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Effect of Limited Modal and Noise Information on Structural Damage Detection

J. ALBERTO ESCOBAR, RAMSÉS RODRÍGUEZ AND ROBERTO GÓMEZ
Instituto de Ingeniería.
Universidad Nacional Autónoma de México.
Ciudad Universitaria, Apdo. Postal 70-472, Coyoacán, 04510.
MÉXICO, D.F.

Abstract: The effect of limited modal and noise information on the Transformation Matrix Method is presented. This method detects structural damage and operates on the global stiffness matrix of a structure to condense on the primary degrees of freedom. The method is based on the fact that the transformation matrix for the damaged state can be initially estimated from the corresponding to the non damaged state, by using an iterative procedure.

As data, the method is based on the condensed stiffness matrix of the structure that requires modal shapes and vibration frequencies identified from acceleration records. Initially, the localization of seismic instruments is defined. Afterwards, the acceleration records coming from the instrumented floors of the structure are obtained. From these, the dynamic characteristics of the structure are identified to reconstruct the condensed stiffness matrix. This matrix corresponds to the damaged state of the structure.

Finally, using the adjusted condensed stiffness matrix with the dynamic characteristics of the structure and the analytically computed condensed stiffness matrix, the Transformation Matrix method locates damage in structural elements as the percentage of loss of stiffness.

In this paper, the effect of ignoring all modal shapes and vibration frequencies in the damage detection of structures is studied. Examples are presented and advantages and disadvantages of the Transformation Matrix Method are discussed.

The final objective of this work is to present this new formulation applying it to the study of building structures.

Introduction

Many of recent human and economic big losses in the world, have been caused by earthquakes. During history, it has been observed that these losses have been caused by a faulty seismic behaviour of structures. This behaviour causes partial failures and even total collapse of the structures, as well as fires or explosions that can increase the losses.

In order to prevent from such disasters, there are some methods for damage detection in structures that produce satisfactory results (Berman and Nagy, 1983; Hassiotis and Jeong, 1995; Cobb and Liebst, 1997; Sohn and Law, 1997; etc). These are necessary for different load conditions of the buildings, such as earthquakes, over load, wind, machinery induced vibrations, corrosion, thermal effects, etc.

In addition, in spite that current construction codes are improved every year, engineers are aware of catastrophic losses that could be caused by severe earthquakes, even in those countries in which studies on seismic engineering are a high-priority research activity.

In this paper, the effect of limited modal information on structural damage detection based on the Transformation Matrix method that operates on the global stiffness matrix is studied. The method is based on the condensed stiffness matrix of the structure which requires modal shapes and vibration frequencies, identified from acceleration records which are recorded at seismic instruments located on the structure. From these, the dynamic characteristics of the structure are identified, and consequently the condensed stiffness matrix is derived. This matrix corresponds to the damaged state of the structure. Finally, using the condensed stiffness matrix adjusted with the dynamic characteristics of the structure and the condensed stiffness matrix analytically computed, the Transformation Matrix method locates damage in the structural elements as the percentage of the loss of stiffness.

In this paper, the effect of ignoring all modal shapes and vibration frequencies in the detection of damage of structures is studied. Examples are presented and the advantages and disadvantages of the Transformation Matrix method are discussed.

The Transformation Matrix Method

When a structure is subjected to seismic accelerations, measurement of its effects provides useful information to study and to evaluate its dynamic response as well as the historical evolution of its behaviour. When a structure is instrumented it is possible to get its modal shapes and vibration frequencies that can be used to estimate its damage state.

The damage detection method presented in this paper is known as the Transformation Matrix method, TMM, because it is based on the transformation matrix that operates on the global stiffness matrix to condense the primary degrees of freedom (Sosa et al., 1998; Escobar, et al., 2001, 2004, 2005). This reduction is carried out based on modal shapes and vibration frequencies identified from seismic records.

Damage detection in plane frames

The global stiffness matrix of a plane frame for a damaged state $[K_d]$, can be written as:

$$[K_d] = [K_{wd}] - \sum_{i=1}^{nej} dk_i [K]_i \qquad (1)$$

where $[K_{wd}]$ is the stiffness matrix of the structure without damage; *nej*, is the number of elements of the frame; dk_i is a nondimensional parameter that represents the degradation of stiffness of the *i-th* element ($0 < dk_i < 1$); $[K]_j$ is the stiffness matrix without damage of the element *i* of the frame. The lateral stiffness matrix $[\bar{K}_d]$ corresponding to a state of damage of the frame is calculated as:

$$[\bar{K}_d] = [\bar{K}_{wd}] - \sum_{i=1}^{nej} dk_i [\bar{K}]_i \qquad (2)$$

where $[\bar{K}_d] = [T_d]^T [K_d][T_d]$; $[T_d]$ is the transformation matrix, and a function of the partition performed in the global stiffness matrix among primary and secondary degrees of freedom; $[\bar{K}_{wd}] = [T_{wd}]^T [K_{wd}][T_{wd}]$ and $[\bar{K}]_i = [T_{wd}]^T [K][T_{wd}]$. Sub indexes *d* and *wd* correspond to the damaged and non damaged state, respectively.

