

A Novel Approach for Extending Quantitative Feedback Theory on Nonlinear MIMO Systems

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Abstract: An approach to robust control design for a nonlinear multi-input/multi-output (MIMO) plant using linearization theory and quantitative feedback theory (QFT) is presented and applied to the design of a MIMO nonlinear robot control system. This method is named Generalized Quantitative Feedback Theory (GQFT). GQFT techniques are introduced to give a feedback control design for the plant model under the parameter uncertainty. The QFT method for single-input/single-output (SISO) plants is used to obtain robust stability under the given plant-parameter uncertainties. The design results demonstrate the good performance and features of the proposed GQFT approach that may be achieved.

Key-Words: - Robust control- Nonlinear control - Quantitative feedback theory -Robot manipulator

1 Introduction

Many practical systems are characterized by uncertainty which makes it difficult to maintain good stability margins and performance properties for the closed-loop system. There are two general design methodologies for dealing with the effects of uncertainty:

1. "Adaptive Control", in which the parameters of plant or some other appropriate structure are identified online and the information obtained is then used to 'tune' the controller.
2. "Robust Control", which typically involves a 'worst-case' design, approaches for a family of plants representing the uncertainty using a single fixed controller, [12,9,15].

Quantitative feedback theory (QFT) is a robust-control method developed during the last two decades which deals with the effects of uncertainty systematically. It has been successfully applied to the design of both SISO and MIMO systems; it has also been extended to the nonlinear and the time varying case [1,2,3, and 6]. In comparison to other optimization-based robust control approaches, QFT offers a number of advantages. These include:

- a. The ability to assess quantitatively the 'cost of feedback' [5 and 7].
- b. The ability to take into account phase information in the design process (which is lost if, for example, singular values are used as the design parameters).
- c. The ability to provide 'design transparency', that

is, clear tradeoff criteria between controllers complexity and feasibility of the design objectives.

d. Note that (c) implies in practice that QFT often results in simple controllers which are easy to implement.

For the purpose of QFT, the feedback system is normally described by two degrees-of-freedom structure shown in Fig. 1. In this case R is an input, F is prefilter transfer function, G is a cascade compensator, and P represents a set of transfer functions which describes area of plant-parameter uncertainty. QFT takes into account 'quantitative' information on the plant's variability (uncertainty), requirements for robust performance, tracking-performance specifications, expected disturbance amplitude and requirements for its attenuation. The output $y(t)$ is required to track the command input signal $r(t)$ and to reject the external disturbances $d_1(t)$ and $d_2(t)$, [18,19,14].

The compensator is designed so that the variations of $y(t)$ to the uncertainty in the plant P are within acceptable tolerances and the effects of the disturbances $d_1(t)$ and $d_2(t)$ on $y(t)$ are small. The prefilter properties of $F(s)$ must be designed for the desired tracking of the reference $r(t)$.

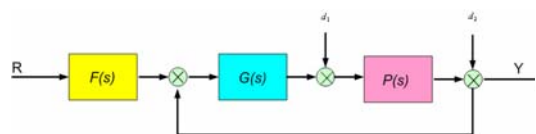


Fig. 1. The fundamental QFT design.

2 Introduction To QFT Method for Nonlinear MIMO Systems

In designing a robust controller for MIMO nonlinear systems, a combination of two approaches is used as follows:

- I. Designing a robust controller for SISO nonlinear system, [8].
- II. Designing a robust controller for linear time-invariant system.

Due to this aim, firstly, the MIMO nonlinear system must be transformed to a set of linear time-invariant systems furthering disturbances. Then, applying robust controller approach (QFT) for equivalent MIMO linear time-invariant ones, as a solution of the MIMO nonlinear control systems will be designed. The following example is prepared for more understanding, [10,11,13].

Example 1: Consider a MIMO nonlinear system as shown in Fig.2, which is formulized by a dynamic equation as (1).

