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EMPIRICAL STUDY OF THE Q-FACTOR OF INTEGRATED SQUARE SPIRAL INDUCTORS ON SILICON-ON-SAPPHIRE

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Abstract- Design of integrated inductors is a time consuming process usually requiring significant amount of electromagnetic simulation. During this process a designer simulates an inductor geometry that will provide the requisite inductance and usually iterates the design to achieve the maximum quality factor (Q). Due to the complexity and the lack of formulae this process is an art and the designer is guided by past designs and experience. This paper investigates the effect of geometric parameters on the Q (quality factor) of an inductor.

Key-Words: - Inductor, Quality Factor, Integrated Circuit

INTRODUCTION

During the on-chip passive component design, the integrated circuit designer seeks to build an inductor with a given inductance and the best performance. In many cases the best performance is the inductor with the highest Q-Factor. An initial design to meet the requisite inductance, based on geometric parameters, is well guided by prior work in the literature providing formulae relating inductance to the geometry of an inductor. The formulae in the literature are based on physical [2], [4], [5], [6], semi-empirical [7], [9], and empirical [7], [8] expressions for the inductance calculation.

The Q factor of an inductor can significantly affect the performance of circuits built with them. With the exception of [1][4] there is limited literature providing guidance to the RFIC designer addressing the problem of quality factor of an inductor to its geometric parameters. Hence to date a designer needs to iteratively modify and simulate the design until an inductor with requisite performance is found.

In reference [1] the quality factor is defined to equal to

$$Q = 2\pi \frac{|\text{Peak Magnetic Energy} - \text{Peak Electric Energy}|}{\text{Energy Loss in One Oscillation Cycle}} = \frac{\omega L_s}{R_s} \times \frac{R_p}{R_p + \left[(\omega L_s / R_s)^2 + 1\right] \cdot R_s} \times \left(1 - \frac{R_s^2 C_o}{L_s} - \omega^2 L_s C_o\right) \quad (1.1)$$

where R_s is the series resistance, L_s is the series inductance, R_p and C_p represent the lumped impedance and reactance between the structure and substrate.

Fig. 1 illustrates an on-chip spiral inductor and the corresponding lumped components used to model the inductors performance. From the figure it is apparent that in the inductor the magnetic energy stored in the magnetic field generated by the flow of current in the metal tracks, the inductance L relates the voltage across the inductor to the rate of change of current flowing in the same tracks. The stored electric field energy depends on the parasitic capacitances (C_s, C_{ox}, and C_{sub}) where C_s is the capacitance between adjacent tracks, C_{ox} is the capacitance between the structure and the oxide and C_{sub} is the capacitance to the substrate. The energy lost in the structure is modelled by the parasitic resistances (R_s, R_{sub}). The component R_s is meant to model the resistive loss in the tracks due to skin effect, proximity effect, current crowding and radiation. R_{sub} is the loss in the substrate. In practice however it is difficult to determine the lumped circuit equivalent components without significant amounts of simulation due to the complicated physics governing these processes.

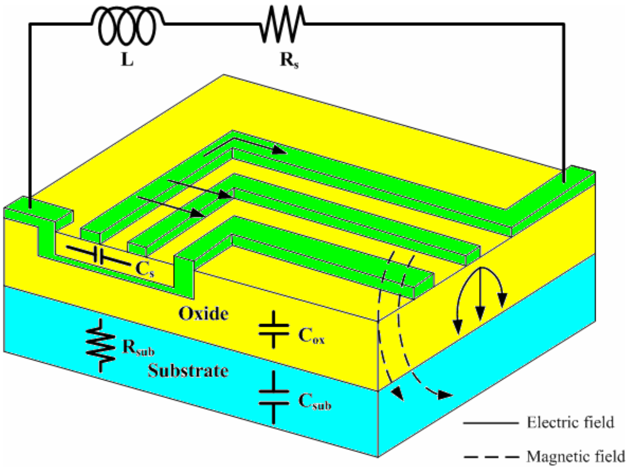

Fig. 1

Equivalent circuit representation of Square Spiral Inductors

This paper extends the work published in [1] by building empirical models relating the quality factor of an inductor to its geometric properties. That is to mitigate the need to determine, a usually difficult to extract, the lumped equivalent circuit components during the design phase. Empirical models for inductors built on Silicon on Insulator technology known as Silicon on Sapphire [10], [11] are presented in this paper. These formulae serve to guide the designer through the design process reducing the amount of full wave electromagnetic simulation required to produce an inductor with the requisite Q.