In order to calculate the lateral stiffness matrix of the structure in equation (2), as a first approach, it can be assumed that the transformation matrix for the damaged state $[T_d]$, does not differ from the one corresponding to the non-damaged state $[T_{wd}]$. In this way, an iterative procedure allows to detect the damaged elements by successive approaches (Escobar, et al., 2001, 2004, 2005).

The lateral stiffness matrix of the damaged structure is of order *mxm*, and due to its symmetry, it has $nti=m(m+1)/2$ independent terms. Developing equation (2) for each one of the terms of each matrix, it is obtained:

$$\{\bar{k}_{wd}\} - \{\bar{k}_d\} = [S_k]\{dk\} \qquad (3)$$

where $\{\bar{k}_{wd}\}$, $\{\bar{k}_d\}$, and $\{dk\}$ are vectors of order ntix1 that contain: the independent terms of the matrix of lateral stiffness without damage, the independent terms of the damaged lateral stiffness matrix, and the stiffness degradation of the structural elements, respectively; and $[S_k]$ is a matrix of order *ntixnej* that contains the terms $\bar{k}_{ij}$.

Because in general, the number of equations *nti* is different from the number of unknowns *nej*, the previous system of equations is non-consistent. A vector that provides an exact solution for the terms of the left side of equation (3) does not probably exist; in other words, it is probable that vector $\{\bar{k}_{wd}\}$ - $\{\bar{k}_d\}$ it is not a linear combination of the columns of $[S_k]$.

Damage detection in three-dimensional structures

The global stiffness matrix corresponding to a damage state in frame *j* of a structure is:

$$[K_d]_j = [K_{wd}]_j - \sum_{i=1}^{nej} dk_{ij} [K]_{ij} \qquad (4)$$

where dk_{ij} is the stiffness degradation of the element *i* of frame *j*. In this case, the condensed stiffness matrix of the three-dimensional structure corresponding to a damage state is obtained as:

$$[\overline{Kt}_d] = [\overline{Kt}_{wd}] - \sum_{r=1}^{Nr} dk_r [\overline{K}_d]_r \qquad (5)$$

where:

$$[\overline{Kt}_{wd}] = \sum_{j=1}^{Nm} [C]_j^T [T_d]_j^T [K_{wd}][T_d]_j [C]_j$$

$$[\overline{K}_d]_r = \sum_{\substack{j=1 \\ r \in j}}^{Nm} [C]_j^T [T_d]_j^T [K]_{rj} [T_d]_j [C]_j$$

In the previous equations, N_r is the number of elements in the structure; N_m is the number of frames; and $[C]_j$ is the transformation matrix of displacements. This matrix defines a relationship between the lateral degrees of freedom of the frame *j* with the primary degrees of freedom of the three-dimensional structure. From equation (5) it is possible to establish a system of linear equations when developing an equation for each matrix term different from zero. This is:

$$\{\overline{kt}_{wd}\} - \{\overline{kt}_d\} = [S_k]\{dk\} \qquad (6)$$

where $[S_k]$ is a matrix formed by the $\overline{k}_{dr}$ terms. Because the displacement transformation matrices are independent of the state of damage of a frame, the procedure to solve the equation (6) is similar to the one used for plane frames. As an initial approach for the solution, it is considered that the transformation matrices correspond to the non-damaged state.

Algorithm

The TMM can be summarized as the next iterative procedure:

1. For the non-damaged state, matrices $[K]_i$, $[K_{wd}]$ and $[T]$, are computed

2. Transformed matrices $[\overline{K}] = [T]^T [K_{wd}][T]$ and $[\overline{K}]_i = [T]^T [K]_i [T]$, are computed.

3. Vector $\{\overline{k}_{wd}\}$ and matrix $[S_k]$ of independent terms are formed.

4. Solve the system of equations $\{\overline{k}_{wd}\} - \{\overline{k}_d\}_{known} = [S_k]\{dk\}$ for $\{dk\}$.

5. For the obtained damage vector $\{dk\}$, the new global stiffness matrix $[K_d]$ and its corresponding new transformation matrix $[T]$ are computed.

