Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp46-51)

Image Compression in the Wavelet Domain Using an AR Texture Model with Compressed Initial Conditions

M. H. Perng and H. H. Lin
Department of Power Mechanical Engineering,
National Tsing Hua University,
101, Sec. 2, Kung Foo Road, Hsinchu,
Taiwan, R. O. C.
http://ppsc.nthu.edu.tw

Abstract: - This paper present a texture compression technique for still images based on the wavelet transform and the auto-regressive (AR) texture model in order to increase the compression ratio with a minimal loss of image quality. First the influences of the initial condition and the order of an AR model on the resulting texture model are investigated to serve as a theoretical foundation for the proposed approach. To further the compression ratio, this paper also presents a texture compressing technique using an auto-regressive texture model with compressed initial conditions. Results show that the AR model is better than a random texture model when the order of the AR model is adequately chosen, and compression of the initial conditions in the AR model can significantly improve the compression ratio without a noticeable loss of image quality.

Key-Words: - image compression, AR model, texture model, initial conditions, order

1 Introduction

Due to the unceasing demand for a larger and larger compression ratio with satisfactory image quality, texture modeling has gained increasing interest from researchers in the field of image compression [1-4].

In an early attempt to combine the wavelet compression with a texture model [5], an image is first partitioned into texture and non-texture regions. The AR model is then applied to represent the information in the medium and high frequency ranges of texture regions, while the rest of the image is compressed using the traditional wavelet codec. The approach suffers from two drawback: (1)The boundaries of texture and non-texture regions are usually highly irregular, not to mentioned that it is not always easy to distinguish 'texture regions' from 'non-texture regions'. (2) Medium and high frequency ranges of an image usually contain non-texture information (such as edges).

An alternative approach by Debure and Kubota [6] has an entirely different view of 'texture'. In this approach, 'texture' is perceived as the difference between the original image and a compressed image, to be referred to as the 'residual image' in this paper, and the AR model is applied to represent the residual image for the sake of improvng image quality with a minimal cost of data bits. However, this approach does not gain much in the compression ratio.

Most recently, Nadenau and his colleagues [7] treat the information contained on the least significant bit planes as stochastic texture while applying the conventional wavelet codec to compress information on the other bit planes.

However, a stochastic texture model assuming uniformly distributed texture imposes a very strong restriction so that only the three least significant bit planes can be treated as texture. To compress more bit planes in a texture model, we need to resort to texture modeling technqiues that can deals with non-uniform distribution of texture.

Among various texture models, such as those in review papers [8-9], the auto-regressive (AR) model can better handle the non-uniform distribution of texture through its initial conditions, hence it shows very attractive performance in texture modeling [10].

Unfortunately, most existing techniques for AR texture models and wavelet compression with an AR texture model suffer from two major drawbacks: (1) It has been suggested that the order of an AR model can be chosen based on an optimization criterion [11-13], however, there is lack of explicit research reports on the influences of the initial condition and the order of an AR model on it performance in texture modeling. (2)The initial condition occupies the largest portion of bit-rate in an AR model, usually exceeds 10 times more than model parameters, but it seems that there has been no report on how and when the initial condition can be further compressed.

These two issues will be addressed in this paper and the proposed technique to push forward the existing limit of compression ratios.

2 The Influences of the Order and Initial Conditions of an AR Model

In this section we shall first investigate the influence of the order of an AR model on the modeling accuracy, followed by the investigations into the effect of initial conditions before proposing a technique for the compression of initial conditions of a texture image in an AR model. Consider the following AR model

$$y(i,j)=\sum_{s=0}^{m}\sum_{t=0}^{n}\alpha_{st}y(i-s,\ j-t)+u(i,j) \qquad (1)$$

where m and n represent the order of the model while the initial conditions consists of the first m rows and n columns, $s=t\neq 0$, α_{st} is a model parameter solved from a least square problem to minimize the cost function [14]

$$J\triangleq\left\{y(i,j)-\sum_{s=0}^{m}\sum_{t=0}^{n}\alpha_{st}y(i-s,\ j-t)\right\}^2 \qquad (2)$$

$u(i,j)$ is a zero mean white Gaussian noise with and a standard deviation σ to model the following difference [14]

$$y(i,j)-\sum_{s=0}^{m}\sum_{t=0}^{n}\alpha_{st}y(i-s,\ j-t) \qquad (3)$$

It is shown in [15] that the solution to Eq. (1) consists of two parts, a homogeneous solution $y_H(i,j)$ resulting from the least square problem given in Eq. (2) and a particular solution $y_P(i,j)$ due to the existence of the white Gaussian noise $u(i,j)$.

