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Uncertainty-Driven Synchronization of a Hyper-chaotic System

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Abstract: This paper presents a novel idea of uncertainty-driven synchronization for a hyper-chaotic system. The idea is described in a general sense and a feedback control method is proposed for performing the chaotic synchronization based on nonlinear feedback by analyzing the stability of a nonlinear system in which the Lyapunov function is built. Owing to the existence of uncertain parameters in a nonlinear system, an adaptive law is applied for deduction. Synchronization with unequal parameters of a driving and responding system is achieved in this paper. The validity of the proposed approach is proved by computer simulations in a hyper-chaotic Rössler system.

Key-Words: Chaotic synchronization, uncertainty driven, hyper-chaotic system, uncertain system parameter, feedback control

1. Introduction

In recent few years, chaotic synchronization in nonlinear complex system, an area of tremendous interest in nonlinear science and crossed fields, has potential applications in secure communication, laser light and biology systems. Chaotic synchronization has been widely studied since Pecora and Carroll [1] firstly introduced it. For the various approaches developed in these years, they can be classified into driving-responding system and coupled method.

Linear feedback control synchronization is of simple structure and easy to be done, therefore it can be applied in many practical systems [2][3][4][5]. To design such systems, the key is selection of feedback gain or coupling coefficient. The design of feedback coefficient for the given system has been discussed in previous contributions, whereas the universal methods for the common chaotic systems are little. In addition, most studies are under the ideal circumstance, i.e., parameters of two systems being exactly identical, while in practice, it is hard to get the same parameters of two systems [6][7]. Besides, simulations of current works usually focus on the simple chaotic system. Hyperchaos, however, is able to make communication more practical and to improve the security as well.

In this paper, we propose a method on parameter design for chaotic synchronization with linear feedback control according to the Taylor expansion and Lyapunov stability theory, and give a parameter adaptive formula. Simulations on such a hyper-chaotic system show that the method can well achieve synchronization in uncertain systems.

2. Adaptive Unidirectional Feedback Synchronization Method and Its Stable Condition

Consider a chaotic system

$$\dot{x} = Ax + f(x) + g(x)\theta \qquad (1)$$

where $x \in R^n$ is the state vector, $A \in R^{n \times n}$ is a constant matrix, $f(x)$ is continuous nonlinear function and $g(x)\theta$ is system structure parameters.

Suppose

$$\begin{aligned} f(x) - f(x_r) &= M_f(x - x_r) \\ g(x)\theta - g(x_r)\theta &= M_g(x - x_r) \end{aligned} \qquad (2)$$

where M_f is the bounded matrix determined by $x - x_r$, and M_g is the bounded matrix determined by state vectors x, x_r. Many chaotic systems can meet this condition, e.g. Lur'e nonlinear systems and Lipschitz nonlinear systems.

The equation of responding system is given by

$$\dot{x}_r = Ax_r + f(x_r) + g(x_r)\theta_r + K(x - x_r) \qquad (3)$$

where $K \in R^{n \times n}$ is the control matrix. Meanwhile, define the system state error being $e(t) = x - x_r$.

Subtracting Eq.(1) from Eq.(3) yieds the system state error,

$$\begin{aligned} \dot{e} &= Ae + f(x_r) - f(x) + g(x_r)\theta_r - g(x)\theta - Ke \\ &= (A + M_f + M_g - K)e \end{aligned} \qquad (4)$$

Theorem 1. If there exist two positive constant matrixes *P* and *Q* and a positive constant γ, which satisfy

$$(A-K)^T P + P(A-K) + 2PM_f + 2PM_g = -Q \tag{5}$$

and

$$2[g(x_r)\theta_r - g(x_r)\theta]^T Pe - \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r = 0, \tag{6}$$

then the error system (4) is globally asymptotically stable (i.e. the two chaotic systems reach synchronization).

[*Proof*] Construct a Lyapunov function as

$$V = e^T Pe + \frac{1}{\gamma}(\theta-\theta_r)^T(\theta-\theta_r) \tag{7}$$

where $P \in R^{n\times n}$ are positive constant matrix and γ is a positive constant, respectively.