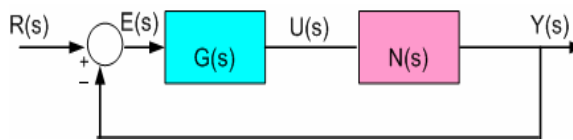


Fig. .2. The block diagram of a MIMO nonlinear system

$$\dot{y}_1 + Ay_1^3 + B(y_1 + 1)y_2 = k_1u_1 \tag{1}$$

$$\dot{y}_2(1 + Cy_1) + Ey_1^2 + Dy_2 = k_2u_2$$

The purposed system is supposed to be a MIMO nonlinear case with two outputs, $[y_1, y_2]^T$ and two inputs, $[u_1, u_2]^T$. Furthermore, all of the initial conditions are zero. The parameters, k_2, k_1, E, D, C, B and A are as follows:

$$A = [0.04, 0.05], B = [0.08, 0.12], C = [0.08, 0.12],$$

$$D = [0.8, 1.2], E = [0.8, 1.2], k_1 = [0.5, 2.5],$$

$$k_2 = [0.5, 2.5]$$

To design the robust controller by using QFT, the system outputs should satisfy the following inequality:

$$|y_i(w) - y_i^\circ(w)| \leq e_i(w) \quad , \quad i = 1, 2 \tag{2}$$

$$y_i^\circ(s) = \frac{2}{s(s^2 + 3s + 2)} \quad , \quad y_2^\circ(s) = \frac{2}{s(s + 2)} \tag{3}$$

The desired performance and time response characteristics, which are simulated on Fig.3, shows that the output deviation of first channel from

$y_1^\circ(w)$ shouldn't be more than quantitative amount of $e_1(w)$ and for the second channel deviation of the output from $y_2^\circ(w)$ shouldn't exceed $e_2(w)$.

The design procedure is as follows:

1. Transforming the nonlinear system into a linear system considering disturbances on the system output which satisfies below equation:

$$y = Nu = P_{N,y} + y'_{N,y} + d_{N,y} \tag{4}$$

To meet this purpose, the Taylor extension for the input $u(t)$ and the output $y(t)$ should be calculated as follows: (with regard to $y_{10}, y_{20} = 0$):

$$u(t) = \begin{bmatrix} u_{10} + u_{11}t + u_{12}t^2 + \dots \\ u_{20} + u_{21}t + u_{22}t^2 + \dots \end{bmatrix} \tag{5}$$

$$y(t) = \begin{bmatrix} y_{11}t + y_{12}t^2 + \dots \\ y_{21}t + y_{22}t^3 + \dots \end{bmatrix} \tag{6}$$

Substituting the above equations into the system equations, we have:

$$y_{11} + 2y_{12}t + By_{21}t = K_1u_{10} + K_1u_{11}t \tag{7}$$

$$y_{21} + 2y_{22}t + Cy_{11}y_{21}t + Dy_{21}t = K_2u_{20} + K_2u_{21}t$$

Then:

$$y_{11} + (2y_{12} + By_{21})t = K_1u_{10} + K_1u_{11}t \tag{8}$$

$$y_{21} + (2y_{22} + Cy_{11}y_{21} + Dy_{21})t = K_2u_{20} + K_2u_{21}t$$

Where:

$$u_{10} = y_{11}/K_1 \quad , \quad u_{11} = By_{21}/K_1 \tag{9}$$

$$u_{20} = y_{21}/K_2 \quad , \quad u_{21} = By_{21}/K_2$$

It can be found that:

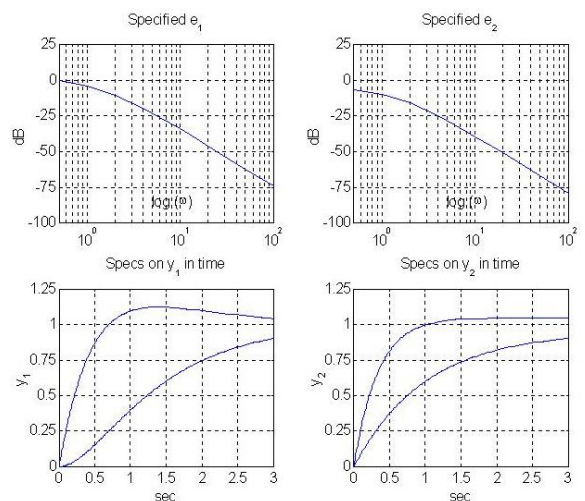


Fig. 3. The frequency and time characteristic of the closed loop system of e_1 and e_2 .

