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A Identification Algorithm of Freeway Steady-state Speed-Density Balance Relational Expression Based on the Power Series Expansion and the Least Squares Method

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Abstract: - This paper proposes a novel identification algorithm to the freeway steady-state speed-density balance relational expression of the Markos Papageorgiou's freeway traffic flow model. Two stages are involved. First, use the power series principle to transform this relational expression from nonlinear model into linear model. Second,use the least squares method to identify the linear model and further get all unknown parameters of the nonlinear model. So,the identification to the freeway steady-state speed-density balance relational expression can be achieved. Both theory analysis and simulation research show that ,comparing with conventional nonlinear identification algorithm,this algorithm highly improves operating speed of model identification and can achieve arbitrary identification accuracy.

Key-Words: - Traffic flow model,Freeway steady state speed-density balance relational expression, Power series principle, Least squares method

1 Introduction

The traffic flow model plays an important role in the design,analysis,simulation and operation of the traffic control system. According to different classification criteria, traffic flow model can be classified as microcosmic model and macroscopical model, static model and dynamic model, deterministic model and random model. Among these models, macroscopical, dynamic, deterministic model is suitable for traffic control and simulation. Many scholars research the identification problem of the traffic flow model deeply and have gotten some import outcomes.

This paper researches the identification problem to the freeway steady-state speed-density balance relational expression of the Markos Papageorgiou's freeway traffic flow model[1]. Reference [2,3] solved this kind of problem with a direct search method(complex method). However, this algorithm need mass iterative computation and has not very ideal identification effect. Reference [4] adopted the neural networks modeling method.But this kind method usually has the defect of poor generalization capability.On the base of analysing the freeway steady-state speed-density balance relational expression detailedly, this paper proposes a novel identification algorithm which transforms nonlinear model into linear model and uses the least squares method to identify the linear model and further gets all unknown parameters of the nonlinear model. Comparing with conventional nonlinear identification algorithm,this algorithm highly improves operating speed of model identification and can achieve arbitrary identification accuracy. This algorithm solves the problem of identifying the relational expression in engineering and has great promotion value.

2 Problem Description

Divide a freeway into N sections and each section's length Δ_i is several hundreds meters. In each section, the traffic state can be considered uniform approximatively. Namely, the traffic volume, the vehicle flow speed and the vehicle flow density in a section are constant.In each section, the lane number is constant and the maximum number of entrance or exit is one.The traffic parameters, such as traffic volume, vehicle flow speed and vehicle flow density, can all be measured. Detection period T is about dozens of seconds.

The traffic flow model[1] which was proposed by MARKOS PAPAGEORGIOU is as following:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\Delta_i}[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)] \quad (1)$$

$$q_i(k) = \alpha\rho_i(k)v_i(k) + (1-\alpha)[\rho_{i+1}(k)v_{i+1}(k) - r_{i+1}(k)] - s_i(k) \quad (2)$$

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}\{v[\rho_i(k)] - v_i(k)\} + \frac{T\xi}{\Delta_i} v_i(k)[v_{i-1}(k) - v_i(k)] - \frac{\gamma T}{\tau \Delta_i} \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k+1) + \lambda} \tag{3}$$

where

$$v(\rho) = v_f \exp[-(1/b)(\rho/\rho_{cr})^b] \tag{4}$$

T sampling period;
$v_i(k)$ the vehicle flow speed of i th section at kT time;
$\rho_i(k)$ the vehicle flow density of i th section at kT time;
$q_i(k)$ the traffic volume from i th section to $(i+1)$ th section at kT time;
Δ_i the length of i th section;
$r_i(k)$ the on-ramp traffic volume for i th section;
$s_i(k)$ the off-ramp traffic volume for i th section.

Model parameters are $v_f, \rho_{cr}, b, \tau, \gamma, \xi, \lambda, \alpha$.

Formula (4) is the freeway steady-state speed-density balance relational expression. This paper's task is how to identify the relational expression. The parameters which should be identified are v_f , b , ρ_{cr} . The input of the identification algorithm are ρ , $v(\rho)$ which can be measured by detector.

Two stages are involved in the identification algorithm.

(1)Use power series principle to transform the freeway steady-state speed-density balance relational expression from nonlinear model into linear model.

(2)Use the least squares method to identify the linear model and further get all unknown parameters of the nonlinear model. So,the identification to the freeway steady-state speed-density balance relational expression can be achieved.