Calculate its derivative, we have

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} - \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r$$
$$= [Ae + f(x_r) - f(x) + g(x_r)\theta_r - g(x)\theta - Ke]^T Pe$$
$$+ e^T P[Ae + f(x_r) - f(x) + g(x_r)\theta_r$$
$$- g(x)\theta - Ke] - \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r$$

$$= e^T[(A-K)^T P + P(A-K)]e$$
$$+ 2e^T P[f(x_r) - f(x)]$$
$$+ 2[g(x_r)\theta_r - g(x)\theta]^T Pe - \frac{2}{\gamma}(\theta-\theta_r)\dot{\theta}_r$$

$$= e^T[(A-K)^T P + P(A-K)]e + 2e^T PM_f e$$
$$+ 2[g(x_r)\theta_r - g(x_r)\theta + g(x_r)\theta - g(x)\theta]^T Pe$$
$$- \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r$$

$$= e^T[(A-K)^T P + P(A-K)]e + 2e^T PM_f e$$
$$+ 2e^T P[g(x_r)\theta - g(x)\theta] + 2[g(x_r)\theta_r$$
$$- g(x_r)\theta]^T Pe - \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r$$

$$= e^T[(A-K)^T P + P(A-K)]e + 2e^T PM_f e$$
$$+ 2e^T PM_g e + 2[g(x_r)\theta_r - g(x_r)\theta]^T Pe$$
$$- \frac{2}{\gamma}(\theta-\theta_r)^T \dot{\theta}_r$$
$$= -e^T Qe \tag{8}$$

In terms of Lyapunov stability theory, this makes Eq.(4) globally and asymptotically stable.

3. Simulation

This section investigates a typical example to test a hyperchaotic Rössler system formulated in this paper. Simulations are carried out with the same and variant parameters, respectively. The hyperchaotic Rössler system is described as:

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay + w \\ \dot{z} &= xz + d \\ w &= -cz + bw \end{aligned} \tag{9}$$

where x, y, z, w are state variables and a, b, c, d are system parameters. There is only one nonlinear term included in the system. In fact, when $a = 0.25$, $b = 0.05$, $c = 0.5$, $d = 3$, the system produces high-dimensional chaotic phenomenon.

3.1 With Certain Parameters

The hyperchaotic Rössler system in Eq.(9) may be expressed as:

$$\dot{x} = Ax + f(x) \tag{10}$$

The subsystem is defined as:

$$\dot{x}_r = Ax_r + f(x_r) + K(x - x_r) \tag{11}$$

where $A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0.25 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.05 \end{bmatrix}$,

$f(x) = \begin{bmatrix} 0 \\ 0 \\ xz + d \\ 0 \end{bmatrix}$, $f(x_r) = \begin{bmatrix} 0 \\ 0 \\ x_r z_r + d \\ 0 \end{bmatrix}$,

$$K = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$$

A and $f(x)$ correspond to the linear and nonlinear part of system and K represents the linear feedback parameter.

Furthermore, from Eq.(2), we get

$$f(x)-f(x_r)=\begin{bmatrix}0\\0\\xz+3\\0\end{bmatrix}-\begin{bmatrix}0\\0\\x_r z_r+3\\0\end{bmatrix}=M_f\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix} \quad (12)$$

where,

$$M_f=\begin{bmatrix}0&0&0&0\\0&0&0&0\\z_r&0&x&0\\0&0&0&0\end{bmatrix} \quad (13)$$

Substitute A, M_f and K into Eq.(4) and the error state equation of system is determined by