To explore the influence of the order and the initial condition on the AR model, the following discussions will focus on a 1D signal for simplicity but without the loss of generality. For a general 2D case, please refer to [15]. Let the AR model of a 1D discrete system be

$$y(i)=\sum_{s=1}^{m}\alpha_s y(i-s)+u(i) \qquad (4)$$

whose solution consists of a homogeneous solution defined by

$$y_H(i)=\sum_{s=1}^{m}\alpha_s y_H(i-s) \qquad (5)$$

and a noise driven particular solution $y_P(i)$ so that

$$y(i)=y_H(i)+y_P(i) \qquad (6)$$

For the convenience of discussions, let the characteristic equation of this system be given by

$$f(r)=r^i-\sum_{s=1}^{m}\alpha_s r^{i-s}=0 \qquad (7)$$

that has no repeated roots, and $r=\overline{r_s}\triangleq\beta_s e^{j\theta_s}$, $s = 1, 2, \ldots, m$, be a root of Eq. (7), which is also a pole of the discrete system given in Eq. (4), then the homogeneous solution to Eq. (4) is given by [15]

$$y_H(i)=\sum_{s=1}^{m}c_s\left(\overline{r_s}\right)^i\triangleq\sum_{s=1}^{m}c_s y_H^{(s)}(i) \qquad (8)$$

where c_s, $s = 1, 2, \ldots, m$, is determined by the initial condition of Eq. (4), while

$$y_H^{(s)}(i)\triangleq\left(\overline{r_s}\right)^i \qquad (9)$$

is the sth special solution to Eq. (4), which is independent of the initial condition.

In the special case where the 1D signal has a period T such that

$$y(i)=y(i-T) \qquad (10)$$

where $T\leq i$, the special solutions given in Eq. (9) reduces to

$$y_H^{(s)}(i)=e^{j\left(\frac{2\pi}{T}s\right)i},\ s = 0, 1, 2, \ldots, m \qquad (11)$$

and the corresponding homogeneous solution becomes

$$y_H(i)=\sum_{s=1}^{m}c_s e^{j\left(\frac{2\pi}{T}s\right)i} \qquad (12)$$

When the order $m=T$, the homogeneous solution $y_H(i)$ given in Eq. (12) becomes the Discrete Fourier Transform (DFT) of a discrete signal with a period T, and can exactly represent the later. However, if the order $m<T$, the homogeneous solution $y_H(i)$ given in Eq. (12) may fail to exactly represent some discrete signals with a period T.

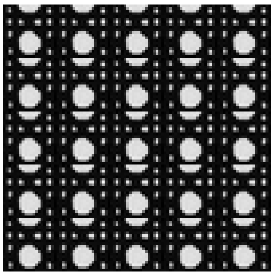
It is therefore suggested that, to simulate a discrete periodic signal with a period of T, the minimal order of the AR model is $i\geq T$.

On the other hand, when the 1D signal is a random signal, the noise driven particular solution $y_P(i)$ dominates the resultant AR model. In such a case, the performance of an AR model is insensitive to its order.

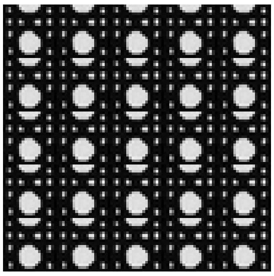
In most real applications, the texture is neither periodic nor purely random. In such a case, an AR model can be perceived as a linear combination of a homogeneous solution to approximate the periodic change in the intensity of a given image and a particular solution to approximate the random change in the intensity. With this notion, it is suggested that the order of the AR model be selected to approximate the periodic component of a given image.

Next, let us consider the influence of the initial condition on the resultant AR model in order to derive a suitable approach for compressing the initial condition.

