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Measuring Moment of Inertia Based on Identification of Nonlinear System Featuring Naught Excitation

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Abstract: For large ammunitions and spacecrafts, friction moment and air resistance are major error resources in measuring moment of inertia (MOI). The paper proposes a novel measurement method based on compound pendulum, in which the whole measurement is considered as a problem of system identification, while the friction moment, the air resistance, and the to-be-measured MOI are considered as the system parameters to identify. A compound pendulum is a nonlinear kinetic system, and what is special, its excitation function is zero. Modern System identification technology could identify an unknown system by analyzing the relation between different responses and excitations. But to zero excitation, the above technology could no longer work. So the paper resorted to phase-plane analysis and identifies the zero excitation nonlinear system successfully. With this unique method, the influences of both friction moment and air resistance on the compound pendulum have already been taken into consideration without estimating or the measuring them in advance.

Key-words: Compound pendulum, Moment of inertia, Friction moment, Air resistance, Nonlinear system, System identification, Naught excitation

1 Introduction

Moment of inertia (MOI) represents the measure of inertia of a rotating rigid body. It is of considerable significance in the study, design, and manufacturing of ammunitions, satellites, and other spacecrafts.

Torsional pendulum method[1-13] and string pendulum method[14-19] are widely used in measuring MOI. For large ammunitions and spacecrafts, often heavy in weight and many of them have aerofoil and rudder mounted, friction moment and air resistance are major error resources. Methods have been taken to eliminate the both resistances, like gas bearings and vacuum cabin. But these two techniques bear the disadvantages of complex in structure and expensive in cost. So the paper tries to find a new measurement method.

Because torsional pendulum method and string pendulum method require the axes of the measured object being placed vertical to the horizontal surface; they are suitable for large-scale object for the sake of safety, especially those of large length/diameter ratio. So we adopt compound pendulum.

2 Composition of the measurement equipment

As shown in Fig. 1, the instrument consists of chassis, swing shaft, bearings, connecting arms, supporting tray, and angular displacement sensor. Q_1 represents the axes of the swing shaft. Q_2 represents the axes, with respect to which the MOI is measured, usually the axes of a cylinder-shaped object. L represents the distance from Q_1 to Q_2.

Exert a moment onto the supporting tray and then release it suddenly. The deviation from the balance position makes the tray, together with the connecting arms and the to-be-measured object swing back and forth; that is to say, they are making a compound pendulum movement.

We denote the sum MOI of the tray, arms and the to-be-measured object with respect to Q_1 by J_Z, the sum MOI of the arms and the tray with respect to Q_1 by J_T, the MOI of the to-be-measured object with respect to Q_1 by J_D, the MOI of the to-be-measured object with respect to Q_2 by J_S. The angular displacement sensor recodes the variation of angle with the time and the recorded data are stored in a computer. With these data, we can obtain J_Z (for detailed calculation, please read Section 2 and

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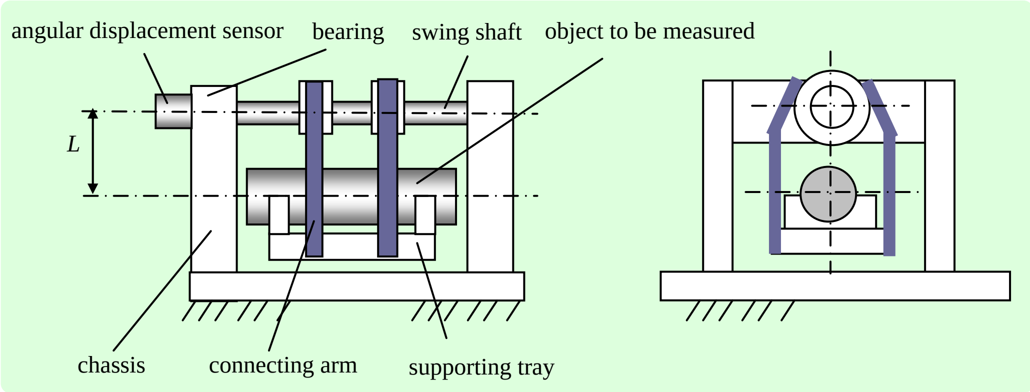

Fig.1 Composition of the measurement Equipment

Section 3). Subtract J_T from J_Z, we have

$$J_D = J_Z - J_T \quad (1)$$

Further, we can get

$$J_S = J_D - m_D \bullet L^2 \quad (2)$$

In which, J_T is a known quantity; m_D represents the mass of the to-be-measured object, which has usually been measured in advance.

For large-scale ammunitions or spacecrafts, the friction moment of bearings and the air resistance, acting as damping that prolong the swing period and reduce the swing angle, must not be ignored. In Section 3, we establish the kinetic model of pendulum system considering friction moment and air resistance, and any conclusion drawn in Section 3 suits the equipment illustrated in Fig. 1.

