Proceedings of the 5th WSEAS Int. Conf. on Signal Processing, Robotics and Automation, Madrid, Spain, February 15-17, 2006 (pp323-328)

Accurate Synchronised Position Cross-Coupling Motion Controller for Two-Wheel Mobile Robot

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Abstract: The position accuracy of current mobile robots that are driven by independent motors has not been yet achieved to full satisfaction due to the difference in the angular positions of motor rotors that is caused by external and internal disturbances. Thus, an Accurate Synchronised Position Cross-Coupling Motion Controller (ASPC-CMC) for a Two-Wheel Mobile Robot that guarantees zero steady state and zero transient response errors is proposed in this paper to achieve higher accuracy in the robot desired position. Previous Cross-Coupling controllers are based on the consideration of only internal disturbances by using only feed-back signals to correct the system speed errors. The technique presented in this paper is based on the consideration of both internal and external disturbances by employing feed-back and feed-forward signals at the same time to generate an error position corrective signal to improve the accuracy in mobile robot desired position. The proposed technique has been mathematically modelled, analysed and simulated by using Digital Signal Processing (DSP) and MATLAB-SIMULINK tools. Simulation results verify the theoretical analysis and show that the suggested technique is robust and yields zero transient response and zero steady state errors. Hence, the proposed technique has improved the accuracy of the mobile robot desired position, for straight or curve of motion, even with the existence of continuous internal and external disturbances that affect both motor control loops.

Key-Words: - Position Cross-Coupling motion control, Mobile robot, DSP, BLDC Motor.

1 Introduction

Two-wheel mobile robots that are driven by two independently controlled motors have a lack of full coordination between motor control loops [1] [2] [3] [4]. To solve the coordination problem between the two motor-control loops, speed cross-coupling controllers were introduced in [1] [2] [5]. The cross-coupling controllers introduced in [1] [2], were based on the consideration of the internal disturbances and used the speed feedback signals to correct system errors. These types of cross-coupling controllers are good to guarantee zero steady-state errors in the system speed output [1] [2].

However, lack of accuracy in the desired position of mobile robots still occurs. This lack of accuracy is due to external and internal disturbances that affect each control loop. Therefore, to improve the accuracy in the mobile robot desired position, external and internal disturbances must be considered in order to achieve zero transient and zero steady state errors in the system position output response. This will lead to satisfactory accuracy in mobile robot desired position.

This paper presents a proposed accurate position cross-coupling motion control technique to improve the accuracy in the mobile robot desired position. This technique is based on the consideration of both internal and external disturbances that affect each control loop.

It is known that internal and external disturbances that affect one motor control loop may differ from disturbances that affect the other loop [2][5]. This difference will lead to non-zero transient response and non-zero steady state errors in the system position output response. Thus, to eliminate these errors and to improve the accuracy in the whole system desired position, this difference must be considered and shared between both motor control loops. This difference can be considered and shared between both motor control loops by implementing the proposed technique.

This paper consists of six sections: section 2 shows motion system position errors; section 3 illustrates the position cross-coupling motion control technique. Also, it shows how the suggested idea will improve the motion control performance of the system, section 4 shows the mathematical analysis of the system and the suggested position Cross-coupling Algorithm, in section 5, simulation results are presented, a conclusion is provided in section 6.

2 Motion System Position Errors

External and internal disturbances that affect the robot motion system cause errors in the system desired output [2] [5] [6]. These errors are reflected on the system transient and steady state responses [5]. As a result, they change the system's behaviour from what is desired and cause an error in the mobile robot desired position. For more details about the main sources of external and internal disturbances see [5].

When the robot moves in a direct line, external and internal disturbances may cause position errors such as: [4][5]

- Tracking error (e_t), which is the distance between the actual and desired position in the direction of travel,
- Orientation tracking error (e_θ), which is the displacement about the desired path in terms of angle (θ) as shown in Fig.1 below.

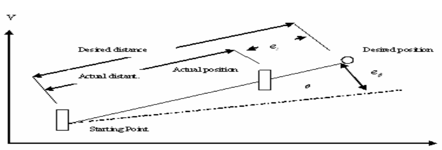

Fig.1 Direct Line Motion Error

Similarly, when the robot moves in a curved path, external and internal disturbances may cause another types of position errors: [2] [5] [7]

- Contouring error, (e_c): Which is the perpendicular distance between the actual contouring path and the desired one as shown in Fig.2 below.
- Orientation error, (e_θ): which is the error in the direction angle or the angular displacement about the actual curved path as shown in Fig.2 .
- Tracking error, (e_t): which is the distance between the actual and desired position in the direction of travel as shown in Fig.2 below.