6. Matrix $[K_d]$ is condensed and a vector $\{\overline{k}_d\}_{approx}$ of independent terms is formed.

7. If the difference between $\{\overline{k}_d\}_{known}$ and $\{\overline{k}_d\}_{approx}$ is less than a tolerance value, the process is halted; if not, process returns to step 3.

In order to finish the process it is necessary to define a criterion to measure the refinement reached by the solution after each iteration. Initially, if there is no damage in the structure (step 1 of the algorithm), the iterative procedure may converge to the damage state defined by the vector $\{\overline{k}_d\}_{known}$. This can be achieved if the transformation matrix used in step 3 of the *n+1* iteration is computed for a fraction of the sum of the damage states obtained in previous iterations *n* and *n-1*, for example:

$$dk_{n+1} = \beta\, dk_n + (1-\beta)\, dk_{n+1} \qquad (7)$$

Through an equivalent steepest-descent method, in each iteration the proposed algorithm finds an optimal value of β (from the values proposed by the user). In this way, the transformation matrix shows a gradual change that allows the detection of the damaged elements by successive approximations. In order to measure the approximation of the solution in each iteration, the following equation is used

$$e = min \left\| \{\overline{k}_d\}_{known} - \{\overline{k}_d\}_{approx} \right\|^2 \qquad (8)$$

Another way to finish the process is to establish a tolerance for the maximum value obtained when comparing the terms of the vectors in step 7. In structures with few structural elements, it is possible in the first iteration to find and quantify the damaged elements. Additionally, since a structural element can present only a damaged or non-damaged state, it is possible to improve the straight solution of the system of equations of step 4 assigning zero damage to the elements with a value of stiffness degradation smaller than a specific value. This criterion improves the location of the damaged elements using the TMM proposed. In order to reconstruct the lateral stiffness matrix, corresponding to the damaged state of the structure, modal parameters are needed. This procedure is described as follows and is named adjustment of the stiffness matrix.

Adjustment of the stiffness matrix

With the values of the known modal shapes and vibration frequencies, the stiffness matrix of the frame was fitted by using the Baruch and Bar-Itzhack algorithm (Baruch and Bar-Itzhack, 1978), this is

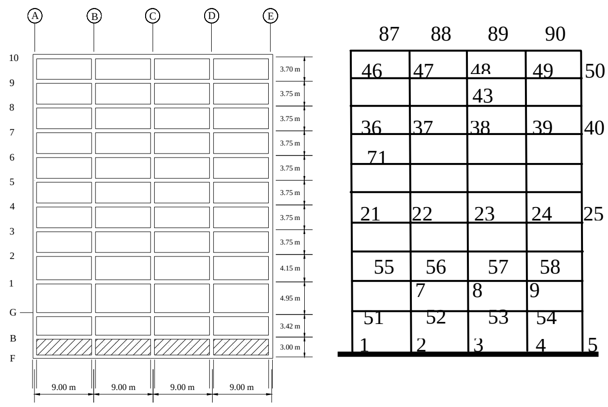
$$[\overline{K}_d] = ([\overline{K}] - [M][Z])[H] + [M][X][\Omega]^2[X]^T[M] \qquad (9)$$

Where $[X] = [\phi]([\phi]^T[M[\phi]])^{-1/2}$; $[H] = [I] - [Y]$; $[Y] = [X][X]^T[M]$; $[Z] = [X][X]^T[\overline{K}]$; $[\phi]$ is a known modal matrix; $[I]$ is the identity matrix; and $[\Omega]$ is a diagonal matrix containing the square of known natural frequencies.

To evaluate the effects of limited modal information on the TMM, it was applied to the STC building (figure 1), studied for Ávila and Meli (1987). The lateral stiffness matrix of the damaged structure was computed using Baruch and Bar-Itzhack algorithm (equation 9). Table 1 shows the results of relative error values, in percentage, of the diagonal terms of the simulated lateral stiffness matrix of the damaged structure computed with equation (9) with respect to the number of modes used, for damage cases A, B and C.

In Table 1, it can be observed that, in order to obtain relative error values (lower than 10%), six mode shapes must be utilized to reconstruct the condensed stiffness matrix.

For damage cases A and B at least eight modes were needed to get relative error values smaller than 2%, and for damage case C all modes were required. On the other hand, for damage case A, when the first three vibration modes of the STC building were utilized in equation (9), relative error values for all diagonal terms were smaller than 10%, except for the one corresponding to the damaged storey.



a) head frames   b) structural elements

Fig. 1, The STC Building studied (Ávila and Meli, 1987).