3 Transform Nonlinear Model into Linear Model

For convenience,we substitute v for $v(\rho)$.Then the formula (4) can be expressed as following:

$$v = v_f \exp[-(1/b)(\rho/\rho_{cr})^b] \tag{5}$$

Take the natural logarithm of both sides of the equation (5) and rearrange to get equation (6) .

$$\ln(v_f) - \ln(v) = 1/b(\rho/\rho_{cr})^b \tag{6}$$

Take the natural logarithm of both sides of the equation (6) to get equation (7) .

$$\ln[\ln(v_f) - \ln(v)] = \ln(1/b) + b\ln(\rho) - b\ln(\rho_{cr}) \tag{7}$$

The left side of the equation (7)=

$$\ln\{\ln(vf)[1 - \frac{\ln(v)}{\ln(v_f)}]\} = \ln[\ln(v_f)] + \ln[1 - \frac{\ln(v)}{\ln(v_f)}] \tag{8}$$

Let $a = \frac{\ln(v)}{\ln(v_f)}$

In the process of identification model ,we let

$$\frac{1}{v_f} < v < v_f ,$$

therefore, $-1 < \frac{\ln(v)}{\ln(v_f)} < 1$.

Namely, $-1 < a < 1$.

$\ln(1-a)$ can be expand as the sum of N terms of the power series.

$$\ln(1-a) = -a - \frac{a^2}{2} - \cdots - \frac{a^n}{n} \qquad (n \to \infty) \tag{9}$$

Usually,when n is a big enough positive integer,the first N terms of the power series expansion can substitute for $\ln(1-a)$ approximately. More n , more accuracy.So,we can achieve arbitrary identification accuracy with big enough n .

Substitution of formula (9) into formula (8) yields

$$\text{formula (8)} = \ln[\ln(v_f)] - a - \frac{a^2}{2} - \cdots - \frac{a^n}{n} \tag{10}$$

According to equation (7),yield

$$\ln[\ln(v_f)] - a - \frac{a^2}{2} - \frac{a^3}{3} - \cdots - \frac{a^n}{n} = \ln(1/b) + b\ln(\rho) - b\ln(\rho_{cr}) \tag{11}$$

Substitution of $a = \frac{\ln(v)}{\ln(v_f)}$ into formula (11) yields

$$\ln[\ln(v_f)] - [\frac{1}{\ln(v_f)}]\ln(v) - \frac{1}{2}[\frac{1}{\ln(v_f)}]^2[\ln(v)]^2 - \cdots - \frac{1}{n}[\frac{1}{\ln(v_f)}]^n[\ln(v)]^n = \ln(1/b) + b\ln(\rho) - b\ln(\rho_{cr}) \tag{12}$$

Rearrangement yields

$$\frac{\ln[\ln(v_f)]}{b} + \frac{\ln(b)}{b} + \ln(\rho_{cr}) + [\frac{1}{b\ln(v_f)}][-\ln(v)] + \{\frac{1}{b}[\frac{1}{\ln(v_f)}]^2\}\{-\frac{[\ln(v)]^2}{2}\} + \cdots + \{\frac{1}{b}[\frac{1}{\ln(v_f)}]^n\}\{-\frac{[\ln(v)]^n}{n}\} = \ln(\rho) \tag{13}$$

Let $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_{n+1}]^T$

$$\theta_1 = \frac{\ln[\ln(v_f)]}{b} + \frac{\ln(b)}{b} + \ln(\rho_{cr}), \theta_2 = [\frac{1}{b\ln(v_f)}], \theta_3 = \{\frac{1}{b}[\frac{1}{\ln(v_f)}]^2\}, \cdots, \theta_{n+1} = \{\frac{1}{b}[\frac{1}{\ln(v_f)}]^n\} \tag{14}$$

Let $d = \ln(v)$,Then, $d(k) = \ln[v(k)]$, $(k = 1,2,\cdots,M)$, M is the sample number.

M

$$\varphi^T(k) = [1, -d(k), -\frac{[d(k)]^2}{2}, -\frac{[d(k)]^3}{3}, \cdots, -\frac{[d(k)]^n}{n}] \quad (15)$$

Let

$$\Phi = \begin{bmatrix} \varphi^T(1) \\ \varphi^T(2) \\ \vdots \\ \varphi^T(M) \end{bmatrix} \quad (16)$$

$$Y = \begin{bmatrix} Y(1) \\ Y(2) \\ \vdots \\ Y(M) \end{bmatrix} = \begin{bmatrix} \ln[\rho(1)] \\ \ln[\rho(2)] \\ \vdots \\ \ln[\rho(M)] \end{bmatrix} \quad (17)$$

The equation (13) can be expressed as

$$Y = \Phi\theta \quad (18)$$

Observing the formula (18), θ is constant ,then the relationship between the Y and Φ is linear. Therefore,we transform the nonlinear model(formula (4)) into linear model(formula (18)).We can get the unknown parameters in the linear model by the least squares method.