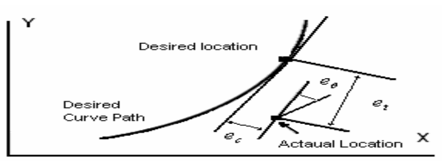

Fig.2 Curved Line Motion Errors

As previously mentioned, external and internal disturbances cause errors that affect the mobile robot motion system causing an error in the mobile robot desired position. Thus, to defeat such problem and to improve the accuracy in the mobile robot desired output position despite continuous internal and external disturbances, the following Accurate Synchronized Position Cross-Coupling Motion Controller (ASPC-CMC) technique is proposed and analysed.

3 Position Cross-Coupling Motion Control Technique

The cross-coupling motion control technique that proposed in this paper is to coordinate and distribute undesired disturbance signals between both motor control loops. The proposed technique is a new approach for Accurate Position Cross-Coupling Motion Controller of a Two-Wheeled Mobile Robot. This technique is suggested to improve the accuracy in the mobile desired output position by achieving zero transient response and zero steady state errors.

The proposed idea is based on the consideration of external and internal disturbances that affect both control loops. As mentioned before, disturbances that affect one control loop may differ from those affect the other loop. This difference will lead to non-zero transient response and non-zero steady state errors. As a result, a variation between the two motor rotors angular position will occur, which will affect the robot desired output position. Thus, to overcome such problems, this disturbance difference must be considered and shared between both control loops as shown in the system block diagram, Fig.3.

In this technique, disturbance errors are considered and distributed between both motor control loops at the same time by employing feed-back and feed-forward signals as an input for the cross-coupling controller as shown in Fig.3. A PI controller is used to control the resultant error signal. The corrective signal from the output of the PI controller is fed again to the input of each motor position control loop to keep them coordinated, synchronised and balanced at any time even in the presence of internal and external disturbances.

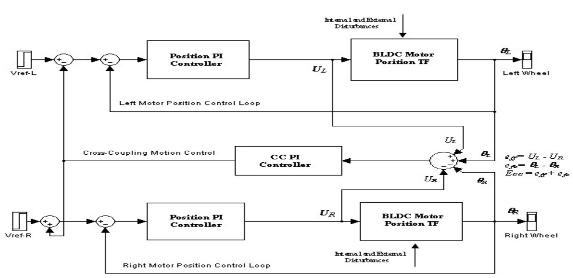

Fig.3 System Block Diagram

As shown in Fig.3 above, two expected errors are shared between both loops to improve the system accuracy and obtain zero transient response and zero steady state errors. These two expected errors are:

- Feed-Forward Error Signal (e_{ff}), which is the difference between the motors input signals (U_L and U_R) that may be affected by external disturbances. This difference must be considered and it is measured by feed-forward signals, as: $\boldsymbol{e_{ff} = U_L - U_R}$.
- Feed-Back Error Signal (e_{fb}), which is the difference between motors position output signals (θ_L and θ_R) that may be affected by internal and external disturbances. This difference must be considered and it is measured by feed-back signals, as: $\boldsymbol{e_{fb} = \theta_L - \theta_R}$.

The over-all cross-coupling error signal (E_{CC}) is obtained from both errors, as: $\boldsymbol{E_{CC} = e_{ff} + e_{fb}}$

This error signal (E_{CC}) will be fed into the cross-coupling PI controller to generate an error corrective signal to be distributed between both motor position control loops to eliminate synchronisation errors and hence to achieve zero transient response and zero steady state errors, in the system position output response.

As a result of implementing the suggested technique, the following features are expected:

a- Zero transient response and steady state errors.

b- Correct system coordination, synchronisation and balance at any given time.

c- An improvement in the accuracy of the robot desired output position with the presence of continuous disturbances that affect the control system, from either external or internal sources.

4 Position Cross-Coupling Motion Controller and System Analysis

The motion control system of the two-wheeled mobile robot that is considered in this paper is composed of two independent motor control loops. The suggested cross-coupling control idea is used to couple and coordinate between both of them. Each motor control loop has its own PI controller. The new cross-coupling control idea has dual feedback and feed-forward signals with comparator and PI controller as has been shown in Fig.3 before. All control modules are modelled in the discrete domain using Digital Signal Processing (DSP).

