Accurate Doppler Prediction Scheme for Satellite Orbits

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Abstract: - In satellite communications particular in low earth and elliptical orbits, Doppler frequency shift is one of the most important problems in communication channels. This paper scrutiny Doppler frequency shift in various satellite orbits. For this purpose an orbit generator is used for accurate simulation of satellite orbits. The nonspherical mass distribution turbulence considered in the estimator. Simulation results of Doppler frequency shift for an elliptical LEO[1] satellite in L-band also presented.

Key-Words: - Doppler estimation, orbit generator, elliptical satellite orbit, LEO

1 Introduction

In communication system that receiver and transmitter are not fixed and have significant relative velocity, Doppler frequency shift is high. For proper design of various parts of transceivers like input filters in receivers and PLL parts, need to accurately calculate carrier frequency shift and compensate.

With the presented simulator, receivers can calculate Doppler frequency in each time and also time interval of viewing satellite. Accurate Doppler information have influence in improvement of pashed lock loop performance, also ground segment with notice of satellite viewing only in moments that transmitting is possible, turn on the power and this reduce the power consumption.

There are many researches that focused on methodology to compensate for Doppler shifts but for accurate estimating of this parameter in [1] the author characterized Doppler time curves in the simple case of circular in the equatorial plane and Doppler observed by points on the equator. In [2] the author derived analytical expression of Doppler shift for circular LEO satellites, the presented scheme include any satellite orbit.

2 Orbit dynamics

Among many kinds of nongeostationary orbits, there are two typical orbits for the communication satellites; highly elliptical and low-altitude orbits. Since the nongeostationary satellites changes their relative positions to earth stations, the signal suffer from Doppler shift whose value and drift rate are very large compared with that in the system with geostationary satellites or the terrestrial systems. One consequence of the use of elliptical orbits is the existence of Doppler shifts in all satellite/ ground link.

To calculate the accurate Doppler frequency shift, we need the relative velocity between satellite and ground terminal. The relationship between the relative velocity and Doppler shift is given by:

$f_d = \dfrac{f_0 v_t}{c}$ Where f_0 is the carrier frequency and

$v_t = \dfrac{d\left(P_s - P_e\right)}{dt}$ in spherical coordinates

Where P_s and P_e are the position of satellite and earth transceiver respectively.

For this purpose an orbit generator present here that calculates this relative velocity. This estimator uses Kepler’s equation to determine position and velocity of satellite in the orbit. To determine an earth orbit we need six parameters, the parameters are defined at some reference time or epoch (t_0): *semimajor axis* (a) and *eccentricity* (e) that define the size and shape of the orbit, the *inclination* (i) and *right ascension of the ascending node* (Ω) that define the orbit plane, the rotation of the orbit within the plane that is defined by the *argument of perigee* (ω), finally the *mean anomaly* (M) specifies the position of the satellite in its orbit at the epoch time.

[1]) Low Earth Orbit

In figure 1 if the plane of paper is the reference plane and the dashed part of orbit is below the paper, then the nodes are as illustrated.

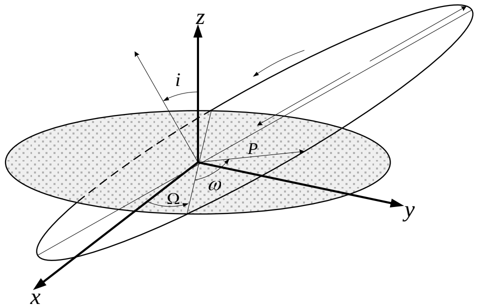

Figure 1- Keplerian Orbital elements in ECI coordinate

To determine the relative velocity we need to calculate velocity of satellite and ground station in a reference coordinates. For this purpose the satellite velocity calculates in ECI coordinates and then converts to ECF, also velocity of ground station determines in ECF coordinates.