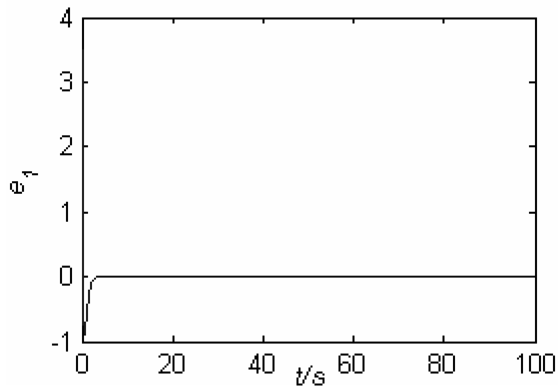
$$\dot{e}=(A+M_f-K)e$$
$$=\begin{bmatrix}-k_1&-1&-1&0\\1&0.25-k_2&0&1\\z_r&0&x-k_3&0\\0&0&-0.5&0.05-k_4\end{bmatrix}e\,. \quad (14)$$

Since chaotic attractor is bounded, from Fig.1(a)-(d), ranges of state variables of the hyperchaotic Rössler system are $-75.54<x<11.05$, $-47.73<y<34.58$, $0.04<z<113.06$ and $9.05<w<40.05$ respectively.

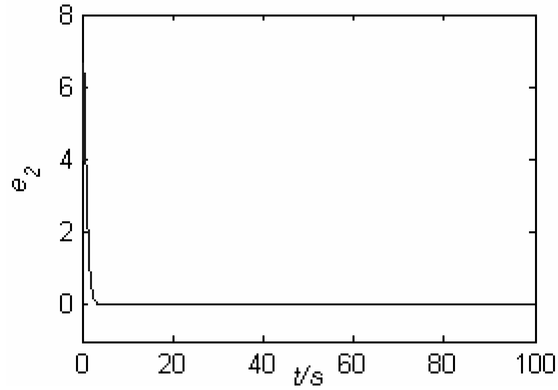
From Eq.(5), we obtain

$$K=\begin{bmatrix}6.35&0&0&0\\0&2&0&0\\0&0&5.3&0\\0&0&0&2.78\end{bmatrix}. \quad (15)$$

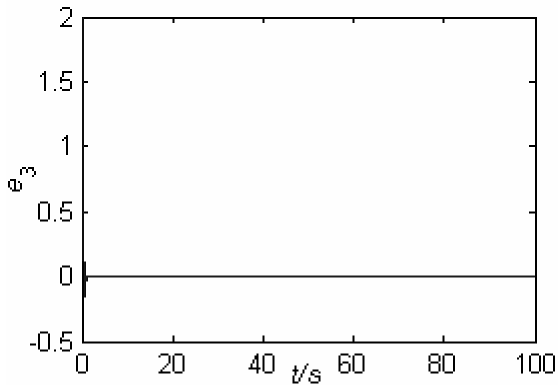
Using four-order Runge–Kutta integral, the initial conditions of driving and responding system are $[1.5,1,2,25]^T$ and $[-2,-3,0,-2]^T$, respectively. The simulation results are given in Fig.1, which illustrates two systems running the fast synchronization after a short time adjusting.



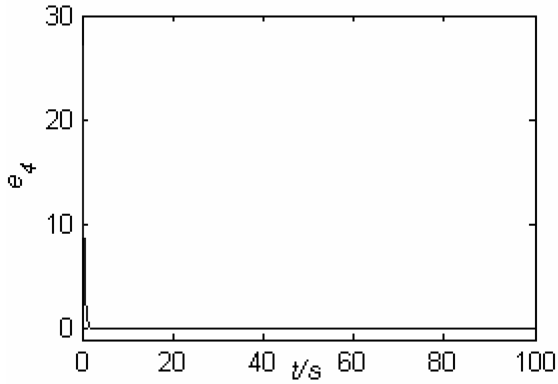
(a) $e_1=x-x_r$



(b) $e_2=y-y_r$



(c) $e_1=z-z_r$



(d) $e_4=w-w_r$

Fig.1 synchronization state errors

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3.2 With Unknown Parameters

The implementation of system synchronization with the given parameters has been discussed. Practically, parameters of chaotic systems cannot be obtained in prior or might be different as circumstance changes, thus the above-mentioned approach is not feasible. It is necessary to analyze the synchronization problem under uncertain parameters.