4.1 BLDC Motor Position Transfer Function

A three-phase permanent magnet brushless DC motor is considered. The BLDC motor consists of two parts, electrical and mechanical. The electric circuit and free-body diagram of the 3-Phase BLDC motor are shown in Fig.4 [8].

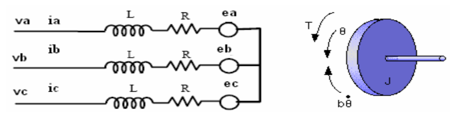

Fig.4 BLDCM electric circuit & Free-Body diagram

However, in this type of motor, only two of the three phases conduct simultaneously at any time. Therefore, the equivalent electrical circuit for BLDC motor model is as shown in Fig.5 below.

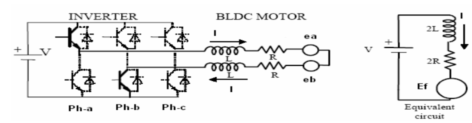

Fig.5 BLDC Motor equivalent circuit when only 2-pheses conducting

From the circuit above, the BLDC Motor Position Transfer Function in S-domain can be derived, which looks like:

$$\frac{\theta(s)}{V} = \frac{K_t}{S[2(R+LS)(b+JS)+K_e K_t]} \qquad (1)$$

Where the angular position (θ) is the output and the voltage (V) is the input, R & L are the winding resistance and inductance respectively, J is the rotor inertia, b is the damping ratio of the mechanical system, K_e is the motor constant and K_t is the torque constant. For more details about BLDC Motor Model, see [4] [5].

4.2 Position Cross-Coupling Motion Control Technique and System Analysis

In this section, the mathematical description of the suggested technique is derived and discussed. Since external and internal disturbances that affect two independent motors are considered, the suggested position cross-coupling control technique shown in Fig.6 below is studied and analysed.

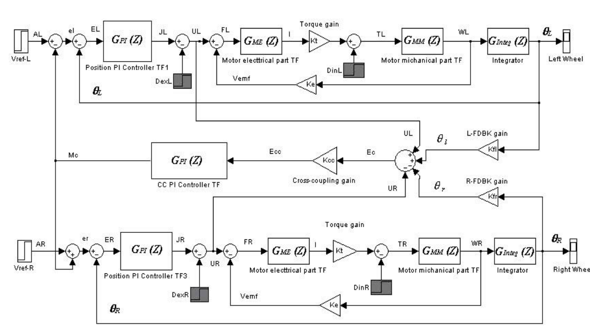

Fig.6 System mathematical components with External & Internal Disturbance parameters.

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By analysing the system in Fig.6, then:

$$J_L = E_L.G_{PI}(Z) = [A_L - M_C - \theta_L].G_{PI}(Z) \quad (2)$$

Where A_L is the reference input to the left control loop, M_C is the digital cross-coupling corrective signal fed to both loops, θ_L is the left loop output angular position, E_L is the tracking error signal sent to the position PI controller input, J_L is the output of the position PI controller signal and $G_{PI}(Z)$ is the PI controller discrete Transfer Function (TF).

After exertion the effect of external and internal disturbances, as shown in Fig.6, following formula can be obtained:

$$\begin{aligned} U_L &= J_L - D_{exL} \\ F_L &= U_L - K_e.W_L \end{aligned} \quad (3)$$

Where D_{exL} is the external disturbance affecting the left loop, U_L is the motor input effected signal, K_e is the motor constant and W_L is the left control loop output speed, which looks like:

$$W_L = F_L.K_t.G_{ME}(Z).G_{MM}(Z) - D_{inL}.G_{MM}(Z) \quad (4)$$

Where K_t is the torque constant, $G_{ME}(Z)$ is the motor electrical part discrete TF; $G_{MM}(Z)$ is the motor mechanical part discrete TF and D_{inL} is the internal disturbance affecting the left loop.

The angular output position of the left loop can be expressed as:

$$\theta_L = W_L \cdot G_{Integr}(Z) = F_L.K_t.G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z) - D_{inL}.G_{MM}(Z).G_{Integr}(Z) \quad (5)$$

Where θ_L is the angular output position of the left motor control loop and $G_{Integ}(Z)$ is the integrator transfer function.