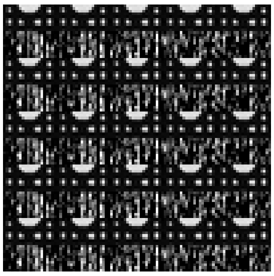
First let us consider a periodic pattern and assume that the order of the AR model is sufficient to represent the image. In such a case, it can be derived from Eq. (8) that the resultant AR model largely depends on the coefficients c_s, s = 1, 2, …, m, in the homogeneous solution, which are completely determined by initial conditions. As a result, the resultant AR model is generally sensitive to initial conditions. In such a case, there is no room for the compression of initial values.



(a) original image



(b)AR model with exact initial conditions



(c)AR model with approximate initial conditions

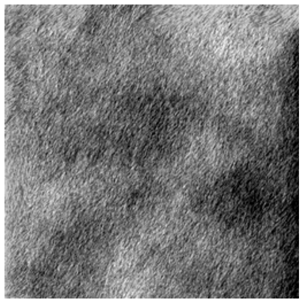
Figure 1. AR models of periodic (repeated) patterns

Figure 1 compares an AR model of a repeated (periodic) pattern with exact initial conditions to that with approximate initial conditions. The original image shown Fig. 1(a) has a period of T = 20 in both vertical and horizontal directions, the AR model with exact initial conditions and the order of $s = 0$, $t = 20$ is shown in Fig. 1(b), while the AR model with approximate initial conditions and the same order is shown in Fig. 1(c), where the approximate initial conditions is resulted from a low order AR model of the exact initial condition. It is clear from Figs. 1(a) and 1(b) that the AR model can exactly describe a repeated pattern when its order is sufficient, and the comparison between Figs. 1(b) and 1(c) confirms that a slight difference in initial conditions may lead to substantial difference in the resultant AR model when the original image is a repeated (periodic) pattern.

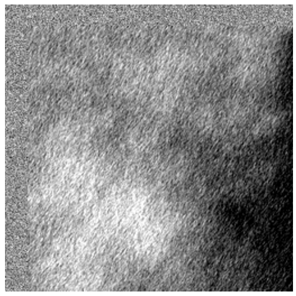
On the other hand, when the image consists of purely random texture, the AR model is dominated by the particular solution and is therefore insensitive to the initial condition.

Figure 2 compares an AR model of a random texture with exact initial conditions to an AR model with approximate initial conditions. Fig. 2(a), 2(b) and 2(c) show the original image, the AR model with exact initial conditions and the AR model whose initial conditions are modeled as white noise, respectively. It is clear from Fig. 2 that a substantial difference in initial conditions does not lead to substantial difference in the performance of the resultant AR model.

(a) Original image



(b) AR model with exact Initial conditions



(c)AR model with approximate initial conditions

Figure 2. AR models of random texture

However, it is also clear from Fig. 2 that there is a noticeable difference between an image and its AR model unless the original image is a repeated pattern. Therefore, a AR model should not be applied to represent the whole image except minute details in the lower bit planes of high frequency channels, which are referred to as the ‘texture image’ hereafter in this paper.

It is therefore suggested that, when the texture of an image to be represented by an AR model is closer to a random texture than a repeated pattern, there is a good chance to compress the initial conditions with an acceptable loss in image quality.

This is especially true when the texture of an image is specifically and restrictedly referred to the lower bit planes in the high frequency channels of a wavelet transformed domain. A technique for image compression in the wavelet domain will therefore be proposed in the next section to make use of the AR model with compressed initial conditions for an increased compression ratio without a substantial loss in the quality of the image.

As regard the technique for the compression of initial conditions, it is suggested based on our experience that down sampling be adopted for compression (coding) and linear interpolation for up-sampling (decoding). The cubic interpolation is not recommended because it is more time consuming without a noticeable improvement in the resultant quality. The other alternative is to model the initial conditions into 1-dimensional white noise. Experiments show that both of them give similar results.

3 Image Compression Using an AR Texture Model with Compressed Initial Conditions

This section presents a technique for image compression in the wavelet domain using the AR texture model with compressed initial conditions.