$$P \times \begin{bmatrix} \frac{y_{11}}{K_1 s} + \frac{B y_{21}}{K_1 s^2} \\ \frac{y_{21}}{K_2 s} + \frac{D y_{21}}{K_2 s^2} \end{bmatrix} = \frac{P}{s^2} \begin{bmatrix} \frac{1}{K_1} S & \frac{1}{K_1} B \\ 0 & \frac{(S+D)}{K_2} \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} \quad (10)$$

$$\rightarrow \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} \times \frac{1}{s^2}$$

Finally, the linear system can be determined as follows :

$$P_{N,y} = \begin{bmatrix} \frac{s}{k_1} & \frac{B}{k_1} \\ 0 & \frac{(s+D)}{k_2} \end{bmatrix} \quad (11)$$

Disturbance $d_{N,y}$ for the system output sets of a second order plant can be determined as follows:

$$y_1(s) = \frac{(\lambda\sigma - \lambda\tau + \tau)s + \sigma\tau}{s(s^2 + (\sigma + \tau)s + \sigma\tau)}, \quad \lambda = [1.5, 2], \quad \sigma = [0.5, 1] \quad (12)$$

$$\tau = [1, 2], \quad y_2(s) = \frac{\alpha}{s(s + \beta)}, \quad \alpha = [0.95, 1.05], \quad \beta = [1, 3]$$

The steps of determining $d_{N,y}$ are as follows:

a. Choosing the parameters A, B, C, D, E, k_2 and k_1 , and transforming y_2 and y_1 into time - domain functions.

b. Defining u_2 and u_1 according to (1), and determining transfer function.

c. Determining $d_{N,y}$ according to (4), which satisfies initial equation $y'_{N,y} = 0$, and $d_{N,y} = y - P_{N,y} u$.

d. Repeating the steps a, b and c for all values on A, B, C, D, E, k_2 and k_1 , and choosing upper bounds on y_2 and y_1 to create sets of couples $\{P_{N,y}, d_{N,y}\}$, Which satisfy desired outputs equations.

2. Then the system inputs can be calculated from relation $r = G y_0 + u_0$ where $y_0 = [y_1^\circ, y_2^\circ]^T$ arises from (3), and u_0 can be calculated choosing $A_0, B_0, C_0, D_0, E_0, k_2$ and k_1 , which are the average amounts of A, B, C, D, E, k_2 and k_1 , respectively.

3. According to the approach which is verified on designing controller for MIMO systems, the controller g_1 should be designed in such a way that the first inequality from (2) would be satisfied. The bounds and the nominal loop gain function are shown in Fig.4. The controller transfer function is

determined as follows:

$$g_1 = \frac{36s + 223}{s^2 + 82s + 43} \quad (13)$$

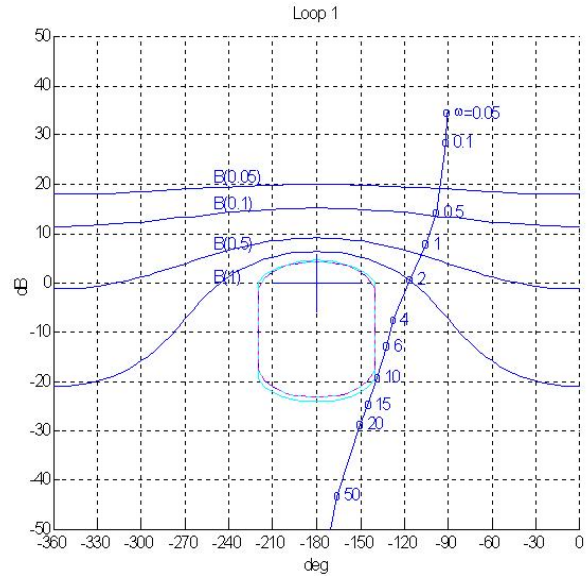


Fig.4. The forming of the nominal gain function and the frequency bounds for the first channel.