Different perturbing forces like Nonhomogeneity and oblateness of the earth affect the orbit parameters as follow:

Rate range of orbit parameters influenced by perturbing forces represent by Gauss planetary equations along the axes of a moving Cartesian frame defined in the following way: R along the radius vector r; S in the local plane of osculating orbit, perpendicular to R, and in direction of satellite motion; W perpendicular to both R and S, in the direction of the momentum vector $R \times S$. any perturbing force can the be expressed as:

$$\gamma_p = R\mathbf{R} + S\mathbf{S} + W\mathbf{W} \quad (4)$$

$$K = -1.5 * \mu * J_2 * R_E^2 / r^4 \quad (5)$$

$$R = K\left(1 - 3\sin^2(\omega + \nu)\sin^2(i)\right) \quad (6)$$

$$S = K\sin\left(2(\omega + \nu)\sin^2(i)\right) \quad (7)$$

$$W = K\sin(\omega + \nu)\sin(2i) \quad (8)$$

The resulting Gauss equations are:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(eR\sin\upsilon + (1 + e\cos\upsilon)S\right) \quad (9)$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left(R\sin\upsilon + (\cos E + \cos\upsilon)S\right) \quad (10)$$

$$\frac{di}{dt} = \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\cos(\upsilon + \omega)W \quad (11)$$

$$\frac{d\Omega}{dt} = \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\frac{\sin(\upsilon + \omega)}{\sin i}W \quad (12)$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\left(-R\cos\upsilon + \left(1 + \frac{1}{1 + e\cos\upsilon}\right)S\sin\upsilon - \frac{d\Omega}{dt}\cos i\right) \quad (13)$$

$$\frac{dM}{dt} = n + \frac{1-e^2}{nae}\left(R\left(\frac{-2e}{1 + e\cos\upsilon} + \cos\upsilon\right) - S\left(1 + \frac{1}{1 + e\cos\upsilon}\right)\sin\upsilon\right) \quad (14)$$

Equations 9-14 show if the perturbing force vector is known then the differential changes of all six orbit parameters can be calculated analytically.

3 orbit generator

satellite position and velocity can be derived from orbit parameters.

The mean motion computes from semi major axis:

$$n = \sqrt{\frac{\mu}{a^3}} \quad , \quad \mu = 3.986005\text{e}14 \quad (15)$$

Mean anomaly in term of mean motion is:

$$M = n(t - T) \quad (16)$$

Eccentric anomaly can be computed from mean anomaly and eccentricity.

$$E = M + e\sin M + \frac{1}{2}e^2\sin 2M + \frac{1}{8}e^3\left(3\sin 3M - \sin M\right) + \ldots \quad (17)$$

With knowing the eccentric anomaly, true anomaly calculates from the equation below:

$$\tan\frac{\nu}{2} = \left[\frac{(1+e)}{(1-e)}\right]^{1/2}\tan\frac{E}{2} \quad (18)$$

distance of satellite from the center of earth is:

$$r = \frac{a\left(1 - e^2\right)}{\left(1 + e\cos(\upsilon)\right)} \quad (19)$$

With the droved parameters, satellite position in ECI coordinates represents as bellow:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = r\begin{bmatrix} \cos(\omega + \upsilon)\cos(\Omega) - \sin(\omega + \upsilon)\sin(\Omega)\cos(i) \\ \cos(\omega + \upsilon)\sin(\Omega) + \sin(\omega + \upsilon)\cos(\Omega)\cos(i) \\ \sin(\omega + \upsilon)\sin(i) \end{bmatrix} \quad (20)$$

Satellite ECI velocity drives from differentiation of position respect to time.

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \frac{na}{r}\begin{bmatrix} bl_2\cos E - al_1\sin E \\ bm_2\cos E - am_1\sin E \\ bn_2\cos E - an_1\sin E \end{bmatrix} \quad (21)$$

Where:

$$b = a\left(1 - e^2\right)^{1/2} \quad (22)$$

$$l_1 = \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i \quad (23)$$

$$m_1 = \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i \quad (24)$$

$$n_1 = \sin\omega\sin i \quad (25)$$

$$l_2 = -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i \quad (26)$$

$$m_2 = -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i \quad (27)$$

$$n_2 = \cos\omega \sin i \tag{28}$$

4 Doppler equations

To calculate the relative velocity we need to have velocity vector of satellite and ground station in the same coordinate system. position of ground station in ECF coordinates can be derive easily by knowing the longitude and latitude of station and hence by transforming the satellite coordinate from ECI to ECF we reach the goal.