The paper is organized as follows: In Section titled Experimental Setup a description the Peregrine 0.25um GC silicon on sapphire process is provided (Fig. 2). A geometric description of inductor properties is presented in conjunction with the simulation methodology. Because of the impractical nature of building a significant sample set of inductors, greater than sixty two thousand (62,000) inductors in this study, a simulation study was undertaken and details are provided in this section. In subsequent sections the quality factor versus inductor outer length, width, spacing and number of turns are investigated respectively. In the experimental results section a proportion of inductors is selected and are fabricated. Experimentally measured values of the Quality factor are compared with the simulation results.

Experimental Setup

The aim of this paper is to establish relationships between the quality factor of an inductor and its geometric properties such as length (namely outer length), width, spacing, and number of turns. These are denoted as Len, W, S, and N respectively, and are shown in Fig. 3.

In order to determine the requisite relationship between Q and geometry a large number of inductors is required. This is impractical. Instead in this work a simulation study was undertaken. In this study 62,000 inductors square spiral inductors were simulated using ASITIC [2]. The inductors simulated varied in length, (Len): 200-500um with a step size of 10um, varied in width, (W) between 5-150um with a step size of 5um, and spacing, (S) varied between 5-70um with a step size of 5um. Variations were subject to constraint (1.2) to ensure geometries specified corresponded to real inductors.

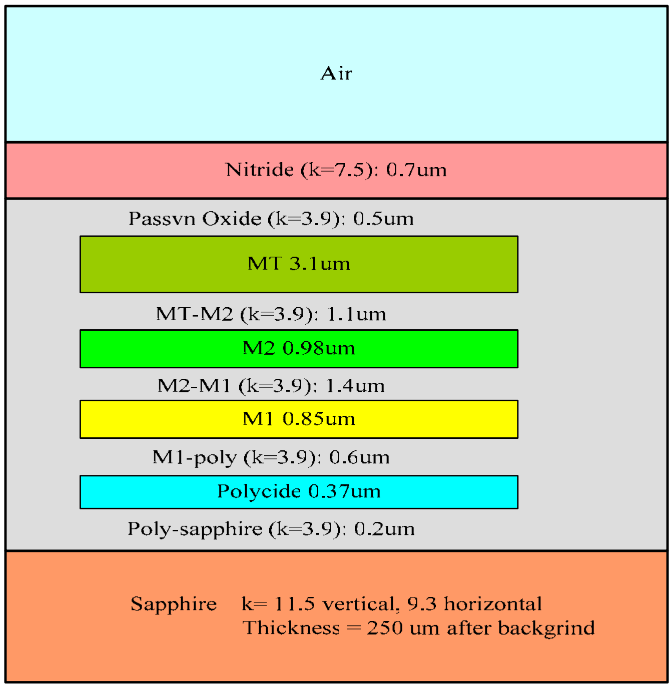

Fig. 2

Stack up of the 0.25um Peregrine GC process used in this study.

Equation (1.2) represents the minimum structural prerequisite in the design of square spiral inductors. Any inductor not satisfying this equation cannot be actually built as a square spiral inductor. Many inductors simulated that had a self resonance frequency of less than 2GHz were also discarded. Sixty two thousand nine hundred and ninety seven 62,997 inductors satisfying equation (1.2) remained which had a self-resonance frequencies of greater than 2GHz. For this simulation study the simulation chip size was set to 900umx900um and the FFT size was set to 512. All inductors in this simulation study are built on the top thick metal layer MT.

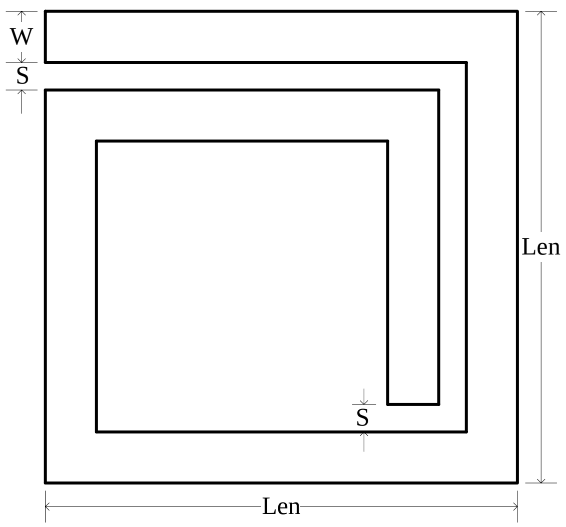

Fig. 3

Geometric parameters defining a square spiral inductor. In this figure an N=1.5turns; Len: Length (or Out Length); W: Width; S: Spacing inductor is illustrated.