The controller g_2 must be designed, so that the first inequality from (2) be satisfied. The transfer function for this loop can be determined as follows.

$$g_2 = \frac{49s + 502}{s^2 + 32s + 48} \quad (14)$$

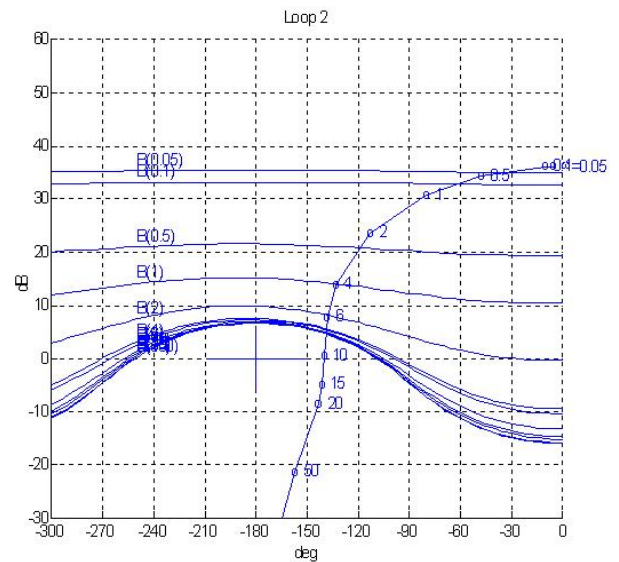


Fig.5. The Forming of the nominal loop gain function and the frequency Bounds for second channel

The frequency bounds and the nominal loop gain for second loop are shown in Fig. 5

The results of simulations for the closed loop system using designed controllers are shown in Fig.6. The time response in both channels of the nonlinear

system for the domain of parameter variations is located between the desired bounds as shown in Fig.6. The undesired mutual effects of the channels on the outputs of each others isn't too much. On the other hand, the QFT controller can handle this amount of uncertainties. The control efforts simulations illustrate acceptable behaviors in Fig.7.

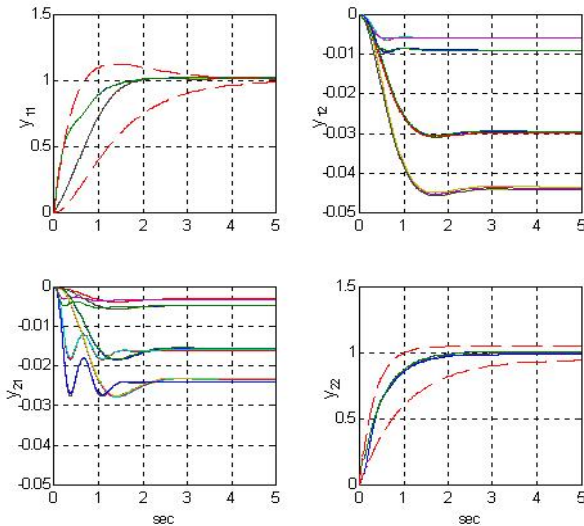


Fig. 6. The time response for the closed loop system.

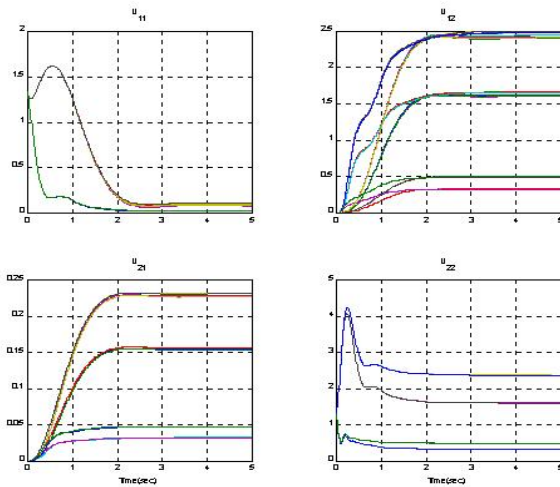


Fig.7. Control signals for closed loop system

3 Manipulator Control Using GQFT Method

In this section, to present the practical application of this method on real industrial systems, the robust controllers are purposed for the position and angle control of a planner robot on sketching, planning, and other similar tasks, in such a way it is faced with high degree of uncertainty, disturbances and nonlinearity.