The transformation of an ECI position vector r_{ECI} to an ECF position vector r_{ECF} is given by the following vector-matrix operation $r_{ECF} = [T]\, r_{ECI}$.

where the elements of the transformation matrix $[T]$ are given by

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{29}$$

where θ is the Greenwich sidereal time at the moment of interest. Greenwich sidereal time is given by the following expression:

$$\theta = \theta_{g0} + \omega_e t \tag{30}$$

where θ_{g0} is the Greenwich sidereal time at 0 hours UT, ω_e is the inertial rotation rate of the Earth, and t is the elapsed time since 0 hours UT.

The ECF velocity vector is determined by differentiating this expression:

$$V_{ECF} = [T]\dot{r}_{ECI} + [\dot{T}]r_{ECI} = [T]V_{ECI} + [\dot{T}]r_{ECI} \tag{31}$$

The elements of the $[\dot{T}]$ matrix are as follows:

$$[\dot{T}] = \begin{bmatrix} -\omega_e \sin\theta & \omega_e \cos\theta & 0 \\ -\omega_e \cos\theta & -\omega_e \sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{32}$$

if latitude and longitude of ground segment represent as φ, λ then the ground segment position in ECF is as bellow :

$$\begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix} = R_E \begin{bmatrix} \cos\lambda \cos\varphi \\ \sin\lambda \cos\varphi \\ \sin\varphi \end{bmatrix} \tag{33}$$

With knowing the position vector of satellite and earth terminal in ECF coordinates, the position vector of satellite with terminal reference can derived. With this vector and the satellite velocity vector, relative velocity of satellite respect to earth terminal in line of sight view can derive.

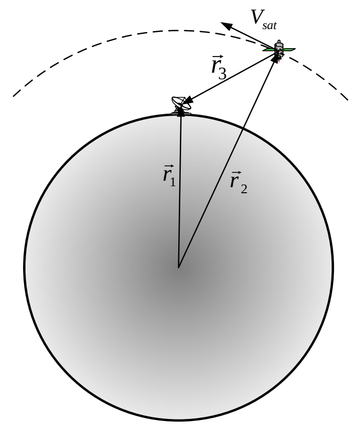

Figure 2- Position and velocity vectors of satellite and earth terminal

$\vec{r}_1$ is the earth terminal position vector and $\vec{r}_2$ is the satellite position vector. Hence $\vec{r}_3$ determine the relative position of satellite respect to earth terminal in ECF coordinates. Relative velocity between satellite and ground terminal is: $\vec{V}_{rel} = \vec{V}_{sat} \cdot \hat{\vec{r}}_3$ where $\hat{\vec{r}}_3 = \frac{\vec{r}_3}{|\vec{r}_3|}$ and $\vec{r}_3 = \vec{r}_1 - \vec{r}_2$

$$\vec{r}_1 = \begin{bmatrix} R_E \cos\lambda \cos\phi \\ R_E \sin\lambda \cos\phi \\ R_E \sin\phi \end{bmatrix}^T \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{34}$$

$$\vec{r}_2 = r \begin{bmatrix} \cos(\omega+\upsilon)\cos(\Omega+\theta) - \sin(\omega+\upsilon)\sin(\Omega+\theta)\cos i \\ \cos(\omega+\upsilon)\sin(\Omega+\theta) + \sin(\omega+\upsilon)\cos(\Omega+\theta)\cos i \\ \sin(\omega+\upsilon)\sin i \end{bmatrix}^T \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{35}$$

In the above equations $\vec{V}_{sat}$ is satellite velocity in ECF coordinates. With knowing the accurate relative velocity, Doppler frequency shift can be calculated.

An other problem that seams in the above equations is that the Doppler drives for the whole times but we want to compute this parameter in the visible time intervals. For this purpose with using the minimum earth elevation angle, the satellite visibility intervals can be determined.