$$\left[\frac{N-0.5}{0.25}\right] \leq \frac{Len-W}{W+S} \tag{1.2}$$

A select proportion of inductors were fabricated utilizing the Peregrine 0.25um GC Silicon on Sapphire process. The experimental results were used to validate the simulation results. The Peregrine 0.25um GC Silicon on Sapphire process comprises of a back-grinded 250 micrometer anisotropic sapphire substrate with a vertical and horizontal relative dielectric constant of 11.5 and 9.3 respectively. There are three aluminium metal layers, M1, M2 and MT of thickness 0.85um, 0.98um and 3.1um respectively. These layers are separated by silicon dioxide with thicknesses of substrate-M1, in the absence of polycide, being 0.8um, M1-M2 being 1.4um, M2-MT being 1.1um. The Thick top metal layer has a layer of silicon dioxide of thickness of 0.5um. This is covered by a nitride passivation layer of thickness of 0.7um. The relevant parameters of this process are also illustrated in Fig. 2.

In each of the subsequent sections, our analysis comprised of three steps: The first step involved plotting Q as a function of one of the geometric variables. This produced one curve. By varying a second variable a family of curves was produced. These families of curves were used to determine a mathematical relationship between Q and the geometric parameters. Equations representing the relationship between Q versus length, Q versus width, and Q versus spacing were determined. The equations were validated by changing other parameter and determining the predictive capability of the equation. Lastly a physical explanation of the observed behaviour is provided.

ANALYSIS OF Q VERSUS LENGTH

In this section the Q of an inductor versus outer length with the width spacing and number of turns fixed is analyzed.

From the data and with the assistance of curve fitting software, the parametric equation is established

$$Q = a + b \times Len^{2.5} + c \times \ln(Len) \tag{1.3}$$

where a, b, c are parameters depending on values of width, W, spacing ,S, and number of turns ,N, is able to accurately represent the variation of Q versus length for all simulated inductors.

In order to demonstrate the validity of (1.3) two arbitrary sets of samples with fixed W, S, and N were used. The one displayed in Fig. 4 is for small N and geometric parameters equal to N=1.5, W=20um, S=5um; Using least square the parameter were determined and

$$\begin{aligned} Q = &-32.6574 - 1.4564 \times 10^{-6} \times Len^{2.5} \\ &+ 8.7429 \times \ln\left(Len\right) \end{aligned} \tag{1.4}$$

for Fig. 4.

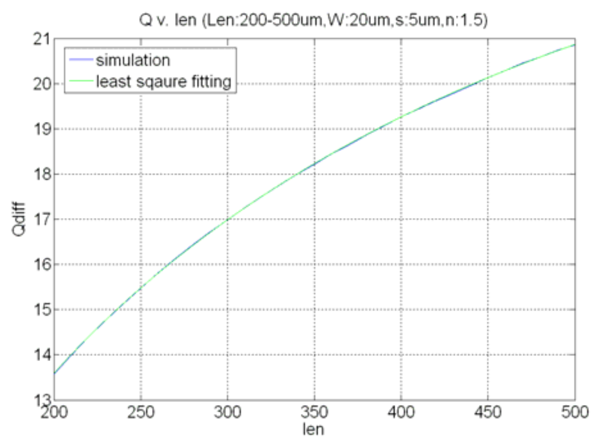

Fig. 4

Comparison of curves of Q versus Length between simulated data and equation calculation – small N (N=1.5, W=20um, S=5um)

In the absence of effects such as: proximity effect, current crowding and radiation losses, when the outer length of an inductor, Len (outer length) increases, the energy stored in the magnetic field $E_{magnetic}$ increases because a larger flux capturing area is enclosed. The energy lost, $E_{loss/second}$, increases proportionally to the total length of the metal track increase due to the increase in metal resistance. This is consistent with [4] where it is reported that that as metal length increases, the metal's self-inductance is increasing faster than its resistance.

The results however show that increasing length does not always lead to larger Quality factor. Inductors with large number of turns, N, simulated in this section exhibit a parabolic dependence of Q versus length. The drop of Q is attributed to the increase in capacitance between tracks, currents flowing near each other exhibit forces that further reduce the current carrying component of the track as well as edges that cause current crowding and losses due to radiation.

Q VERSUS WIDTH

In this section the Q of an inductor versus width with the length spacing and number of turns fixed is analyzed.