The wavelet transform is a well developed technique in image compression, and has become standards [16-17]. Wavelet based compression is particularly efficient in the compression of smoothly varying regions of image data. It is the discontinuities located between the regions, along with high contrast textural regions, which cause large transform coefficients and erroneous artifacts, such as blurring or starring, at high compression ratios [6].

In theory, a homogeneous region of pixels can be described with a more compact representation than a non-homogeneous one, hence a texture oriented wavelet image compression scheme has been proposed [6] using an AR texture segmentation technique and an AR texture model to solve the aforementioned drawback of the wavelet transform codecs. However, as mentioned before, the 'residual image' to be modeled by the AR model is defined as the difference between the original image and the wavelet compressed image, hence it does not gain much in the compression ratio. Besides, the initial condition of the 'residual image' is not compressed.

To achieve a better compression ratio, we propose a wavelet transform compression technique which consists of the following steps: (1)First, the discrete wavelet transform (DWT) is applied to the original image; (2)The wavelet domain is divided into the high frequency region and the low frequency range, with the size of the low frequency range being only one fifteenth of that of the high frequency range, as is shown in Fig. 3; (3)Bit planes in the high frequency range is divided into higher bit planes and lower bit planes with a case dependent threshold value T_B; (4)Data in the high frequency region and those in higher planes of the low frequency region are coded using prevailing coding techniques, such as the entropy coder; (5)As an optinal step, lower bit planes of the high frequency region can be segmented into several regions with homogeneous texture in each region using texture segmentation techniques such as that in [6]; (6)Textures in each texture region are compressed using the AR model, then the initial condition in the AR model are further compressed before coding. Figure 4 shows the complete flow chart of the present technique.

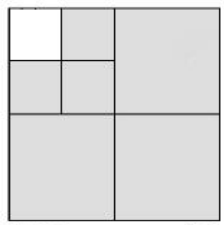


Figure 3. Low and high frequency bands in the wavelet domain: the high frequency region is marked by gray color.

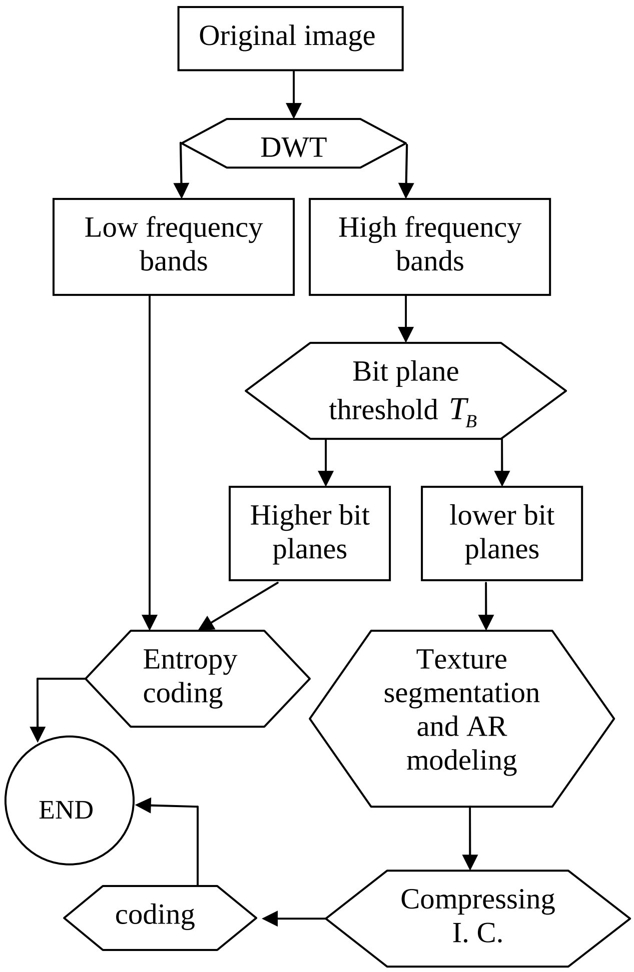


Figure 4. The flow chart of the present technique

Please notice that texture segmentation is optional because it is a computationally demanding process that usually results in irregular texture boundaries mis-matching exact texture boundaries. When this causes troubles to a specific application, it can be dropped.