By substituting equations (2), (3), (4) and (5) together, the following angular output position equation for the left loop is obtained:

$$\theta_L = \frac{K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).[A_L - M_C] - K_t.D_{exL}.G_{ME}.G_{MM}.G_{Integr}(Z) - D_{inL}.G_{MM}(Z).G_{Integr}(Z)}{1 + K_t.G_{ME}(Z).G_{MM}(Z)\left[G_{PI}(Z).G_{Integr}(Z) + K_e\right]} \quad (6)$$

Following the same procedure as before, the angular output position equation of the right loop can be obtained:

$$\theta_R = \frac{K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).[A_R + M_C] - K_t.D_{exR}.G_{ME}.G_{MM}.G_{Integr}(Z) - D_{inR}.G_{MM}(Z).G_{Integr}(Z)}{1 + K_t.G_{ME}(Z).G_{MM}(Z)\left[G_{PI}(Z).G_{Integr}(Z) + K_e\right]} \quad (7)$$

Where θ_R is the angular output position of the right motor control loop, A_R the reference input of the right control loop, D_{exR} is the external disturbance affecting the right loop, D_{inR} is the internal disturbance affecting the right loop and $G_{Integ}(Z)$ is the integrator transfer function.

To see the effect of the suggested position cross-coupling technique on the system, the output signal of the cross-coupling technique (M_C) must be found. The output cross-coupling corrective signal can be found as follows:

$$\begin{aligned} E_C &= (\theta_l - \theta_r) + (U_L - U_R) \\ \theta_l &= K_{fl}.\theta_L \\ \theta_r &= K_{fr}.\theta_R \\ E_{CC} &= K_{CC}.E_C \end{aligned}$$

Where E_C the cross-coupling error signal, and is obtained from the difference between the dual feed signals, K_{fl} is the left loop feedback gain and K_{fr} is the right loop feedback gain. These two gains (K_{fl} & K_{fr}) are introduced to allow the robot to move in curved paths. If the robot moves in a straight line, then both gains will be set to one, $K_{fl} = K_{fr} = 1$.

However, if the robot moves in a curved path, the centre of the robot must move along a circle of radius R. In this case these two gains can be expressed as: $K_{fl} = 1$ and $K_{fr} = \frac{1 - d/2R}{1 + d/2R}$

Where d is the distance between the two wheels and R is the radius of the curved path. The gain K_{CC} is the cross-coupling gain. This gain can be adjusted to obtain a zero difference between the transient responses of both loops in case of continuous disturbances.

Now, the output of the suggested position crosscoupling control technique is:

$$M_C = E_{CC}.G_{PI}(Z)$$

Where E_{CC} is the cross-coupling PI controller input signal and $G_{PI}(Z)$ is the PI controller discrete TF.

By substituting M_C in equations (6) and (7), final angular output position equations for both motor control loops using the suggested position cross-coupling technique can be expressed as:

$$\begin{aligned} \theta_L &= \frac{K_{cc}.K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{PIcc}(Z).G_{Integr}(Z).\left[K_{fr}\theta_R + (U_R - U_L)\right] + G_{MM}(Z).G_{Integr}(Z).\left[A_L.K_t.G_{ME}(Z).G_{PI}(Z) - K_t.D_{exL}.G_{ME} - D_{inL}\right]}{1 + K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z) + K_{cc}.K_t.K_{fl}.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).G_{PIcc}(Z) + K_e.K_t.G_{ME}(Z).G_{MM}(Z)} \\ &= \frac{K_{cc}.K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{PIcc}(Z).G_{Integr}(Z).\left[K_{fr}\theta_R + (J_R - J_L) + (D_{exL} - D_{exR})\right] + G_{MM}(Z).G_{Integr}(Z).\left[A_L.K_t.G_{ME}(Z).G_{PI}(Z) - K_t.D_{exL}.G_{ME} - D_{inL}\right]}{1 + K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z) + K_{cc}.K_t.K_{fl}.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).G_{PIcc}(Z) + K_e.K_t.G_{ME}(Z).G_{MM}(Z)} \end{aligned} \quad (8)$$