3.1 Robot Modeling

Let's assume that the basic equation for motion of a robot arm would be as follows:

$$M(q).\ddot{q} + N(q, \dot{q}) = u \tag{15}$$

q is a $(k \times 1)$ position vector,

\dot{q} is a $(k \times 1)$ velocity vector,

\ddot{q} is a $(k \times 1)$ acceleration vector,

$M(q)$ is a $(k \times k)$ matrix of inertia (invertible),

$N(q, \dot{q})$ is a $(k \times 1)$ vector of damping centrifugal coriolis gravitational force, (u) is a $(k \times 1)$ vector of generalized forces and torques.

Furthermore, let's suppose that the open loop system mentioned in (15) has k degree of freedom (d.o.f). Normally, the amount of k is equal to 6, so the number of d.o.f in the Cartesian space is R_q^k . The control problem is to follow a given trajectory $q^d(t)$ and to produce a torque vector u such that the tracking error approaches to the acceptable value (zero) as $t \rightarrow \infty$, [13].

3.2 The Motion Equation

In this part, we concentrate on both the equation of motion of a two-link robot arm and the computation of the robust controller. Consider the following two-link robot arm which its masses concentrated at the ends of the links and the motor inertias are neglected (Fig. 8).

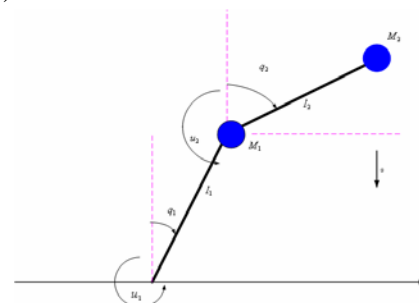


Fig. 8. The scheme of a two-link robot manipulator.

According to (15), we will have the equations of motion as:

$$M(q).\ddot{q} = -N(q, \dot{q}) + u$$

In this equation:

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} = \begin{bmatrix} (m_1 + m_2)l_1^2 & m_2 l_1 l_2 \cdot \cos(q_1 - q_2) \\ m_2 l_1 l_2 \cdot \cos(q_1 - q_2) & m_2 l_2^2 \end{bmatrix} \tag{16}$$

And

$$N(q, \dot{q}) = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} \dot{q}_2^2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \text{Sin}(q_1 - q_2) \dots \\ -\dot{q}_1^2 \cdot m_2 \cdot l_1 \cdot l_2 \cdot \text{Sin}(q_1 - q_2) \dots \\ \dots - (m_1 + m_2) \cdot g \cdot l_1 \cdot \text{Sin}q_1 + K_{q_1} \cdot \dot{q}_1 \\ \dots - m_2 \cdot g \cdot l_2 \cdot \text{Sin}q_2 + K_{q_2} \cdot \dot{q}_2 \end{bmatrix}$$

Where K_{q_2} and K_{q_1} are the damping coefficients for the q_2 and q_1 cases, respectively.

3.3 Design of the Controller through GQFT

Considering the uncertainties bound as in Table. 1, If the desired output characteristic is assumed to be the same as in the previous example, control task will be to design the robust controller for the manipulator system, in which controller satisfies the desired closed-loop time response [16,17].

Substituting $g = 9.8$, we have:

$$u_1 = (m_1 + m_2)l_1^2\ddot{q}_1 + m_2l_1l_2 \cos(q_1 - q_2)\ddot{q}_2 + m_2l_1l_2 \sin(q_1 - q_2)\dot{q}_2^2 - (m_1 + m_2)gl_1 \sin q_1 + K_{q_1}\dot{q}_1 \quad (17)$$

$$u_2 = m_2l_1l_2 \cos(q_1 - q_2)\ddot{q}_1 + m_2l_2^2\ddot{q}_2 + m_2l_1l_2 \sin(q_1 - q_2)\dot{q}_1^2 - m_2gl_2 \sin q_2 + K_{q_2}\dot{q}_2 \quad (18)$$