5 Simulation results

Analytical results simulates in MATLAB simulink environment.

Ground terminal has 51.447651 degree north longitude and 35.774475 degree east latitude. Earth elevation angle is 0.

Orbit parameters are as below:

- mean anomaly = 0
- semi major axis = 7678137.085 m
- inclination = 105^o
- eccentricity = 0.1
- argument of perigee = 270^o
- RAAN = 155^o

Figure 3 display the ground terminal and orbit in one cycle.

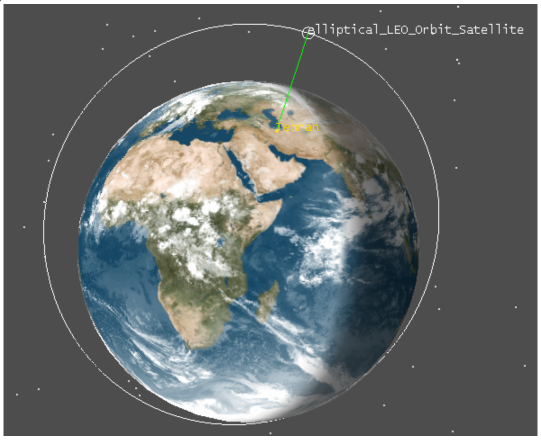

Figure 3- Satellite in the orbit with the determined parameters

For verify the simulation results, some MATLAB simulation results compared with STK results.
Figure 4 shows the ECI satellite position for 2 hours, the maximum difference of two simulation results is less than 2%.

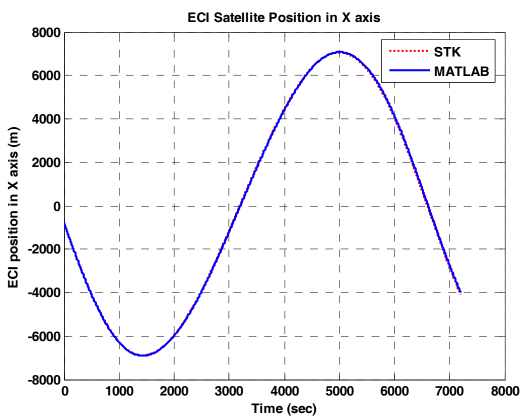

Figure 4- ECI satellite position for 2 hours

ECF satellite velocity in figure 4 shows that error of transformation to ECF is below 1%.

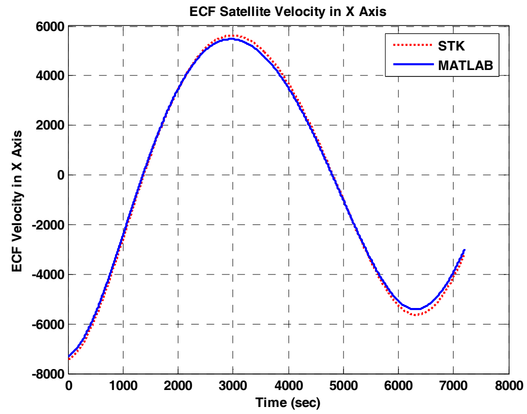

Figure 5- ECF satellite velocity

Doppler frequency in satellite visible durations is shown in figure 6. it shows that in a day in 9 duration intervals satellite can communicate with the ground terminal.

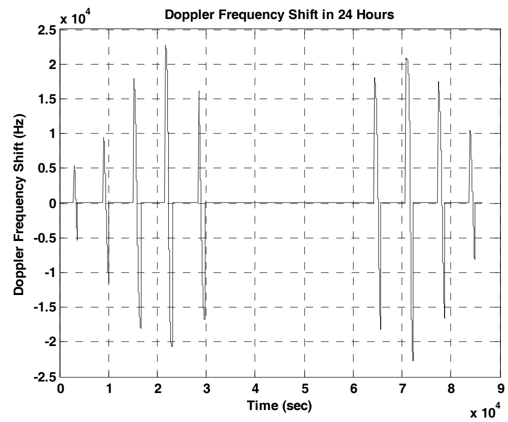

Figure 6- Doppler frequency shift in 24 hours

the forth pass of satellite zoomed in figure below, as you can see because of elliptical orbit, positive and negative Doppler shifts are not same.