$$\begin{aligned} \theta_R &= \frac{K_{cc}.K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{PIcc}(Z).G_{Integr}(Z).\left[K_{fl}\theta_L + (U_L - U_R)\right] + G_{MM}(Z).G_{Integr}(Z).\left[A_R.K_t.G_{ME}(Z).G_{PI}(Z) - K_t.D_{exR}.G_{ME} - D_{inR}\right]}{1 + K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z) - K_{cc}.K_t.K_{fr}.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).G_{PIcc}(Z) + K_e.K_t.G_{ME}(Z).G_{MM}(Z)} \\ &= \frac{K_{cc}.K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{PIcc}(Z).G_{Integr}(Z).\left[K_{fl}\theta_L + (J_L - J_R) + (D_{exR} - D_{exL})\right] + G_{MM}(Z).G_{Integr}(Z).\left[A_R.K_t.G_{ME}(Z).G_{PI}(Z) - K_t.D_{exR}.G_{ME} - D_{inR}\right]}{1 + K_t.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z) - K_{cc}.K_t.K_{fr}.G_{PI}(Z).G_{ME}(Z).G_{MM}(Z).G_{Integr}(Z).G_{PIcc}(Z) + K_e.K_t.G_{ME}(Z).G_{MM}(Z)} \end{aligned} \quad (9)$$

From the output equations, (8) and (9), disturbances (external: D_{exL} & D_{exR}) and (internal: D_{inL} & D_{inR}) are considered. The difference between both loop command, $\pm(J_L - J_R)$, and disturbance, $\pm(D_{exL} - D_{exR})$, signals are coordinated between both motor control loops. Also, equations show that, internal disturbances affect the motor mechanical part and external disturbances affect both electrical and mechanical parts. Moreover, the output of each loop is shared and computed in the other output loop, (θ_L & θ_R). Both equations illustrate that the suggested technique is robust, stable and rejects continuous disturbances (external and internal) that affect either one control loop or both.

5 Simulation Results

The complete system with the suggested position cross-coupling technique is modelled and simulated using MATLAB-SIMULINK. The system, shown in Fig.6, is simulated for the left and right loop position output shape responses. Results of the system output position step response are obtained successfully as the following:

a- The system is simulated without any disturbances. A step reference input signal is applied to the input of each control loop as shown in Fig.6 with zero external and internal disturbances. The position output step response of the system for both control loops is obtained as shown in Fig.7 below.

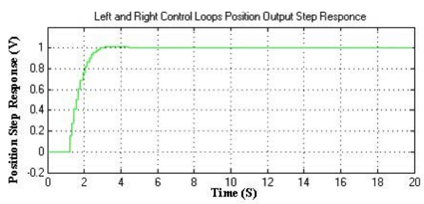


Fig.7 Both loops position output step response together, when Disturbances = 0

Fig.7 shows that the system has zero transient response and zero steady state errors in the case of no disturbances affecting the system.

b- The system is simulated for both control loops encountering external and internal disturbances simultaneously, at t=0s. The simulation result obtained for both loops together as shown in Fig.8.

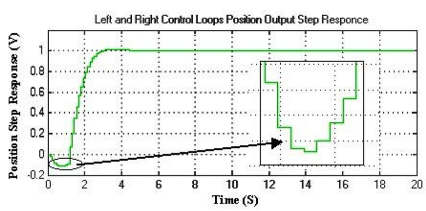


Fig.8 System position output step response when external and internal disturbances affected both loops at t=0s

Fig.8 shows, how robust the suggested technique is. It shows that the system has zero transient response and zero steady state errors despite continuous external and internal disturbances that affect each motor position control loop. Thus, the new cross-coupling control technique coordinates and synchronises both control loops and rejects continuous internal and external disturbances as expected. From the zooming in at t=0s, the suggested technique guarantees zero steady state error and zero difference between the transient responses of both loops (zero synchronization error).

c- For further investigation the system is simulated when both control loops encounter different disturbances at t=10s. Different external and internal disturbances affect both control loops with the new cross-coupling technique. The simulation result obtained is shown in Fig.9 below. By zooming in at t=10s, it can be seen that the suggested technique guarantees zero steady state and zero transient response errors even in presence of continuous external and internal disturbances that affect both motor control loops.