Table 1, Relative error values (in percentage of the total number of modes), of the terms of the condensed stiffness matrix of the damaged structure computed with equation (9) using different number of modes.

Term	Case A		Case B		Case C	
	Modes 1 to 3	Modes 1 to 6	Modes 1 to 3	Modes 1 to 6	Modes 1 to 3	Modes 1 to 6
$K_{1,1}$	14.2	6.1	0.0	0.2	0.0	0.4
$K_{2,2}$	2.9	5.6	0.0	0.0	0.1	0.5
$K_{3,3}$	0.2	0.6	0.0	0.2	3.3	5.6
$K_{4,4}$	0.1	0.3	0.0	0.0	14.2	9.4
$K_{5,5}$	0.0	0.1	0.0	0.4	15.2	8.0
$K_{6,6}$	0.0	0.1	0.1	0.1	3.7	4.9
$K_{7,7}$	0.0	0.2	0.2	1.6	0.1	1.2
$K_{8,8}$	0.0	0.0	3.3	5.6	0.1	0.0
$K_{9,9}$	0.0	0.2	14.5	4.7	0.0	0.3
$K_{10,10}$	0.1	0.1	12.0	1.4	0.1	0.2

In order to generalize results and evaluate the effects on the TMM, several studies for different structures considering limited modal information effects are needed. It is important to mention that these effects are independent of the precision of the proposed method.

Noise effects

The noise in the measurement of the dynamic characteristics in the evaluation of a state of damage of a building complicates damage detection. In this paper, the noise effects were considered through the modal shapes obtained for a state of simulated damage perturbed with different levels of noise; Sohn and Law (1997). Thus, for a modal shape ϕ ,the perturbed modal form $\hat{\phi}$ is built as:

$$\hat{\phi} = \phi\left(1 + \frac{N}{100}R\right) \qquad (10)$$

where N is the level of noise in percentage; R is a random number with normal probability distribution function, with zero mean, and variance one.

Noise effects of the measurements uncertainties on the STC building model were simulated for damage cases A, B and C. In Table 2, results of these damage cases utilizing a 3% of noise level are presented. It is shown that the TMM method identified all the damaged elements with relative error values lower than 15%. On the other hand, the relative error values of the simulated damage, with respect to the calculated one, are greater for beams than for columns for damage case A.

Table 2, Damage (in percentage) and relative error applying the TMM to the STC building with equation (3) of noise level.

Damage case	Damage element	Simulated damage (%)	Computed damage (%)	Relative error (%)
A	1, 5	30	31.4	4.7
	7	20	18.4	-8
	8, 9	20	18.4	-8
	22	10	8.8	-12
	23	10	8.8	-12
	24	10	8.8	-12
	36	20	19.7	-1.5
	(37, 38, 39, 40)	20	19.7	-1.5
	55	25	21.5	-14
	56, 57	25	21.5	-14
	58	25	21.5	-14
B	1, 5	20	19.8	-0.9
	2, 3, 4	20	19.8	-0.9
	51, 53	40	41.6	4
	52, 54	40	41.6	4
C	(46, 47, 48, 49, 50)	20	19.4	-3
	(87, 88, 89, 90)	40	42.9	7.3

Limited modal information and noise effects

In order to evaluate the damage detection method in a more realistic way, both, noise and limited modal information effects were considered to assess damage in the STC Building model.

The noise level used in equation (10) was 3%, and the number of modes utilized to reconstruct the condensed damaged matrix of the building, applying equation (9), varied from 1 to 10. When all modes were used relative error values are smaller than 5%, and smaller than 10% when only the first three modes were utilized. The latter is an advantage because it is common in practice that only a few of the first vibration modes can be identified experimentally. As expected, when all possible modes were utilized relative error values were smaller.

Conclusions

The Transformation Matrix Method, TMM, for damage location and assessment was presented. The method is based on the fact that the transformation matrix for the damaged state of the structure can be initially estimated from the corresponding to the non damaged state, by using an iterative procedure. Particularly, the effect of limited modal and noise information on structural damage detection was studied.

The TMM always locates damaged structural elements. When all the modal parameters are used and no noise is present, the TMM produces high precision to assess damage.

It was shown that in order to better identify the lateral stiffness matrix of the structure, an increase in the the number of modes improves damage identification.

When the noise effects are considered, the TMM method identified structural damage with relative error values smaller than 15%.

On the other hand, when noise effects and complete modal information were considered simultaneously, the TMM located correctly damaged structural elements. In these cases, the maximum relative error values obtained was 5%.

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