Substituting the first term of Taylor extensions of $\sin(q_1 - q_2)$, $\cos(q_1 - q_2)$, $\sin(q_2)$ and $\sin(q_1)$, we will have:

$$\sin q = q - \frac{q^3}{3!} + \frac{q^5}{5!} - \dots \quad (19)$$

$$\cos q = 1 - \frac{q^2}{2!} + \frac{q^4}{4!}$$

and also

$$u_1 = (m_1 + m_2)l_1^2\ddot{q}_1 + m_2l_1l_2\ddot{q}_2 + (m_2l_1l_2(q_1 - q_2)\dot{q}_2^2 - (m_1 + m_2)gl_1q_1 + K_{q_1}\dot{q}_1) \quad (20)$$

$$u_2 = m_2l_1l_2\ddot{q}_1 + m_2l_2^2\ddot{q}_2 - (m_2l_1l_2(q_1 - q_2)\dot{q}_1^2 - (m_2gl_2)q_2 + K_{q_2}\dot{q}_2)$$

$$u_1 = (m_1 + m_2)l_1^2\ddot{q}_1 + m_2l_1l_2\ddot{q}_2 + k_{q_1}\dot{q}_1 + m_2l_1l_2q_1\dot{q}_2 - m_2l_1l_2q_2\dot{q}_2^2 - (m_1 + m_2)gl_1q_1$$

$$u_2 = m_2l_1l_2\ddot{q}_1 + m_2l_2^2\ddot{q}_2 + k_{q_2}\dot{q}_2 - m_2l_1l_2q_1\dot{q}_1^2 + m_2l_1l_2q_2\dot{q}_2^2 - m_2gl_2\dot{q}_2$$

Assuming:

$$\alpha_1 = (m_1 + m_2)l_1^2, \quad \beta_1 = m_2l_1l_2$$

$$\alpha_2 = m_2l_1l_2, \quad \beta_2 = m_2l_1l_2^2$$

If the Taylor extensions for the output, $[q_1, q_2]^T$ and the inputs $[u_1, u_2]^T$, are supposed as the following equations, considering the zero initial conditions ($q_{10} = 0, q_{20} = 0$), we will have:

$$\begin{cases} q_1 = q_{11}t + q_{12}t^2 + q_{13}t^3 + \dots \\ q_2 = q_{21}t + q_{22}t^2 + q_{23}t^3 + \dots \\ u_1 = u_{10} + u_{11}t + u_{12}t^2 + \dots \\ u_2 = u_{20} + u_{21}t + u_{22}t^2 + \dots \end{cases} \quad (21)$$

Neglecting the higher order terms on the above extensions and substituting (21), we will have:

$$\begin{aligned} u_{10} + u_{11}t &= (m_1 + m_2)l_1^2(2q_{12} + 6q_{13}t) \\ &+ m_2l_1l_2(2q_{22} + 6q_{23}t) + (2q_{22} + 6q_{23}t) \\ &+ k_{q_1}(\alpha_{11} + 2q_{12}t) + m_2l_1l_2q_{11}t(q_{21} + 2q_{22}t)^2 \\ &- m_2l_1l_2q_{21}t(q_{21} + 2q_{22}t)^2 - (m_1 + m_2)gl_1q_{11}t \\ u_{20} + u_{21}t &= m_2l_1l_2(2q_{12} + 6q_{13}t) \\ &+ m_2l_2^2(2q_{22} + 6q_{23}t) + k_{q_2}(q_{21} + 2q_{22}t) \\ &- m_2l_1l_2q_{11}t(q_{11} + 2q_{12}t)^2 \\ &+ m_2l_1l_2q_{21}t(q_{11} + 2q_{12}t)^2 - (m_2gl_2)q_{21}t \end{aligned} \quad (22)$$