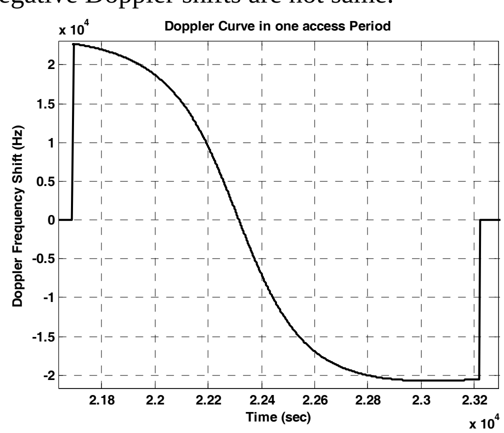

Figure 7- Doppler curve in one path

Comparison of STK and MATLAB results is shown in the figure 8, as it shows accuracy is above 99.5%.

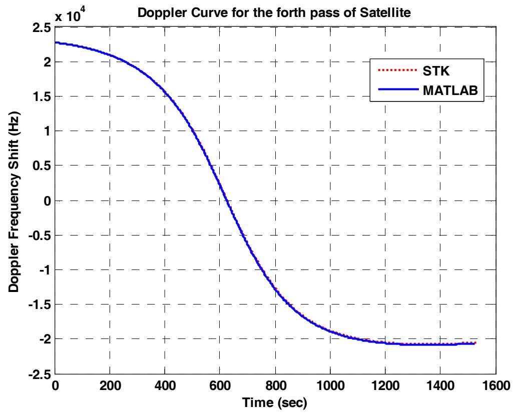

Figure 8- comparison of STK & MATLAB results for the forth pass

With changing the earth elevation angle, satellite viewing durations also changes, in the below figures Doppler for 5^o and 20^o presents. As it seems with increasing the elevation angle visibility intervals decrease.

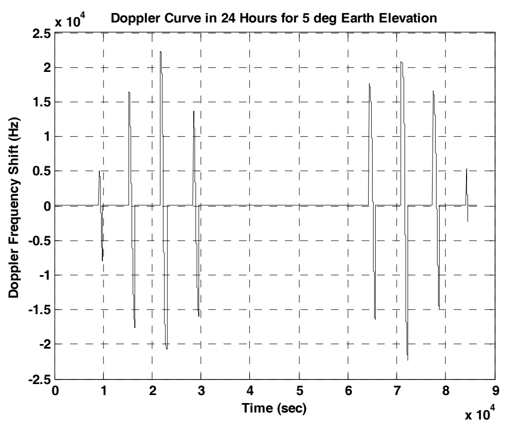

Figure 9- Doppler for 5 degree earth elevation in a day

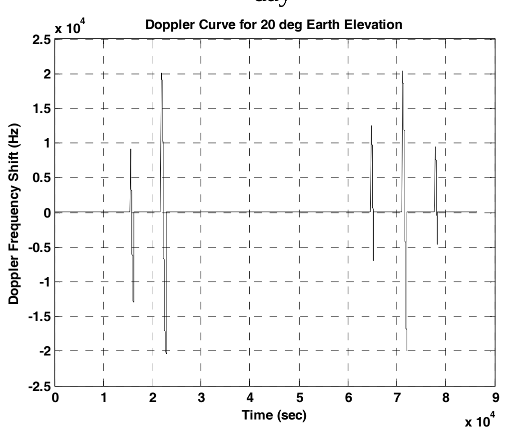

Figure 10- Doppler for 20 degree earth elevation in a day

Maximum value of Doppler shift in term of eccentricity shows in figure 11, with decreasing the eccentricity of this orbit, its circularity increase and satellite altitude in visibility durations decrease and hence Doppler frequency shift increases.