Regarding the selection of the bit plane threshold value T_B, our experiments show that, for a smaller T_B, the remaining texture image is usually very close to a 2-D random signal which can easily be modeled by the AR model in which the noise drive particular solution dominates. On the other hand, when T_B is large, the remaining texture image is usually highly non-homogeneous. In such a case, it is usually advantageous to apply texture segmentation unless it is difficult to model the remaining texture using an AR model.

The corresponding image decoder is briefly described in the followings: First the compressed initial conditions and model parameters are used to reconstruct the texture image, while the compressed low frequency bands and higher bit planes in high frequency bands are reversed by using the inverse discrete wavelet tranform (IDWT), then the two images are summed up to result in the reconstructed image.

4 Experiments

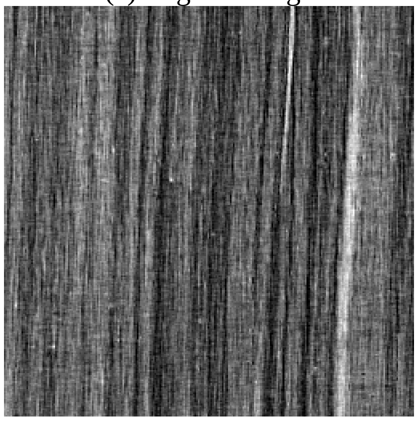
Figure 6 shows the performance of the present approach. The bit plane threshold is $T_B = 6$, and the order of the AR model is $s = t = 20$. Texture segmentation is not adopted in this example.

Fig. 6(a) is the original image, Figs. 6(b), 6(c) and 6(d) are results of the present approach, where Fig. 6(b) uses exact initial conditions, Fig. 6(c) uses zero initial condition to represent an image without a texture model, while Fig. 6(d) uses white noise to compress initial conditions.

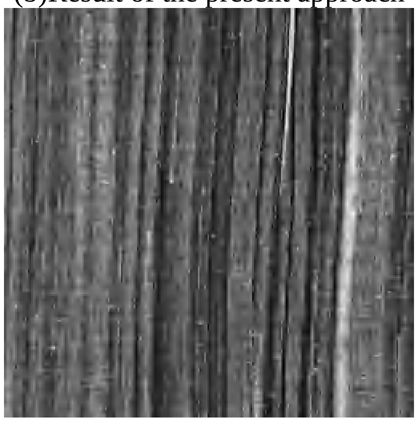
Comparing Figs. 6(b) with 6(a) and 6(c), it is clear that the image with a texture model is significantly better than the one without. Furthermore, comparing Figs. 6(b) and 6(d), the image with compressed initial conditions is as good as the one with exact initial conditions.

Regarding the compression efficiency, the original image is of 256×256 pixels, the lower bit planes in the low frequency channels contains 368,640 bits, corresponding to about 5.62 bits/pixel. The persent approach uses 2,176 bits to model the texture image, corresponding to about 0.03 bits/pixel.

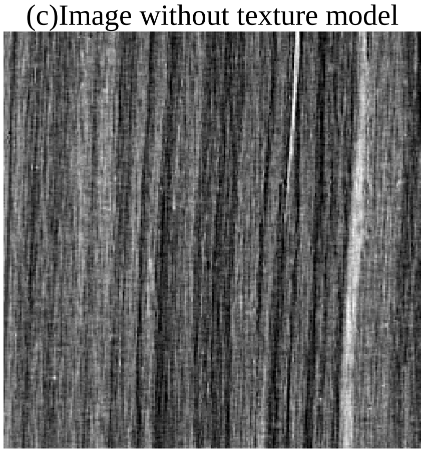
(a)Original image



(b)Result of the present approach



(c)Image without texture model



(d)Image with AR model and exact I. C.

Figure 6. Results of the present technique

The compression ratio of the texture image is quite attractive. However, if exact initial conditions are employed, then it takes 4,647 bits to model the texture image, almost twice the former.

Experiments have also been performed on the Brodatz texture D93, Brodatz texture D105, Brodatz texture D104, and several other images. Most of them show good results similar to the one reported in this paper.

Since the present AR texture model can significantly increase compression ratio without much loss in image quality, and the compression of initial conditions can further improve the compression efficiency without a noticeable deterioration of image quality, we conclude that the present approach may be quite useful in real applications of lossy image compression.