$$\begin{aligned} u_{10} + u_{11}t &= \alpha_1(2q_{12} + 6q_{13}t) + \beta_1(2q_{22} + 6q_{23}t) \\ &+ k_{q_1}(q_{11} + 2q_{12}t) + \beta_1\alpha_1t(q_{21} + 2q_{22}t)^2 \\ &- \beta_1q_{21}t(q_{21} + 2q_{22}t)^2 - (m_1 + m_2)gl_1q_{11}t \end{aligned}$$

$$\begin{aligned} u_{20} + u_{21}t &= \alpha_2(2q_{12} + 6q_{13}t) + \beta_2(2q_{22} + 6q_{23}t) \\ &+ k_{q_2}(q_{21} + 2q_{22}t) - \alpha_2q_{11}t(q_{11} + 2q_{12}t)^2 \\ &+ \alpha_2q_{21}t(q_{11} + 2q_{12}t)^2 - (m_2gl_2)q_{21}t \end{aligned}$$

Where:

$$\begin{aligned} u_{10} &= 2\alpha_1\alpha_{12} + 2\beta_1q_{22} \\ u_{20} &= 2\alpha_2q_{12} + 2\beta_2q_{22} \end{aligned} \quad (23)$$

Then:

$$Px \frac{2}{s} \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix} \begin{bmatrix} q_{12} \\ q_{22} \end{bmatrix} \rightarrow \frac{1}{s^3} \begin{bmatrix} q_{12} \\ q_{22} \end{bmatrix}$$

And finally $P_{N,y}(s)$ can be determined as follows:

$$P_{N,y}(s) = \begin{bmatrix} 2\alpha_1 s^2 & 2\beta_1 s^2 \\ 2\alpha_2 s^2 & 2\beta_2 s^2 \end{bmatrix}^{-1} \quad (24)$$

The next step is obtaining $d_{N,y}(s)$ for all domains of parameters, variations. The controllers g_2 and g_1 should be designed in such a way that the desired output characteristic would be satisfied. The transfer functions of the controllers g_1 and g_2 are obtained according to the nominal transfer gain function, which their simulation results illustrated on Fig.9 and Fig .10 as follows:

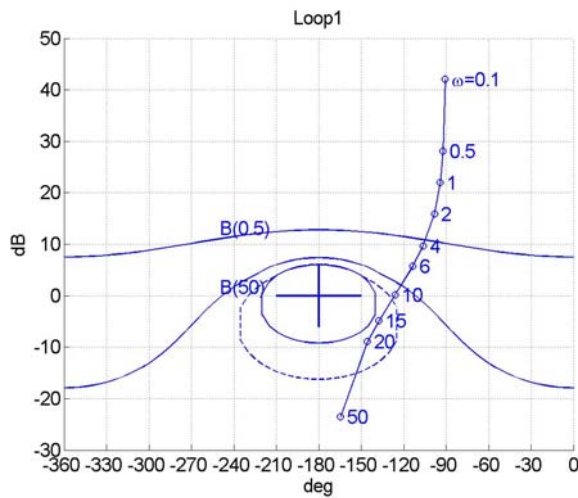


Fig. 9. The frequency bounds and the transfer gain function for the first channel.

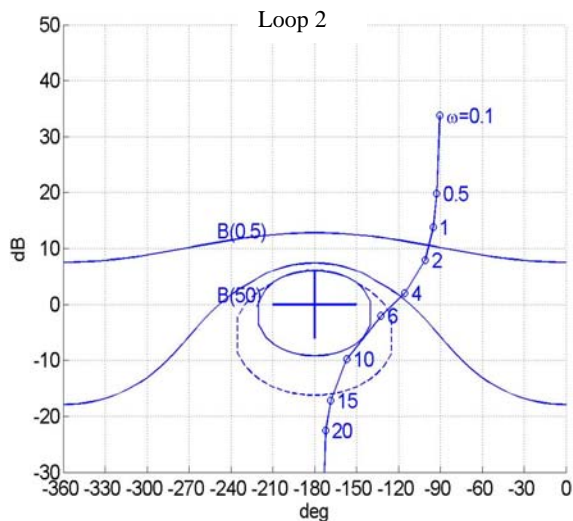


Fig. 10. The frequency bounds and the transfer gain function for the second channel