3 Kinetic model of compound pendulum considering friction moment and air resistance

The compound pendulum is a rigid body swings around an fixed axes that is parallel to the horizontal surface, shown in Fig. 2, in which C is the centroid of the rigid body, O is the intersection point of the fixed axes and the paper (for convenience, we can also use O to represent the fixed axes). We denote the

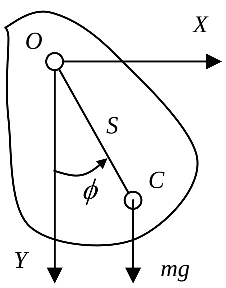

Fig.2A compound pendulum

pendulum's MOI with respect to O by J, the mass of the pendulum by m, the distance from O to C by S, and the included angle between OC and OY by ϕ with anticlockwise direction as positive, the initial angle of OC by ϕ_0 (not shown in Fig. 2). OY is in the same direction as the gravity.

We can describe the compound pendulum movement as the equation

$$J \cdot \frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} = -mgS\sin(\phi) - \mathrm{sgn}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right) \cdot (M + R_x) \quad (3)$$

in which, t represents the time, g the acceleration of gravity, M the friction moment of bearing. R_x represents the air resistance.

When a well-lubricated bearing works under medium load and medium rotate speed, M is proportional to the radial load of the bearing [18], so we have

$$M = \mu \cdot mg \cdot \cos(\phi) \quad (4)$$

in which, μ is a proportional coefficient with unit of m and greater than 0, relating to the type and the pitch circle's diameter of the bearing.

For a body making a translation movement, the air resistance is proportional to the square of its speed [19]. So we can reach the relation

$$R_x \propto v^2 \quad (5)$$

in which, v represents the average speed of all the mass points on the compound pendulum. Because every mass point swings around O, easily we have

$$v \propto \mathrm{d}\phi/\mathrm{d}t \quad (6)$$

Thus, R_x can be expressed as

$$R_x = K \cdot (\mathrm{d}\phi/\mathrm{d}t)^2 \quad (7)$$

in which, K is a parameter relating to current density of air, structure of the instrument, fixture condition and the shape of the to-be-measured object. Because the speed of each mass point is much too much less than the velocity of sound, K is independent of $\mathrm{d}f/\mathrm{d}t$. Both friction moment and air resistance trend to decelerate the compound pendulum movement

Now, from (3) to (7), we can get the complete form of the kinetic model of the compound pendulum system

$$J \cdot \frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} + \mathrm{sgn}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right) \cdot K \cdot \left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^2 + mgS\sin(\phi) + \mathrm{sgn}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right) \cdot \mu \cdot mg \cdot \cos(\phi) = 0 \qquad (8)$$

This is a two order nonlinear system of single variable ϕ respect to time t, in which m, g and S are already known, while K, μ and J are the parameters to identify. We notice that the right side of (8) is equal to zero. That means there is no excitation input to this nonlinear system. The system is only driven by its initial deviation of balance condition. We shall rationally name this kind of system as "Zero excitation nonlinear system" or "Naught excitation nonlinear system". Easily we can also have the concept of "Zero excitation nonlinear system identification" and "Naught excitation nonlinear system identification".

We know that modern system identification technology could identify an unknown system by analyzing the relation between different responses and excitations[20]. But to a zero excitation like (8), the present system identification technology could no longer work. Thus we resorted to phase-plane analysis [21]. Define

$$x = \phi, \qquad y = \mathrm{d}\phi/\mathrm{d}t \qquad (9)$$

Substitute (9) into (8), yields

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-a\sin(x) - b\cos(x) - cy^2}{y} \quad \text{for} \quad y \geq 0 \qquad (10)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-a\sin(x) + b\cos(x) + cy^2}{y} \quad \text{for} \quad y < 0 \qquad (11)$$

where

$$a = mgS\,/\,J, \quad b = \mu\, mg\,/\,J, \quad c = K\,/\,J \qquad (12)$$

When initial condition (x_0, y_0) is given, we can solve the ordinary differential equation (10) and (11) as follows

$$y = \sqrt{\frac{(-2bc + a)\cos(x) - (2ac + b)\sin(x) + C_1 \cdot \mathrm{e}^{-2cx}}{2c^2 + 0.5}}$$

for $y \geq 0$ (13)

$$y = \sqrt{\frac{(-2bc + a)\cos(x) + (2ac + b)\sin(x) + C_2 \cdot \mathrm{e}^{2cx}}{2c^2 + 0.5}}$$

for $y < 0$ (14)

Where

$$C_1 = \frac{\mathrm{e}^{2cx_0}}{4c^2 + 1}\left[\frac{1}{2}y_0^2 + (2bc - a)\cos(x_0) + (2ac + b)\sin(x_0)\right] \qquad (15)$$

$$C_2 = \frac{\mathrm{e}^{-2cx_0}}{4c^2 + 1}\left[\frac{1}{2}y_0^2 + (2bc - a)\cos(x_0) - (2ac + b)\sin(x_0)\right] \qquad (16)$$

Employing so-called "sewing method", we can draw the phase-plain of a compound pendulum movement according to (13) and (14). Fig. 3 is the phase-plain of a compound pendulum movement with g=9.8m · s^{-2}, J = 1000kg · m^2, μ=0.01m, m=500kg, ϕ_0= -0.5rad, S=1m, K=10N · m · s^2. From this spiral-like curve, we see that the sequential spans of angle decrease one by one, and that the compound pendulum movement is a process of energy dispersion.