Firstly, in case of unknown single parameter, assume that the subsystem is

$$\begin{aligned} \dot{x}_r &= -y_r - z_r + u_1 \\ \dot{y}_r &= x_r + ay_r + w_r + u_2 \\ \dot{z}_r &= x_r z_r + d + u_3 \\ w_r &= -cz_r + \hat{b}w_r + u_4 \end{aligned} \tag{16}$$

where $\hat{b}$ is the unknown parameter.

Combining Eq.(9) and (16) gets

$$A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0.25 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ 0 \\ xz + d \\ 0 \end{bmatrix},$$

$$f(x_r) = \begin{bmatrix} 0 \\ 0 \\ x_r z_r + d \\ 0 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix},$$

$$g(x_r) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & w_r & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ \hat{b} \\ c \\ d \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$$

From Eq.(2), we have

$$f(x) - f(x_r) = \begin{bmatrix} 0 \\ 0 \\ xz + d \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ x_r z_r + d \\ 0 \end{bmatrix} = M_f e \tag{17}$$

$$g(x) - g(x_r) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & w_r & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ \hat{b} \\ c \\ d \end{bmatrix} = M_g e \tag{18}$$

Define

$$M_f = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ z_r & 0 & x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_g = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}.$$

From Eq.(4), we get the system error state equation

$$\begin{aligned} \dot{e} &= (A + M_f + M_g - K)e \\ &= \begin{bmatrix} -k_1 & -1 & -1 & 0 \\ 1 & 0.25 - k_2 & 0 & 1 \\ z_r & 0 & x - k_3 & 0 \\ 0 & 0 & -0.5 & b - k_4 \end{bmatrix} e \end{aligned} \tag{19}$$

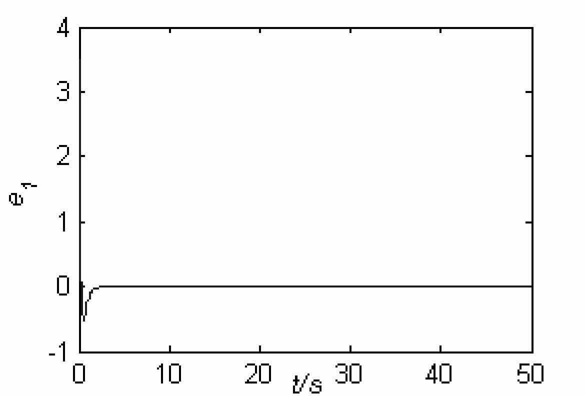
Combine Eq.(5) and (6), and we have

$$K = \begin{bmatrix} 7.35 & 0 & 0 & 0 \\ 0 & 2.06 & 0 & 0 \\ 0 & 0 & 0.92 & 0 \\ 0 & 0 & 0 & 2.78 \end{bmatrix} \tag{20}$$