$$g_1 = 6.61 \frac{(s + 1.4383 * 10^{-3})}{(s + 22.0981 - 24.2315j)} \cdot \frac{(s + 0.031 * 10^{-3})}{(s + 22.0981 + 24.2315j)} \quad (25)$$

$$g_2 = \frac{0.0041 s (s + 3.84 * 10^{-5})}{(s + 13.69)(s + 0.0013)}$$

4 Simulation Results of Applying GQFT Method on Uncertain MIMO Nonlinear System

In this section, The GQFT method will be applied on the manipulator system under the parameters' uncertainties to demonstrate how well they can cope with uncertainty and nonlinearity remaining on the desired efficiency bounds. According to the Fig.11 and Fig.12, it is presented that GQFT approach forces the system states to remain within the desired efficiency bounds with acceptable tracking error and a high robustness. These figures illustrate that generalized quantitative feedback theory as an effective robust method can control the MIMO nonlinear system with high degree of uncertainties. However, one of its important disadvantages is large amount of over design error as shown in Fig.12. The verification of Table . 2 , Table .3, show the efficiency of this method on MIMO nonlinear practical systems.

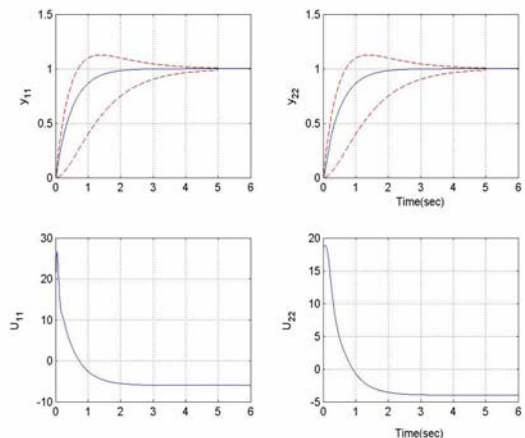


Fig.11. GQFT response to step input for nominal MIMO nonlinear system

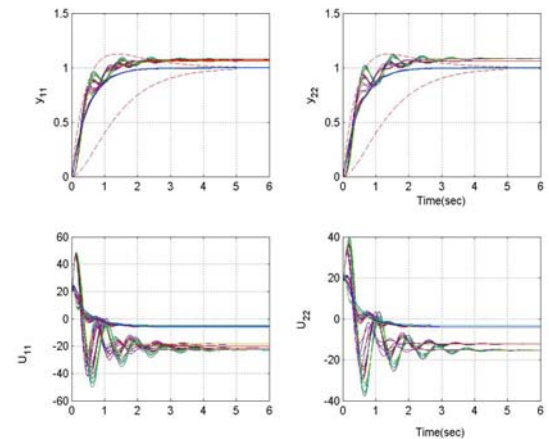


Fig.12. GQFT response to step input for MIMO nonlinear system in presence of high degree of uncertainties

Table.1. The Uncertainty Bounds for MIMO Nonlinear Robot

Link	m_i (kg)	Kq_i (kgm^2)	L_i (m)
1	[0.9, 1.1]	[9, 11]	[0.18, 0.22]
2	[1.8, 2.2]	[9, 11]	[0.18, 0.22]

Table.2. The Efficiency Characteristic for Nominal System Using GQFT Method

Link	Maximum Overshoot	Settling Time	Peak of Control Effort	Time of Simulation	Percentage of Steady State Error
1	0%	2.2	27 N.m	15	0%
2	0%	2.2	19 N.m	15	0%

Table.3. The Efficiency Characteristic in Presence of Uncertainties and Nonlinearities Using GQFT Method

Link	Maximum Overshoot	Settling Time	Peak of Control Effort	Time of Simulation	Percentage of Steady State Error
1	4%	3	48 N.m	30	7%
2	6%	3	40 N.m	30	7%

Conclusion

An effective model based (GQFT) method, have been introduced to generalize QFT for MIMO nonlinear cases. The application of the GQFT technique for the development of a force controller on MIMO nonlinear systems are verified. A parametrically uncertain second-order nonlinear model was developed to represent the relation between the control signal and the force acting on the robot manipulator using GQFT.

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