Using least square fitting techniques a parametric variation of Q versus width is determined to be equal to

$$Q = a + b \times W^{1.7} + c \times W^{0.5} \tag{1.5}$$

where a, b, c depends on the values of length, spacing, and number of turns. We utilized a sample set of inductors that have the same Len of 400um, S of 15um, and N of 2. For this samples set, the parameters are determined to be equal to

$$\begin{aligned} Q = &-3.3033 - 0.0188 \times W^{1.7} \\ &+5.6500 \times W^{0.5} \end{aligned} \tag{1.6}$$

A comparison of the simulation data and the result calculated from equation(1.6) is shown in Fig. 5.

Fig. 5 illustrates that equation (1.5) adequately describes the Q versus Width variation effectively.

In previous work, [10], it has been suggested that metal track be made as wide as possible provided it does not exceed 20um [4]. The motivation has been that increasing the width decreases the resistance losses and will provide a higher quality inductor.

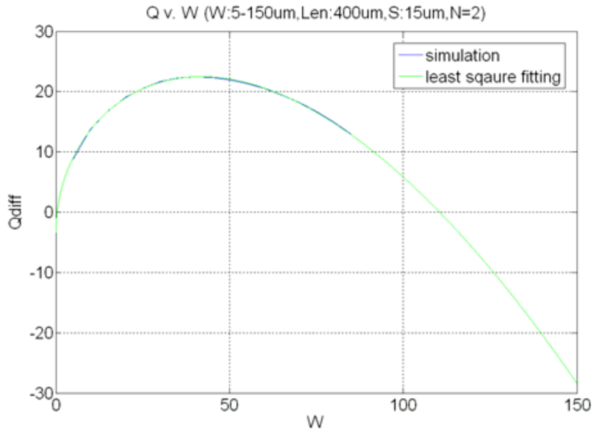

Fig. 5

Comparison of curves of Q versus Width between simulated data and equation calculation (Len=400um, S=15um, N=2)

When W increases from a small value the decrease of resistances plays a positive role on the value of Q increasing its value. As W continues to increasing, the increase of capacitance and the decrease of inductance is greater than the reduction in resistance. This results in a reduction of Q.

Q VERSUS SPACING

In this section the Q of an inductor versus spacing with the length, width and number of turns fixed is analyzed.

$$Q = a \times S^{b} + c \times S^{d} + e \tag{1.7}$$

where the parameters a, b, c, d, e depend on the length, Len, width W, and number of turns N.

For the sample set of inductors with length, Len = 400um, Width W = 60um, and number of turns N = 2. For this samples set the parameters of (1.7) are determined to be

$$\begin{aligned} Q = &-9.9828 \times 10^{-4} \times S^{2.2047} \\ &-2.8070 \times S^{-0.6035} + 21.3930 \end{aligned} \tag{1.8}$$

The results of the simulation and those calculated from equation (1.8) for Q versus spacing are shown in Fig. 6.

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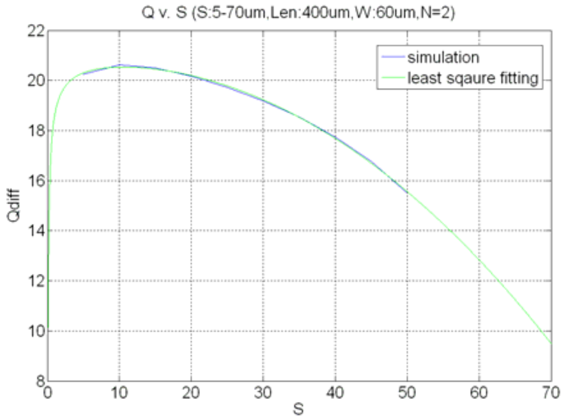

Fig. 6

Comparison of curves of Q versus Spacing between simulated data and equation calculation (Len=400um, W=60um, N=2)

In [4] it is stated that the proximity effect can be neglected for metal tracks built on the same metal layer. For small spacings the proximity effect is significant and cannot be neglected. The reasons why it cannot be neglected are: As spacing is increased the inductance and series capacitive coupling decreases while the proximity resistive loss has a larger decrease. Hence the Q of the inductor begins to initially increase. As spacing continues to increase the proximity loss becomes smaller and then can be neglected. Furthermore as spacing increases the decrease in inductance is greater than the decrease of the series capacitance. Hence the Q decreases as the spacing continues to increase.