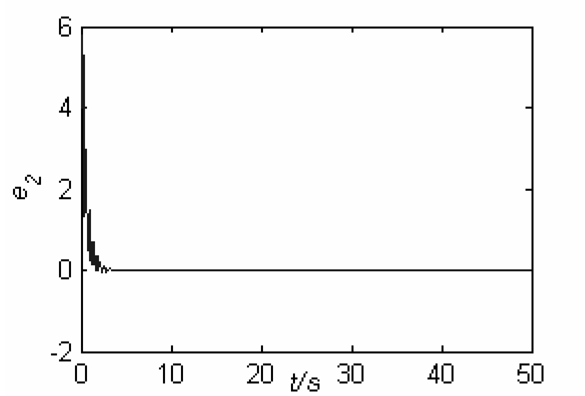
and

$$\dot{\hat{b}} = 10(w - w_r)\,\mathrm{sgn}(w_r). \tag{21}$$

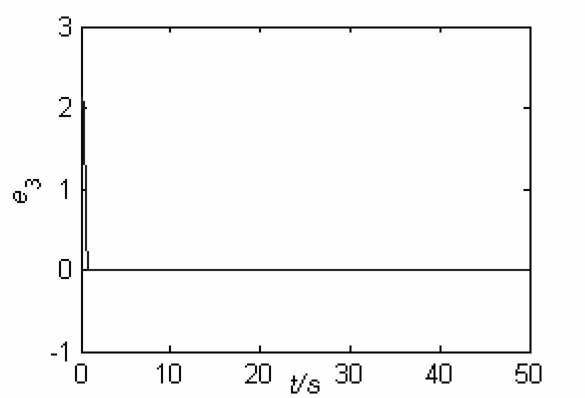
Applying four-order Runge–Kutta integral, set the initial conditions of driving and responding system be $[1.5,1,2,25]^T$ and $[-2,-3,0,-2]^T$, respectively. The simulation results are shown in Fig. 2, where (a)-(d) give the system state error, (e) is the identification process of parameter $\hat{b}$, and (f) is the control effectiveness.



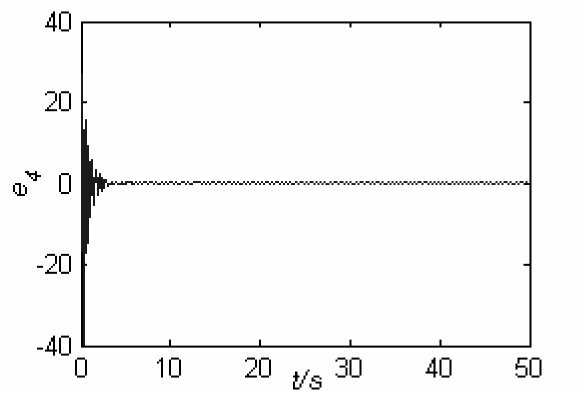
$(a) e_1 = x - x_r$



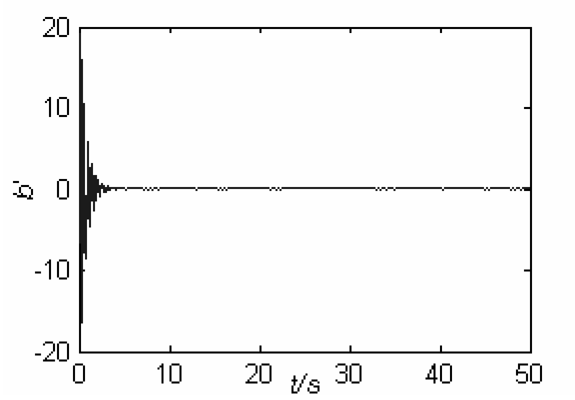
$(b) e_2 = y - y_r$



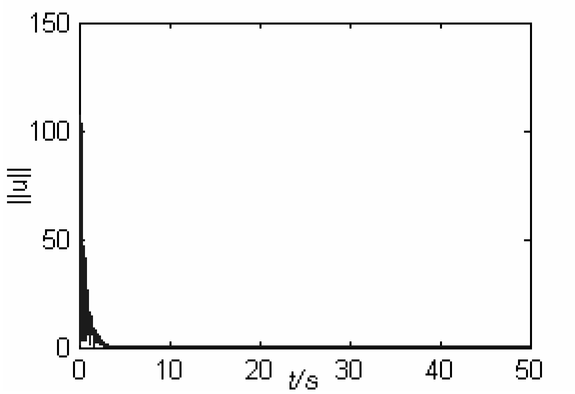
$(c) e_1 = z - z_r$



$(d) e_4 = w - w_r$



(e) parameter $\hat{b}$



(f) control parameter $\|u\|$

Fig.2. synchronization state error, parameter identification and control parameter

4. Conclusion

This paper investigated an uncertainty-driven synchronization approach based on feedback control. With the Taylor expansion and nonlinear stability theory, we proposed a universal method to design good feedback parameters. Moreover, we developed a parameter adaptive law in case of uncertain system parameters. Finally, numerical simulations were provided to demonstrate the effectiveness of the new method.

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