4 The Least Squares Method Identify Parameters

Define performance index function

$$J = \frac{1}{2}\sum_{i=1}^{M}[e_i]^2 \quad (i = 1,2,\cdots,M) \quad (19)$$

where

$$e_i = y(i) - x(i) \quad (20)$$

$y(i)$ is i th output sample practical value

$x(i)$ is i th output's desired value

The θ should let $J = \min$ .Namely,

$$\frac{\partial J}{\partial \theta} = 0 \quad (21)$$

We can get the recurrence least squares algorithm as following:

$$\theta(k+1) = \theta(k) + \theta(k+1)[Y(k+1) - \varphi^T(k+1)\theta(k)] \quad (22)$$

$$D(k+1) = \frac{P(k)\varphi(k+1)}{1 + \varphi^T(k+1)P(k)\varphi(k+1)} \quad (23)$$

$$P(k+1) = [I - D(k+1)\varphi^T(k+1)]P(k) \quad (24)$$

$$(k = 0,1,2,\cdots M)$$

Let $\theta(0) = 0$, $P(0) = \alpha I$,(α is a big enough positive number, I is a unit matrix), Beginning recurrence computation from the first set dada,we can get the θ .

On the base of getting θ ,We can get the unknown parameters v_f , b , ρ_{cr} in formula (4). According to the definition of $\theta_1, \theta_2, \theta_3$ in formula (14),we can get the relationship as following :

$$v_f = \exp(\frac{\theta_2}{\theta_3}) \quad (25)$$

$$b = \frac{\theta_3}{(\theta_2)^2} \quad (26)$$

After getting v_f, b ,according to the definition of θ_1 ,we can further get ρ_{cr} .

$$\rho_{cr} = \exp(\theta_1 - \frac{\ln[\ln(v_f)]}{b} - \frac{\ln(b)}{b}) \quad (27)$$

In the fomula(25),(26),(27), $\exp(x) = e^x$.

Now, we identify the freeway steady-state speed-density balance relational expression successfully. Because the transition from the nonlinear model to linear model is on the base of the power series principle, we can achieve arbitrary identification accuracy if we select big enough terms of the power series expansion. Furthermore, the least squares method belongs to linear optimization algorithm and must have higher operating speed than conventional nonlinear identification algorithm which is on the base of nonlinear optimization algorithm.

5 Simulation Research

The task of the identification is to identify the model parameters v_f, ρ_{cr}, b with the sample data. In the process of the simulation,we firstly suppose the real values of model parameters are known and use freeway steady-state speed-density balance relational expression ((formula (4)) to generate sample data.Then,we use the sample data to identify model parameters and compare them with the real parameter. So,we can verify the validity of the algorithm which is proposed in this paper.

(1)Suppose the real values of the model parameters are $v_f = 98km/h$, $\rho_{cr} = 32veh/km$, $b = 3$.

Let input $\rho = 10:0.1:60$,use formula (4) to generate $v(\rho)$, the sample number is 501.

Identify model parameters with this paper's algorithm and these sample data on the condition of 10 terms of the power series expansion.

We can identify model parameters as following :

$v_f = 105.3km/h, \rho_{cr} = 29.6veh/km, b = 2.77$

Comparing with the real value of model parameters, the relative errors of the 3 parameters are 7.86% , -7.5% , -7.7% .

(2)Suppose the real values of the model parameters are $v_f = 120km/h, \rho_{cr} = 50veh/km, b = 2$.

Let input $\rho = 10:0.1:70$,use formula (4) to generate $v(\rho)$, the sample number is 601.

Identify model parameters with this paper's algorithm and these sample data on the condition of 15 terms of the power series expansion.

We can identify model parameters as following $v_f = 126.3km/h, \rho_{cr} = 47.1veh/km, b = 1.87$.

Comparing with the real value of model parameters, the relative errors of the 3 parameters are 5.3%, -5.8%, -6.5%.

From above two simulation examples,we can get two conclusions as following:

(1)More terms of the power series expansion can achieve higher identification accuracy.

(2)The relative errors can be controlled in the range of $\pm 10\%$ and have obvious engineering meaning.

6 Conclusion

This paper proposes a novel identification algorithm to the freeway steady-state speed-density balance relational expression of the Markos Papageorgiou's freeway traffic flow model. This algorithm transforms this relational expression from nonlinear model into linear model and uses the least squares method to identify the linear model and further gets all unknown parameters of the nonlinear model. Comparing with conventional nonlinear identification algorithm,this algorithm highly improves operating speed of model identification and can achieve arbitrary identification accuracy. This algorithm solves the problem of identifying the relational expression in engineering and has great promotion value.

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References:

[1]PAPAGEORGIOU M, BLOSSEVILLE J M, HAJSALEM H, Macroscopic modeling of traffic flow on the boulevard peripheriquein Paris[J], *Transpn.Res.* ,1989,22B(1):29-47.

[2]Papageorgiou M. Applications of Automatic Control Concepts to Traffic Flow Modeling and Control [M].,*Lecture Notes in Control and Information Sciences*, Spring-Verlag,1983.

[3]Cremer M, Papageorgiou M, Parameter identification for a traffic flow model[J]. *Automatica*, 1981,17(6):837-843.

[4]LUO Zanwen,WU Zhijian,Han Zengjin, Identification approach to macro model of traffic flow in freeway using parameter classification , *Qinghua Daxue Xuebao/Journal of Tsinghua University*, v 41, n 7, July, 2001, p 117-120(in Chinese).