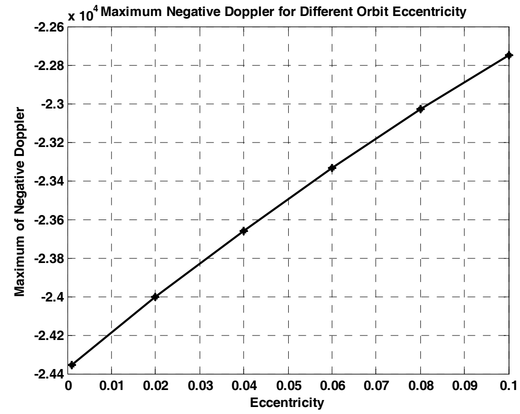

Figure 11- maximum negative Doppler in term of eccentricity

Maximum positive of this parameter also shown in figure 12

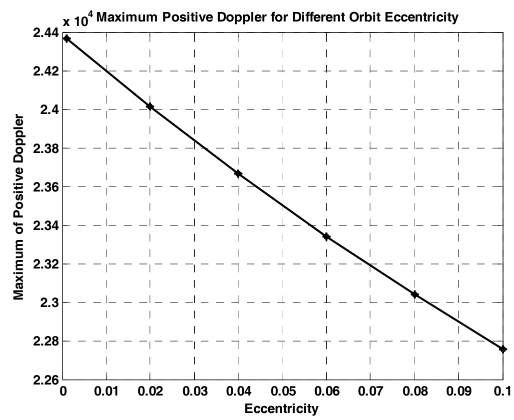

Figure 12- maximum positive Doppler frequency in term of eccentricity

An other important parameters is rate of Doppler shift, figure 13 shows the Doppler rate in the forth pass, as it shows with 1.2 GHz carrier frequency, Doppler rate is below 100 Hz/sec.

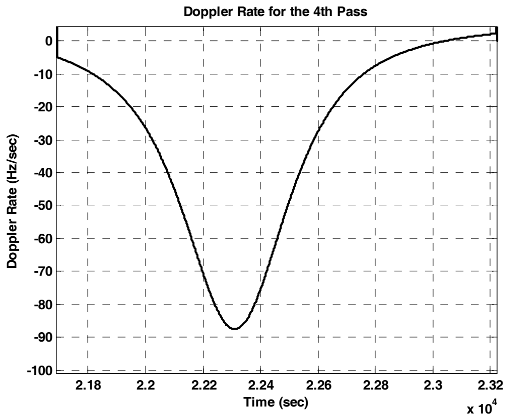

Figure 13- Doppler rate in forth passes

An other parameter that considered in the paper is the visibility durations. The sum of visibility durations in a day in term of various earth elevation angles presents in figure 14.

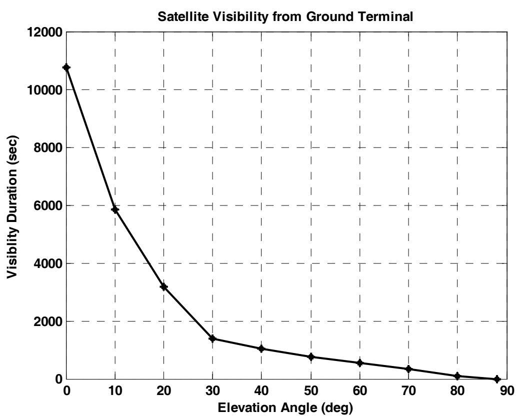

Figure 14- visibility durations in term of earth elevation angle

6 Conclusion

In this paper, we have proposed a Doppler prediction scheme with information of satellite orbital parameters and ground terminal position. This scheme can calculate Doppler shift for various kind of satellite orbits. Because the former Doppler prediction schemes considered only circular orbits, we can't compare the simulation results with the previous schemes and STK software used for verifying the results. As it seems from compared results, the accuracy of this scheme is above 99%. Even this simulator is easier to work than STK because all the orbital and ground station parameters can input easily in simulink MATLAB environment. An other result is that the maximum Doppler rate is about 100Hz for 1.2 GHz carrier frequency and receiver phase lock loops can design properly.

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