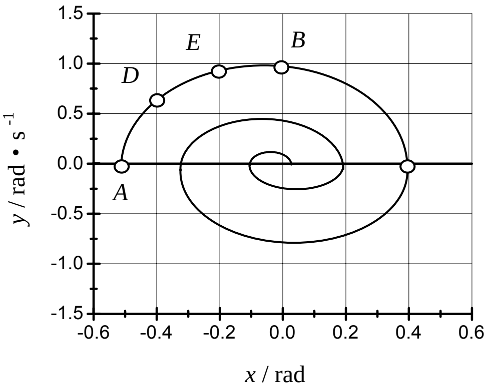

Fig. 3 Phase-plain of compound pendulum

4 Identification of the system

In this section, we discuses the approach to the identification of the system , or the definition of MOI J from the function $\phi(t)$, the variation of angle with the time, acquired by the angular displacement sensor.

Let's consider Fig.3, in which the curve ABC represents the swinging of the compound pendulum from the initial point to the highest point on the other side. A represents the moment when the compound pendulum starts to swing. Abscissa of A is ϕ_0 and ordinate is 0. B represents the very moment when the compound pendulum reaches the balance point. Abscissa of B is 0 and ordinate is ω_B. We define D which abscissa is 0.8ϕ_0 and ordinate is ω_D, and define point E which abscissa is 0.4ϕ_0 and ordinate is ω_E. We know that

$$\omega_B = \frac{\mathrm{d}\phi}{\mathrm{d}t}\Big|_{\phi=0}, \omega_D = \frac{\mathrm{d}\phi}{\mathrm{d}t}\Big|_{\phi=0.8\phi_0}, \omega_E = \frac{\mathrm{d}\phi}{\mathrm{d}t}\Big|_{\phi=0.4\phi_0} \qquad (17)$$

All points on the curve are consistent with (13), so substitute the coordinates of B, D, E, and A into (13), we obtain

$$\omega_B{}^2 = \frac{p_1 + C_1}{p_3} \qquad (18)$$

$$\omega_D^2 = \frac{p_1\cos(0.8\phi_0) - p_2\sin(0.8\phi_0) + C_1 e^{-1.6c\phi_0}}{p_3} \quad (19)$$

$$\omega_E^2 = \frac{p_1\cos(0.4\phi_0) - p_2\sin(0.4\phi_0) + C_1 e^{-0.8c\phi_0}}{p_3} \quad (20)$$

$$p_1\cos(\phi_0) - p_2\sin(\phi_0) + C_1 \cdot e^{-c\phi_0} = 0 \quad (21)$$

Where p_1=(-2bc+a), p_2=(2ac+b), p_3=2c^2+0.5.

Take (18), (19), (20), and (21) as one system of equations. To solve this system for four unknowns, a, b, c, and C_1, we take the following steps:

Step 1: Take (19), (20), and (21) as one system of equations and solve this system for three unknowns, a, b, and C_1. Then a, b, and C_1 can be analytically expressed in terms of c, which is still unknown. (The expressions are too complex to present here.)

Step 2: Substitute the expressions of a, b, and C_1 we obtained in Step 1 into (18), yields a new equation that has only one unknown c.

Step 3: Numerically solve the new equation we obtained in Step 2 for c by using a certain method of computational mathematics, like Newton's method or Gauss-Seidel's method[22].

Substitute c obtained in Step 3 into the expression of a, b, and C_1 obtained in Step 1. Thus we have solved all the equations. According to (12), we can define the MOI of the compound pendulum

$$J = mgS / a \quad (22)$$

The actual phase-plain will be obtained from the experiment and must suffer from a certain amount of error. In order to verify the precision of above method, we conducted numerical simulations[23]. The results of the numerical simulations showed that the measurement method is of a satisfactory precision.

5 Conclusions and discussions

We propose a novel measurement method based on compound pendulum. We consider the whole measurement as a problem of system identification, and consider the friction moment, the air resistance, and the to-be-measured MOI as the system parameters to identify. With this method, we can complete the measurement with high precision, considerable ease and safety. Also, we need not to estimate or measure the friction moment of bearings and the air resistance, while have already taken their influences into consideration.

In this paper, we put forward the new concept of "Zero excitation nonlinear system identification" or "Naught excitation nonlinear system identification". The author believes that this concept is not something eccentric; it may theoretically be a large , even important category in the field of system identification, and it will gradually be noticed by the researchers and professors of system identification. We have managed to accomplish the zero excitation nonlinear system identification of a certain form (equation (8)). The author hopes this effort to be a beneficial start to a thorough research on the whole category of "zero excitation nonlinear system identification".

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