5 Conclusion

This paper presents a technique for image compression in the wavelet domain using the AR texture model with compressed initial conditions. Rules of thumb have been provided as guidance for the selection of orders of the AR model together with two alternative compression techniques for compressing initial conditions. Experiments suggests that the present approach can be applied to a wide class of texture images, and is worthy of further investigations into its limitations.

References:

[1] J. Reichel, and F. Ziliani, Comparison of texture coding methods in a unified motion prediction/ compensation video compression algorithm, *Proceedings of SPIE*, Vol. 5150 III, 2003, pp. 2052-2060.

[2] G. Fahmy, J. Black Jr., and S. Panchanathan, Texture characterization and compression based on human perception in the JPEG2000 framework, *Proceedings ICIP*, Vol. 4, 2004, pp. 2335-2338.

[3] B. Vanhoof, L. Nachtergaele, G. Lafruit, M. Peón, B. Masschelein, F. Catthoor, J. Bormans, and I. Bolsens, A Scalable MPEG-4 wavelet-based visual texture compression system with optimized memory organization, *IEEE Trans. Circuits and Systems for Video Technology*, Vol. 13, No. 4, 2003, pp. 348-357.

[4] T. C. L. Chan, T. C. Hsung, and D. P. Lun, Improved MPEG-4 still texture image coding under noisy environment, *IEEE Trans. Image Processing*, Vol. 12, No. 5, 2003, pp. 500-508.

[5] T. W. Ryan, D. Sanders, H. D. Fisher, and A. E. Iverson, Image compression by texture modeling in the wavelet domain, *IEEE Transaction on Image Processing*, Vol. 5, No. 1, 1996, pp. 26-36.

[6] K. Debure and T. Kubato, Autoregressive texture segmentation and synthesis for wavelet image compression, *Proceeding of Image and Multidimentional Digital Signal Processing*, Alpbach, Austria, July 1998, pp. 131-134.

[7] M. J. Nadenau, J. Reichel, and M. Kunt, Visually improved image compression by combining a conventional wavelet-codec with texture modeling, *IEEE Transactions on Image Processing*, Vol. 11, No. 11, 2002, pp. 1284-1294.

[8] V. Manian, and R. Vasquez, Texture analysis and synthesis: A review of recent advances, *Proceedings of SPIE*, Vol. 5108, 2003, pp. 242-250.

[9] Chaur-Chin Chen and Chien-Chang Chen, Texture synthesis: A review and experiments, *Journal of Information Science and Engineering*, Vol. 19, No. 2, 2003, pp. 371-380.

[10] H. Iversen and T. Lønnestad, An evaluation of stochastic models for analysis and synthesis of gray-scale texture, *Pattern Recognition Letters*, Vol. 15 , No. 6, 1994, pp. 575-585.

[11] P. Stoica and Y. Selen, Model-order selection - a review of information criterion rules, *IEEE Signal Processing Magazine*, Vol. 21, No. 4, 2004, pp.36-47.

[12] O. Alata and C. Olivier, Choice of a 2-D causal autoregressive texture model using information criteria, *Pattern Recognition Letters*, Vol. 24, No. 9-10, 2003, pp.1191-1201.

[13] A. Sarkar, K. M. S. Sharma and R. V. Sonak, A new approach for subset 2-D AR model identification for describing textures, *IEEE Transactions on Image Processing*, Vol. 6, No. 3, 1997, pp. 407-413.

[14] J. D. Edward, R. L. Kashyap and O. R. Mitchell, Image data compression using autoregressive time series models, *Pattern Recognition*, Vol. 11, No. 5-6, 1979, pp. 313-323.

[15] R. E. Mickens, *Difference Equations*. New York: Van Nostrand Reinhold, 1987, pp. 123-150.

[16] C. Cristopolous, A. Skodras, and T. Ebrahimi, The JPEG2000 still image coding systems: An overview, *IEEE Tran. Consumer Electronics*, Vol. 476, No. 4, 2000, pp. 1103-1127.

[17] D. Santa-Cruz, T. Ebrahimi, J. Askelof, M. Larsson, C.A. Christopoulos, JPEG 2000 still image coding versus other standards, *Proc. SPIE*, Vol. 4115, 2000, pp. 446-454.