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Prediction of the Index Fund by Takagi-Sugeno Fuzzy Inference Systems and Feed-Forward Neural Network

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Abstract: - *The paper presents (on the basis of passive investment strategies analysis) the design of the Takagi-Sugeno fuzzy inference system and the feed-forward neural network (with pre-processing of inputs time series) for prediction of the index fund. By means of the Takagi-Sugeno fuzzy inference system and the feed-forward neural network the investor is able to predict the closing price of the index fund.*

Key-Words: - Takagi-Sugeno fuzzy inference systems, feed-forward neural network, prediction, index fund, indicators of technical analysis.

1 Introduction

Decision-making processes on the basis of fuzzy inference systems (FISs) [1,2,3] and neural networks (NNs) [3,4,5,6,7] are moving from technical scientific disciplines to non-technical spheres. An example of this is the prediction of price development of passively constructed portfolios in the field of economics [8], prediction of the gross domestic product development [9,10,11,12] or prediction of the environmental system parameters [13].

Basic types of the investment strategies used in capital markets can be defined on the basis of methodological analysis and approach analysis of the investment strategies. As an example we can mention the construction of the index fund, so that fund returns copy the development of market portfolio returns. The index fund can be modelled by means of FIS, which enables the prediction of price development of the index fund in t+Δt time. The indicators of technical securities analysis are used as the input in the trading system designed on the basis of FIS. Two basic FIS types can be designed on the basis of general structure of FIS [1,2,3], i.e. the Mamdani and the Takagi-Sugeno FISs. Both types differ in determination of the outputs. Different formulation of the outputs results in different construction of the rules. The user can construct the rules (on the basis of his own experience), or the rules can be obtained by extraction from the historical data. The feed-forward NNs [7] and the Takagi-Sugeno FISs [1,2,3,14] (with pre-processing of inputs time series) provide in comparison with econometric methods of prediction the advantage of superior processing of non-linear dependencies which may contribute to the improvement of prediction accuracy. The prediction accuracy can be described by root mean squared error δ.

Next parts of this paper introduce the design of the index fund for the prediction the closing price development of the index fund. The Takagi-Sugeno FIS and the feed-forward NN (with learning on the basis standard back-propagation algorithm) to predict the development of the closing price of the index fund (CPIF) can be realised by passive investment strategy by means of a indexation strategy in the way that CPIF copies the market development. Data pre-processing is realized by means of the indicators of technical analysis. The indicators of technical analysis represent the inputs to the Takagi-Sugeno FIS and the feed-forward NN. The Takagi-Sugeno FIS and the feed-forward NN are designed so that the investor is able to predict CPIF in t+Δt time. Passive investment strategy is suitable for risk-averse investors who plan to achieve the same (or similar) return as that of the market. In this case, passive investment strategy is realized by means of a indexation strategy. Next sections include the analysis designed models results and a short conclusion of the results obtained in this paper, as well as an overview of future research topics.

2 Index Fund Design

The investor tries to mix the portfolio of securities (the index fund), which copy the returns of the market index. It deals with historical prices of the securities of 35 companies when constructing the index fund (Table 1).

Table 1 The base of companies used for the construction of the index fund, where MC is market capitalisation and BF is the beta factor.

Company	MC[1] [bn. USD]	Industry	BF[2]
Alcoa Inc.	31	Metallurgical industry	0.6455
American Express Comp.	53	Financial services	0.0893
AT & T Corp.	**86**	**Telecommunications**	**0.8733**
Boeing Comp.	**54**	**Aviation industry**	**0.7351**
BP Amoco Plc.	190	Oil industry	0.4178
Caterpillar Inc.	15	Machinery	0.7932
Cisco Systems Inc.	**148**	**Network equipment**	**1.7259**
Citigroup Inc.	**240**	**Financial services**	**1.4757**
Coca-Cola Comp.	**128**	**Beverages**	**0.6688**
DuPont & Comp.	47	Chemistry	0.7009
Eastman Kodak Comp.	13	Photographic materials	0.4572
Exxon Mobil Corp.	291	Oil industry	0.4332
General Electric Comp.	**400**	**Electronics**	**1.1333**
General Motors Corp.	**32**	**Car manufacturer**	**0.8307**
Hewlett-Packard Comp.	**58**	**Hardware**	**1.1224**
Home Depot Inc.	**104**	**Retailing of building products**	**1.1164**
Honywell Internt. Inc.	25	Conglomerate	0.9386
IMB Corp.	160	Hardware	1.0544
Intel Corp.	**198**	**CPU manufacturer**	**1.5148**
International Paper Comp.	18	Wood-processing industry	0.6416
Johnson & Johnson	130	Drugs and cosmetics	0.6930
JP Morgan Chase & Comp.	87	Investment banking	0.0681
McDonald's Corp.	36	Fast food chains	0.6422
Merck & Comp.	**170**	**Pharmaceuticals**	**0.8054**
Merrill Lynch & Comp.	**45**	**Investment banking**	**1.6668**
Microsoft Corp.	**277**	**Software**	**0.7560**
Minnesota Mining Comp.	44	Conglomerate	0.5749
Pfizer Inc.	254	Pharmaceuticals	0.9532
Philip Morris Comp.	110	Tobacco industry	0.5979
Procter & Gamble Comp.	**88**	**Cosmetics & Toiletries**	**0.7021**
SBC Communications	151	Telecommunications	0.7076
SUN Microsystems Inc.	55	Network products	1.6707
United Technologies	36	Aviation industry	0.5994
Wal-Mart Stores Inc.	215	Retailing	0.3218
Walt Disney Comp.	**57**	**Entertainment**	**0.7883**

As the given portfolio should copy the development of the market, these companies are selected on the basis of three criteria:

- The BF of the given security is the first and the most important criterion. It measures the sensitivity of securities price development to the market movement. The index fund is constructed in such a way that its BF is approximately one. In this case the portfolio returns precisely copy the market returns.
- Selection of securities from different industries is the essence of the second criterion. The portfolio is constructed so that the companies within the same industry are not repeated in the portfolio.
- Significant position of the company in the industry is the third criterion. The position of the company in the industry is measured by its MC (i.e. number of issued securities multiplied by market price of one security).

On the basis of the given criteria, 15 companies were selected (indicated boldface in Table 1). Selection of these companies presents the construction of the indexation strategy. That is, the BF of the constructed portfolio approximates to value 1, therefore the portfolio returns copy the market returns. The index can be constructed for observation of the securities price development involved in the index fund. The index is constructed on the basis of daily data over the course of 4 years. It is a price-weighted index constructed as a simple average. The value 100 [points] is selected as an initial value of the index. The opening, highest, lowest and closing prices of securities and volume of the security are taken into account when calculating the index. The volume is defined as the number of traded shares on given day. The development of CPIF is shown in Fig. 1.

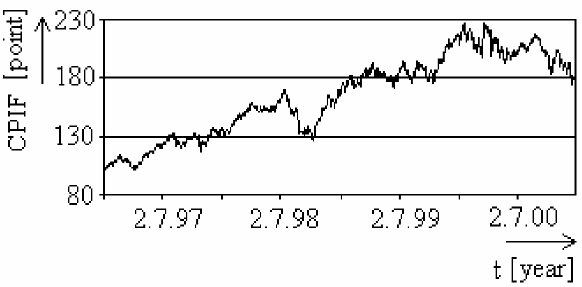

Fig. 1 The development of CPIF

Closing prices of the securities involved in the index fund can be used for calculation of the BF of the index fund. As it has already been mentioned, the entire index fund is constructed so that its BF is approximately 1. The BF of the index fund over the course of the selected period is shown in Fig. 2. It contains the relationship between daily returns of the S&P 500 index (DR_t S&P 500) and daily returns of the CPIF (DR_t CPIF). Generally, the returns of the S&P 500 index can be considered the market returns.

[1] Market capitalisation of companies is from 1.2.2004, www.bloomberg.com.

[2] The BF of the given security is calculated on the basis of daily data over the course of 4 years (from 2.1.2000 to 29.12.2003). The market portfolio return is calculated by means of the S&P 500 index [3].

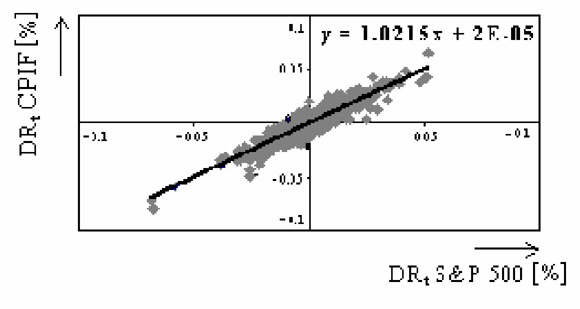

Fig. 2 The BF of the index fund

The BF of the index fund is calculated as a slope of the regression line between daily returns of the S&P 500 index and daily returns of the index fund. As it may be seen from Fig. 2, the BF of the index fund is 1.0215. That is, a 1 % change in the returns of the market portfolio (returns of the S&P 500 index) causes a 1.0215 [%] change in the returns of the index fund. Therefore, it can be concluded that the constructed index fund copies the market returns in compliance with the conditions of construction of the passive investment strategy.

3 CPIF Prediction Models

The closing, highest, lowest and opening prices and the volume of index fund (input time series) can be pre-processed by means of technical analysis indicators. The calculated indicators can be used as the inputs to the Takagi-Sugeno FIS and the feed-forward NN. The output of these models is CPIF in the next week (i.e. $\Delta t = 5$ trading days). Thus the indicators of technical analysis in time t (the inputs to the Takagi-Sugeno FIS and the feed-forward NN) are assigned to CPIF in $t+\Delta t$ time (the output of the Takagi-Sugeno FIS and the feed-forward NN). These models appear to be the most suitable with the following input variables (with pre-processing of inputs time series) [3]:

- indicator based on the calculation of moving average (MA) – 20 day MA of CPIF,
- standard deviation (STD) – 20 day STD,
- momentum (MOM) – 20 day MOM indicator,
- relative strength index (RSI) – 20 day RSI,
- William's % R indicator (W%R) – 20 day W%R.

The first model of the index fund prediction is presented by FIS. General structure of FIS (Fig. 3) contains the fuzzification process by means of input membership functions, construction of base rules (BRs) or automatic extraction of rules from the input data, application of operators (AND, OR, NOT) in rules, implication and aggregation within the rules and the defuzzification process of the obtained outputs to the crisp values.

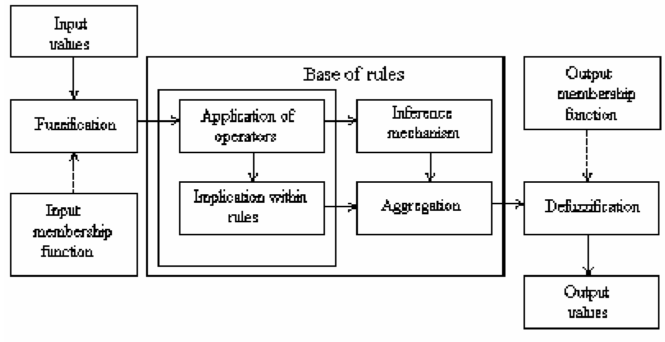

Fig. 3 General structure of FIS

Normalisation within the inputs and their transformation to the range of values of the input membership functions is realized during the fuzzification process. The inference mechanism is based on the operations of fuzzy logic and implication within rules [1,2,3]. Transformation of the outputs of individual rules to the output fuzzy set is realised on the basis of the aggregation process. Conversion of fuzzy values to expected crisp values is realized during the defuzzification process. There exists no universal method for designing the shape, the number and parameters of the input and output membership functions. The input to the fuzzification process is the crisp value given by universe of discourse (reference set). The output of the fuzzification process is the membership function value.

Construction of BRs can be realised by extraction of rules from historical data, if these are at disposal. Various optimisation methods for number of rules in BRs are presented in [1,2,3]. One of the possibilities for optimizing BRs from historical data is the so-called Adaptive Neuro-Fuzzy Inference System (ANFIS) method. The essence of this method is the neuro-adaptive learning process, by means of which it is possible to derive the parameters of membership functions and to extract BRs. The input data are mapped to the output data by means of this method, whereas the parameters of individual membership functions are gradually changed during the learning process, so that the relations between the space of the input variables and the output variables are described in the best possible way.

Let $x_1, x_2, \ldots, x_i, \ldots, x_n$ be the input variables defined in the reference sets $X_1, X_2, \ldots, X_i, \ldots, X_n$ and let y be the output variable defined in the reference set Y. Then FIS has n input variables and one output variable. Every set X_i, $i = 1, \ldots, n$, can be divided into p_j, $j = 1, \ldots, m$, the fuzzy sets $\mu_1^{(i)}(x)$, $\mu_2^{(i)}(x)$, ... ,$\mu_{pj}^{(i)}(x)$, ... ,$\mu_m^{(i)}(x)$. The individual fuzzy sets $\mu_1^{(i)}(x)$, $\mu_2^{(i)}(x)$, ... ,$\mu_{pj}^{(i)}(x)$, ... ,$\mu_m^{(i)}(x)$, $i = 1, \ldots, n$; $j = 1, \ldots, m$ represent assignment of linguistic variables relating to sets X_i. The set Y is also divided into p_k, k = 1, ... ,o the fuzzy sets $\mu_1(y)$, $\mu_2(y)$, ... ,$\mu_{pk}(y)$, ... ,$\mu_o(y)$. The fuzzy sets $\mu_1(y)$, $\mu_2(y)$, ... ,$\mu_{pk}(y)$, ... ,$\mu_o(y)$ represent the assignment of linguistic variables for the set Y. Then rule of the Mamdani FIS can be given as follows

IF x_1 is $A_1^{(i)}$ AND x_2 is $A_2^{(i)}$ AND ... AND x_n is $A_{pj}^{(i)}$ THEN y is B, (1)

where $A_1^{(i)}$, $A_2^{(i)}$, ... ,$A_{pj}^{(i)}$ represent linguistic variables corresponding to the fuzzy sets $\mu_1^{(i)}(x)$, $\mu_2^{(i)}(x)$, ... ,$\mu_{pj}^{(i)}(x)$, ..., $\mu_m^{(i)}(x)$, i = 1, ... ,n; j = 1, ... ,m and B represents the linguistic variable corresponding to the fuzzy sets $\mu_1(y)$, $\mu_2(y)$, ... ,$\mu_{pk}(y)$, ... ,$\mu_o(y)$, k = 1, ... ,o.

The Takagi-Sugeno FIS can be acquired by modification of the Mamdani FIS. These two types of FISs differ in the specification of the output membership functions. These membership functions are constant, linear or polynomial in the case of the Takagi-Sugeno FIS. The division of sets X_i, i = 1, ... ,n, into the fuzzy sets $\mu_1^{(i)}(x)$, $\mu_2^{(i)}(x)$, ... ,$\mu_{pj}^{(i)}(x)$, ... ,$\mu_m^{(i)}(x)$, i = 1, ... ,n; j = 1, ... ,m is the same in both types of FISs. Then the rule of the Takagi-Sugeno FIS can be given in the following way

IF x_1 is $A_1^{(i)}$ AND x_2 is $A_2^{(i)}$ AND ... AND x_n is $A_{pj}^{(i)}$ THEN y = h, (2)

where h is the constant. In this case the output membership functions are singletons. Fuzzy inference system containing rules defined by relation (2) is known as the Takagi-Sugeno FIS a zero-order. If the output membership functions are linear, than the rules of the Takagi-Sugeno FIS are the the following

IF x_1 is $A_1^{(i)}$ AND x_2 is $A_2^{(i)}$ AND ... AND x_n is $A_{pj}^{(i)}$ THEN y = f(x_1, ... ,x_n), (3)

where f(x_1, ... ,x_n) is the linear function. Fuzzy inference system containing rules defined by relation (3) is known as the Takagi-Sugeno FIS a first-order. In the case that f(x_1, ... ,x_n) is a polynomial function, it is the Takagi-Sugeno FIS a second-order.

The second model of the index fund prediction is presented by the feed-forward NN (Fig. 4), where Y is the output of the feed-forward NN, α is the synapse weight vector among neurons in the hidden layer and output neuron, β is the synapse weight vector among the input neurons and the neurons in the hidden layer, d is the activation function, X is the input vector.. The number of neurons in the input layer of the feed-forward NN depend on the input variable number (MA, STD, MOM, RSI, W%R). The neurons used in hidden layers consist of frequently applied logistic functions with the slope parameter equal to 1. The output neuron is linear.

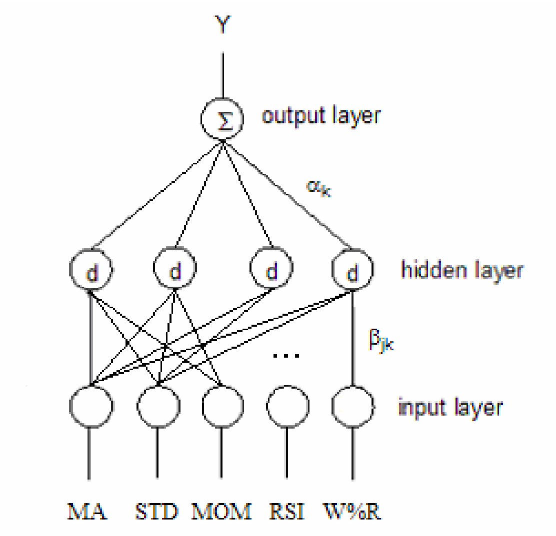

Fig. 4 The feed-forward NN

The membership functions of these input variables (MA, STD, MOM, RSI, W%R) have suitable economic interpretation. The output of the Takagi-Sugeno FIS and the feed-forward NN is the predicted closing price of the index fund (PCPIF) in 5 trading days. A FIS is shown in Fig. 5.

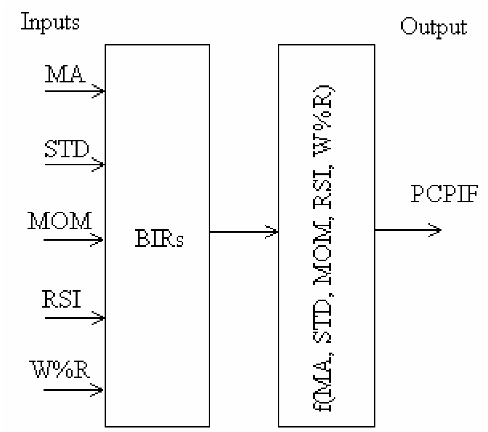

Fig. 5 The Takagi-Sugeno FIS

It is the Takagi-Sugeno FIS with 5 input variables (MA, STD, MOM, RSI, W%R), 3 rules (which are the result of BIRs extraction from the historical data within the optimisation process of FIS) and 1 output variable PCPIF. The input variable 20 day MA in time t is represented by means of 3 membership functions. They are bell membership functions. Individual membership functions are described by means of the linguistic variables value:

- Low_value_of_MA,

- Average_value_of_MA,
- High_value_of_MA.

The membership functions are shown in Fig. 6 The other input variables of the Takagi-Sugeno FIS, i.e. STD, MOM, RSI and W%R are similarly represented.

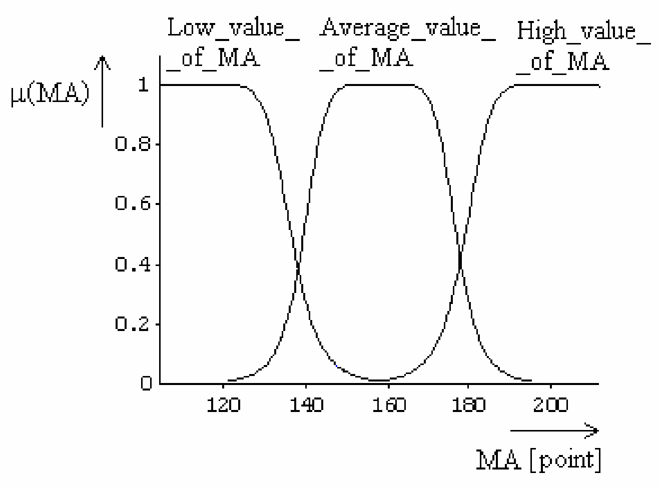

Fig. 6 The membership functions of 20 day MA

The output variable (PCPIF in time t+5) can be described by means of 3 membership functions. These membership functions are linear, because the FIS is of the Takagi-Sugeno. The coefficients of the output membership functions of the designed FIS are optimized by the ANFIS method. The output membership functions can be defined as follows:

μ_1(PCPIF): PCPIF = 1.001 * MA + 0.6824 * STD + 0.2329 * MOM – 0.1547 * RSI + 0.1478 * W%R – 9.202,

μ_2(PCPIF): PCPIF = 1.027 * MA + 0.496 * STD + 0.6416 * MOM – 0.1916 * RSI + 0.1006 * W%R – 56.01,

μ_3(PCPIF): PCPIF = 1.09 * MA – 0.4351 * STD – 0.7775 * MOM + 0.5834 * RSI + 0.03815 * W%R + 32.95.

The BIRs of the designed FIS consists of 3 rules extracted from the historical data. The individual rules are weighted by value 1 which means that all rules have the same influence on the output variable. The rules of the Takagi-Sugeno FIS are (for example) in the following form:

IF (MA is Low_value_of_MA) **AND** (STD is Low_value_of_STD) **AND**
(MOM is Average_value_of_MOM) **AND** (RSI is Average_value_of_RSI) **AND**
(W%R is High_value_of_W%R) **THEN** (PCPIF is μ_1(PCPIF)),

IF (MA is Average_value_of_MA) **AND** (STD is High_value_of_STD) **AND**
(MOM is High_value_of_MOM) **AND** (RSI is High_value_of_RSI) **AND**
(W%R is Average_value_of_W%R) **THEN** (PCPIF is μ_2(PCPIF)),

IF (MA is High_value_of_MA) **AND** (STD is Average_value_of_MA) **AND**
(MOM is Low_value_of_MOM) **AND** (RSI is Low_value_of_RSI) **AND**
(W%R is Low_value_of_W%R) **THEN** (PCPIF is μ_3(PCPIF)).

4 Analysis of the Results

The designed Takagi-Sugeno FIS and the feed-forward NN can be tested on the historical data. The development of CPIF and PCPIF in t+Δt time is shown in Fig. 7. The quality of the prediction is measured by means of the root mean square error δ and it reaching the value of 3.1624 [%] for the Takagi-Sugeno FIS and 2.1158 [%] for the feed-forward NN. The value of the indicator δ reveals the existence of several larger individual errors of the prediction.

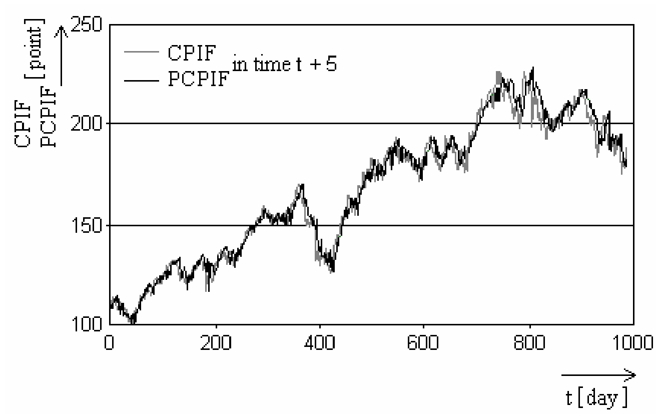

Fig. 7 The development of CPIF and PCPIF

The development over the course of 20 trading days of the month of January in 2004 can be described separately. They are trading days from 2. 1. 2004 to 30. 1. 2004.

The development of CPIF and PCPIF in time t+5 is shown in Fig. 8. The quality of the prediction for the Takagi-Sugeno FIS reaches the value of δ = 1.2043 [%] and for the feed-forward NN reaches the value of δ = 0.8939 [%].

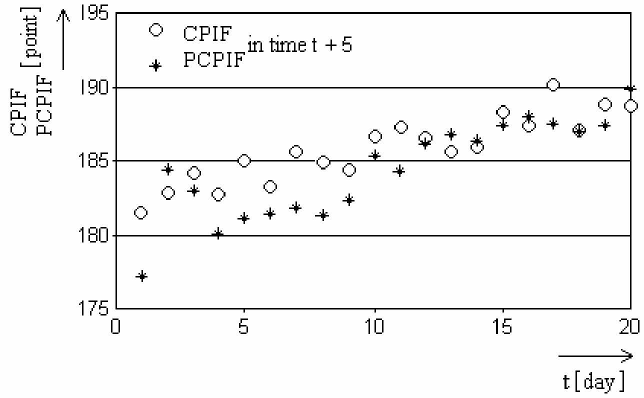

Fig. 8 The development of CPIF and PCPIF in the given period

5 Conclusion

The aim of the paper is to show the options of utilization of the Takagi-Sugeno FIS and the feed-forward NNs with learning by standard back-propagation algorithm (with pre-processing of inputs time series) in prediction of CPIF development by designing prediction models. The Takagi-Sugeno FIS is designed in a MATLAB environment and optimized by means of ANFIS method. The approach of rules exreaction from the historical data is employed when constructing BRs of the designed Takagi-Sugeno FIS. The feed-forward NN is designed in a SNNS-Stuttgart Neural Network Simulator. The presented results of the designed models prediction of the index fund are presenting typical results of the experiments. Further research will be oriented on designing the NN-frontal based models without pre-processing of inputs time series with learning by genetic and eugenic algorithms.

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