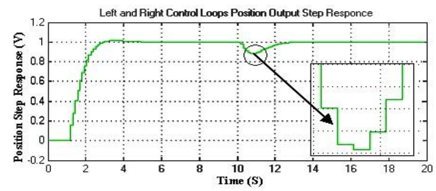
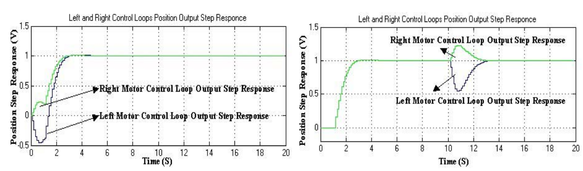


Fig.9 System position output step response when external and internal disturbances affected both loops at the same time at t=10s

d- To illustrate and prove the robustness of the suggested technique and to verify its accuracy, the system is simulated without including the suggested cross-coupling control technique. By repeating steps **b** and **c** without including the suggested Cross-Coupling Control (CCC) idea, simulation results are obtained as shown in Fig.10, (a) and (b).



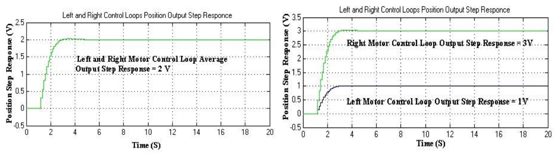
(a) (b)

Fig.10 System position output step response when both loops are encountered different disturbances without including the suggested CCC:
(a): at t=0 s and (b) at t =10s

Fig.10 shows that in presence of continuous external and internal disturbances that affect both motor control loops and without including the new Position CCC technique an error appears between both loops' transient responses. This synchronization error means that both motor rotor angular positions are not equal, which means one motor goes faster than the other, which will affect the robot desired output position accuracy. Fig.8, Fig.9 and Fig.10 verify that the suggested position CCC technique is robust and accurate.

e- To illustrate the effect of the system inputs on its outputs, the system is simulated with two different input set-points. The simulation results obtained are shown in Fig.11 below. From Fig.11, the output of the system using the suggested cross-coupling idea is equal to the formula:

$$V = \frac{V_L + V_R}{2}$$



(a) (b)

Fig.11 System position output step response with two different input set points
(a) With the suggested cross-coupling technique.
(b) Without the suggested cross-coupling technique.

f- As mentioned earlier in section (4.2), the suggested cross-coupling control technique has been designed by taking into consideration the movement of the mobile robot in a curved path by introducing two gains *(K_{fl} & K_{fr})* as shown in Fig.6. If these two gains are equal then the robot will move in a straight line otherwise the robot will move in a curved line according to the following equation.

$$K_{fl} = 1 \quad \text{and} \quad K_{fr} = \frac{1 - d/2R}{1 + d/2R} \qquad (10)$$

Where d is the distance between the two wheels and R is the radius of the curved path. To verify equation (10), the system is simulated to move in a curved path that has a radius *$R = 2$ metre* and the distance between the two wheels *$d = 0.5$ metre.* Fig.12 below shows that by applying equation (10), the angular position of the left motor is higher than the angular position of the right motor. That mains, the left motor goes faster than the left motor, which will cause the mobile robot moves in a curved path with *$R = 2$ metre* to the Right.

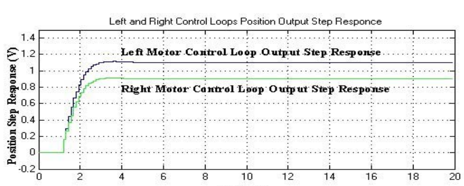


Fig.12 System moves in a curved path

From the simulation results, it can be seen that the system has zero transient response and zero steady state errors with the suggested position cross-coupling control technique. Also, they show that the system is accurate, robust, stable and rejects continuous external and internal disturbances as expected derived from the system mathematical output equations, (8) and (9), in section (4.2).

6 Conclusion

Accurate Synchronised Position Cross-Coupling Motion Controller (ASPC-CMC) for Two-Wheel Mobile Robot is suggested, analysed and modelled. A mathematical model for the proposed technique has been developed and simulated using MATLAB-SIMULIK. Simulation results have been achieved and presented. Simulation results verify the theoretical analysis and show that the suggested technique is accurate, robust and guarantee zero steady state and zero transient response (synchronizing) errors even in presence of continuous external and internal disturbances that affect both control loops. Furthermore, the ASPC-CMC rejects continuous internal and external disturbances and has the ability to control the movement of the mobile robot to all directions. The suggested technique is expected to improve the accuracy in the mobile robot desired position despite continuous disturbances that affect the system from either external or internal sources.

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