EXPERIMENTAL RESULTS

In previous sections the relations between Q and geometric parameters: length, width, spacing and number of turns are investigated and analysed. In this section the inductors built on Silicon on Sapphire, shown in Fig. 7, are compared with the results obtained via ASITIC.

In total 23 square spiral inductors were fabricated. Additional structures were built on the die to permit De-embedding of pads [12]. The inductors built were measured using a Suss-Microtech Probe Station and a 110GHz Anritsu Vector Network Analyser.

After measuring the S-parameter and de-embedding the S-parameter they are converted to Y-parameters. The Q is then determined using the equation

$$Q_{diff} = -\frac{\mathrm{Im}(Y_{11}+Y_{22}-Y_{12}-Y_{21})}{\mathrm{Re}(Y_{11}+Y_{22}-Y_{12}-Y_{21})}. \qquad (1.9)$$

The measured and simulated Q values at 2GHz are compared. The results are presented in Table 1.

Len (um)	W (um)	S (um)	N	L (nH)	f_{SR} (GHz)	2GHz Qdiff-m	Qdiff-as
450	45	15	1.25	1.5	14.2	21.232	22.07
450	40	10	1.25	1.6	13.9	20.551	22.46
450	30	6	1.25	1.8	13.6	20.637	19.7
450	42	15	1.75	2	12.2	19.303	23.6
450	37	15	1.75	2.2	12.0	19.592	22.94
450	30	15	1.75	2.4	11.6	19.153	21.51
450	30	6	1.75	2.6	11.3	19.495	21.57
450	40	12	2.25	2.9	9.8	21.21	24.97
450	38	6	2.25	3.2	9.4	20.265	25.03
450	34	6	2.25	3.5	9.2	18.778	24.67
450	29	6	2.25	3.8	8.9	20.279	23.87
450	23	8	2.25	4.2	8.6	18.437	22.15
450	28	7	2.75	4.6	8.0	19.595	23.89
450	25	6	2.75	5.1	7.7	19.203	23.5
450	23	6	3.25	6.8	6.5	19.976	23.99
450	19	7	3.25	7.5	6.3	19.71	23.14
450	16	7	3.25	8.3	6.1	19.096	21.98
450	18	6	3.75	9.1	5.7	18.392	23.27
450	20	6	4.25	10	5.3	19.047	23.82
450	17	7	4.25	11	5.1	18.726	23.56
450	18	6	5.75	13.3	4.5	16.769	22.67
450	17	6	7.25	17.8	3.8	15.177	21.03
450	16	6	8.25	21.5	3.5	14.22	19.97

Table 1

Measured and simulated Q values at frequencies at 2GHz, for square spiral inductors as a function of geometry.

From the table it is evident the measured and the ASITIC simulated results for quality factor Q correspond reasonably well at 2GHz.

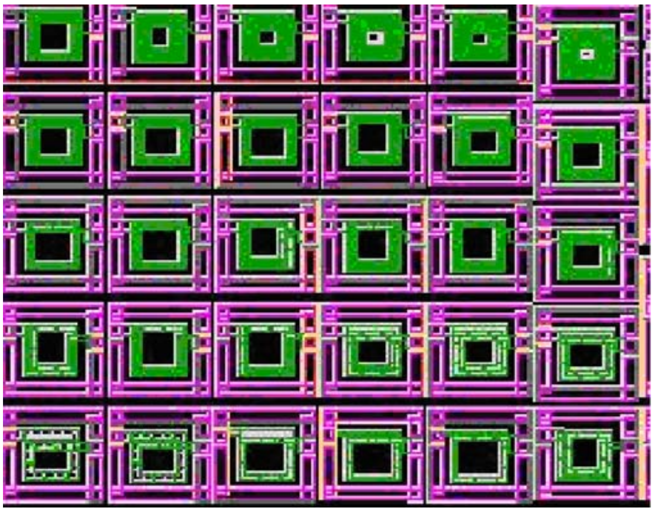

Fig. 7

Picture of Fabricated Inductors

CONCLUSION

An important component in the design of radio frequency integrated circuits is the design and fabrication of integrated on chip inductors. Design of integrated inductors is a time consuming process usually requiring significant amount of electromagnetic simulation. In this paper, we determined empirical formulae that describe the relationship between a square spiral inductor's Q and its geometric parameters. The extracted formulae and proposed rules guide the design of integrated square spiral inductors on silicon on sapphire.

Moreover, it will be more convenient and more intuitive if there is one general equation of Q versus all four geometric parameters, and will be much better to introduce another parameter – operation frequency into the equation, which will be parts of our future work.

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