in this case a failure item is fixed at the first instance.

2.2 Manufacture’s expected profit

The manufacturer produces items with product performance given by parameter \( \theta \). The manufacture cost is \( C_m(\theta) \) per unit. An item manufactured can be none-conforming with probability \( r(\theta) \).

Since failures are repaired minimally and the time to repair is insignificant (relative to the time between failures), the number of failures for an item between the ages \( t_1 \) to \( t_2 \), \( N(t_1, t_2) \), is a random variable and distributed according to a non-homogeneous Poisson process with an intensity function \( r(t, \theta) \), the failure rate of the item. As a result, we have

\[
E[N(t_1, t_2)] = \int_{t_1}^{t_2} r(t, \theta) dt
\]

As a result, the expected profit per item is the difference failure occur between the selling price to the retailer minus the manufacturing cost and the cost to rectify defective items. This can given by

\[
\phi(S, \theta) = [S - C_m(\theta) - \lambda r(\theta)C_d]
\]

The manufacturer’s total expected profit can be given by:

\[
J(S, \theta) = D(P, C, T)\phi(S, \theta)
\]

The optimal decision variables for the manufacturer is the equilibrium points for the many game formulation. Note that the equilibrium point must satisfy \( \lambda^* < P^*, T^* < L^* \). If \( P^*(S, \theta), C^*(S, \theta), T^*(S, \theta) \) (the optimal values of \( P, C, T \), and which maximize \( J(P, C, T) \) for a given \( (S, \theta) \) exist, they can be obtained from the following first-order conditions:

\[
\begin{align*}
\frac{\partial J(P, C, T)}{\partial P} &= 0 \\
\frac{\partial J(P, C, T)}{\partial C} &= 0 \\
\frac{\partial J(P, C, T)}{\partial T} &= 0
\end{align*}
\]

Similarly, if \( S^* \) and \( \theta^* \) exist, with \( 0 < \theta < \theta^* \), they can be obtained from the following first-order conditions:

\[
\begin{align*}
\frac{\partial J(S, \theta)}{\partial S} &= 0 \\
\frac{\partial J(S, \theta)}{\partial \theta} &= 0
\end{align*}
\]

3 Special Case

In this section, we consider the following special case using optimal arithmetic mentioned above, which is aim to the cost-profit and failure modes rectify of \( Y \) series electrical motors.

**Step1:** Define the product (include component) reliability design parameter. The Fig. 2 is a function-structure (HOQ House of Quality) of YD-132M electrical motor. We can get a matrix to mention the relationship of the function and structure in the motor:
The cycle of the product is $t (\text{design value is } 45000h)$ in which there are $18000h\text{is warrant period}$. The probability of product reliability design parameter for the components' failure are $0.012, 0.034, 0.0085, 0.0053, 0.023$ (these parameter are influence the weight of the product reliability design parameter).

**Step 2:** Get the average cost to rectify components each non-conform failure item and cost to repair is $(68, 87, 22, 1.1, 134.5)$, and average lost in function is $(45, 46, 13, 8, 38, 32)$.

**Step 3:** Specially, assume the motor’s failure distribute function is $F(t, \theta) = 1 - e^{-\theta t}, t \geq 0$, there $\theta$ is the integration parameter of the cost to components failure rectify and function lost led to the profit lost, etc. A function of the manufacture cost and profit think of the product’s failure can be made. (formular (3) and (4)), From (5) and (6) we can get the $\theta = 0.61$ denote the optimal values of the motor function and profit by solving differential equation.

4 Conclusion
Now Failure identification are widely used in quality control detection of potential failure modes (includes failure location) during the design stage or product producing. We often aim to find failure modes but neglect the cost of the product influenced by the failure items. Although many methods have given, such as failure modes and effects analysis (FMEA), fault tree analysis (FTA), failure modes, effects and criticality analysis (FMECA), they do not mention the optimization cost and maintain performance constraints for the production. In this paper we developed a simple model formulation for carrying out such a study. The model formulation deals with a monopolistic case involving one manufacturer. The interesting and novel features of the model are (1) the optimization algorithmic modeling for typical censoring structures to identification failure and analysis the reason; (2) the interaction between the different notions of quality and components failure modes; (3) the quality management and economic implications that result from the analysis of the model. Another insight is the commonly seen improvement in design quality by manufacturers as they increase the dreability of products.

![Fig.2 The relationship of the function and structure in YD-132M electrical](image-url)
References:


