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A Computer-Aided Method to Obtain the Actual Maximum Amplitude for Limit Cycles in Recursive Digital Filters

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Abstract: - In this paper we present an alternative to exhaustive search algorithms for calculating the maximum amplitude of limit cycles in fixed-point digital filters. This algorithm allows us to obtain results quite similar to the corresponding exhaustive procedures but taken significantly shorter time of computing. This important improvement is obtained due to the normal behavior of high stability filters, that present all limit cycles in a confined area. The aim is to get this area as soon as possible since all limit cycles are placed there. The good performance of the algorithm is summarized in two different tables that compare the results with classical exhaustive formulations and other early algorithms. They are obtained after analyzing several filters corresponding to different structures, various approximation functions and under different quantizations conditions. This variety shows the general-purpose character of the algorithm.

Key-Words: - Limit cycles, quantization errors, rounding schemes, digital filter structures.

1 Introduction

It is well known that in all practical implementations of digital signal processing algorithms have to cope with finite register length. These lead to impairments of the filter performance that become visible in different ways. One of these ways, in the context of recursive filters realised with fixed-point arithmetic, is a permanent oscillation at the output and inner registers of the filters, called limit cycles.

The occurrence of these parasitic oscillations under zero input conditions is especially critical because the signal-to-noise ratio is affected dramatically. Therefore, predicting the behavior of the final digital filter implementation under finite wordlength conditions must be an integral part of a filter design procedure.

In particular, it is desirable not only to be able to predict if a digital filter is free or not of these limit cycles, i e, (G.A.S) [1] but also to know the maximum amplitude and period of them, in order to evaluate the actual performance of the chosen structure for the application.

However, the main problem of this subject is its random character, since it's impossible to know the inner operations previously.

So far, most of studies are analytic, where upper theoretical bounds have been given [1,2,3,4,7], but they normally lead to quite conservative results and in general are very difficult to obtain by computational methods. Therefore they are useful for certain forms, types of quantization and order of digital filters. In some cases an exhaustive search algorithm is suggested, where all possible initial states of the filters are tested until any of these theoretical bounds [1,7]. In this way the actual upper bound for limit cycles of digital filters is obtained and, it can be shown that in all cases they are quite smaller than its corresponding theoretical ones. This is due to analytic formulation in general considers the worst case analysis. But one serious problem encountered in these exhaustive methods is to carry out the procedure in a reasonably short computation time, since the test of each initial state until a great and conservative theoretical bound take long time. Besides with these treatments initial calculations of filters must be taken so most of them are only useful for certain structures or order of filters.

Other early formulations test only a strategic set of initial vectors for each filter [6].

This paper focus the study of these parasitic oscillations and present an alternative algorithm for detecting the maximum actual energy that the filter is able to keep in their inner registers in some limit cycle. Besides, this algorithm is independent to the order of the filter, type of quantization, structure and do not need any initial theoretical calculation with the filter.

However, this algorithm is able to characterize the impact of limit cycles due to in searching the upper bound, most of limit cycles are detected. So, because the fast performance, this algorithm can be used to take part of other algorithms where some techniques of optimizations are proved and where it is necessary to test several times the behavior of filters and the time is in concern.

In what follows, the algorithm flowchart is shown and at the end, some results with several filters, orders and type of quantizations are presented to confirm the good performance of the algorithm over other early formulations and theoretical standpoints.

2 Maximum Amplitude Algorithm

After several filters analyzed, it has been observed that limit cycles detected in most of them present low amplitudes and in general, are confined in very tight area. For example, the following figure shows that several cases of different structures of filters, under certain quantization conditions present all state vectors belonging to limit cycles confined in a region. Besides, as can be observed, the amplitude in each inner register (normalized by quantization step size $q=2^{B-1}$, where B is the number of bits used) is in general very low (except for filters with low stability margin).

Therefore, in this paper a new fast algorithm is proposed following this characteristic. So the aim is to reach as soon as possible the zone where limit cycles are placed by updating the state vectors to test according to the amplitude stored in limit cycles detected. In this zone it is likely that all limit cycles are detected and mainly the one that is able to store the maximum amplitude at inner registers. This will determine the actual bound searched, instead the theoretical one calculated in [1,2,3,4]. As we can see, the actual performance of the filter under certain quantizations conditions will guide to characterize these parasitic oscillations.

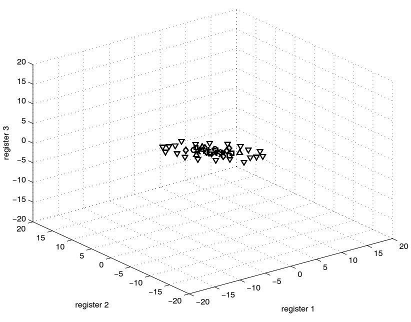

Fig. 1: state vectors reached in all limit cycles detected in a lattice in states space scaled eliptic digital filter under 16 bits in rounding truncation.

2.1 Algorithm formulation.

Due to the generic use of procedure, we represent a filter by its difference equations, which obtain the current value of each internal node of computation. With a convenient ordering of internal nodes we can express the difference equation in each internal node as:

$$x_j[k] = u[k] + \sum_{m=1}^{N}\left(a_{mj}\cdot x_m[k] + b_{mj}\cdot x_m[k-1]\right) \quad (1)$$

Where:

- N: number of internal computable nodes in the filter.
- j: Internal node of computation.
- $x_j[k]$: Amplitude of j^{th} node in the k^{th} iteration.
- b_{mj}, a_{mj}: Transmission coefficients of the branch connecting node m to j.
- $u[k]$: input signal injected in node j.

Working with fixed-point arithmetic precision any stored value is quantified to a integer value multiple of quantization step. So the form (1) turns to:

$$\hat{x}_j[k] = Q\left(\sum_{m=1}^{N}\left(\hat{a}_{mj}\cdot\hat{x}_m[k] + \hat{b}_{mj}\cdot\hat{x}_m[k-1]\right)\right) \quad (2)$$

where the process can be signed-magnitude, two's complement truncation or rounding and a double precision accumulator is considered. Also zero-input is considered.

In this way, any filter and type of quantization can be considered and we'll obtain the actual information about the filter behavior in a processor.

According to the considerations about the limit cycles behavior above, the following algorithm is proposed to detect the maximum bound of the limit cycles and to characterize the performance of the digital filters under any quantization conditions and any type of filter (structure, order, approximation...).

NOMENCLATURE

- B: number of bits used in implementation.
- $q=2^{-B+1}$: quantization step size.
- N: number of internal computable nodes in the filter.
- ℓ : number of storage nodes.
- $\gamma^{(i)} = \{\mathbf{x} \mid x_k \in \{+i,0,-i\}\}$; $\mathbf{x}=\{x_k\}$; $k=1\ldots\ell$, $i=1\ldots2^{B-1}$
- h: free parameter to guarantee the robustness of the algorithms.
- j: orbit formed by all states in limit cycle detected in test j, j=1 up to 2^{B-1}, as in fig. 1.
- $O=\bigcup_j O_j$ j=1 up to a maximum of 2^{B-1}.
- $M_i = \max\{|x_k|\}$; $x=\{x_k\}$, $k=1,2,\ldots\ell$, $x\in O$.
- $\mathbf{M}=\{M_k\}$; $k=1,2,\ldots\ell$. Practical bound calculated.

The flowchart in figure 2 shows the process for the algorithm.

The guided process consist in testing state vectors of increasing norms (only belonging to $\gamma^{(i)}$), form norm 1 as in exhaustive procedures [1,2], but whereas limit cycles are detected, the bound **M** is updated with new information of detected cycles.

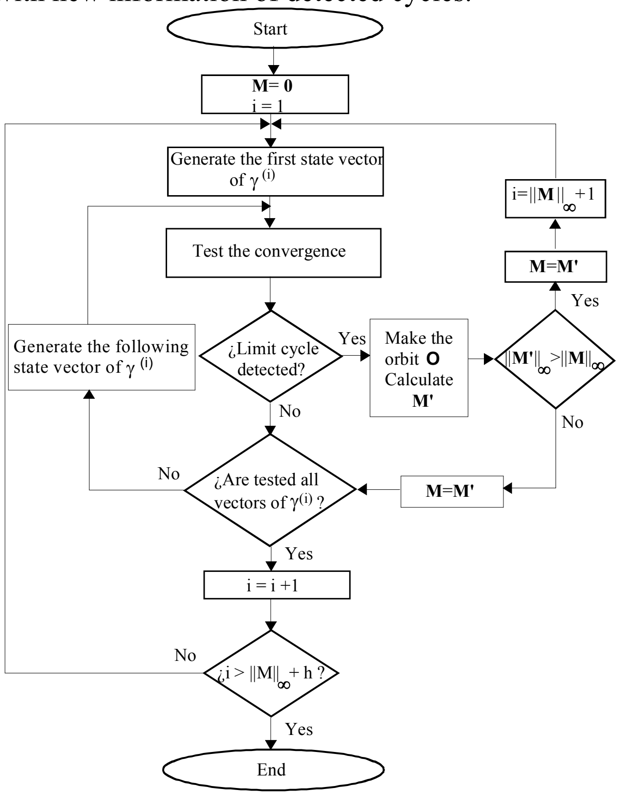

Figure 2: Flowchart of the proposed algorithm.

As show the figure, if the new norm is upper than previous one, is updated and force to prove state vectors of norm $i=||M||_{\infty}+1$. In this way is being tested the maximum amplitude that the filter is able to store with zero input. The process ends when have been proved state vectors of greater norm than the bound **M** plus the free h norms. This is due to, as in [1], all states with upper norm to a certain bound (actual bound of the filter and quantization) map to states lower that bound.

It is worthy of note that as greater the set of state tested of each norm, better the result, but in most cases where limit cycles are very tight is not necessary to test all vectors of each infinite norm and a compromise solution is only to test the initial vectors multiples to norm 1 ($\gamma^{(i)}$).

The box indicated as "Test the convergence" involves to test if the filter map to zero state or a limit cycle from the initial state considered evaluating the difference equation with zero imput. This process is developed by optimized DSP-Oriented algorithm shown in [5].

3 Results

In following tables some results are presented to demonstrate the good performance of the proposed algorithm over an exhaustive search [1] to detect the maximum bound of the limit cycles and even to characterize all limit cycles that appear in the filters, due to the fact that in most cases they are confined in a simple area.

As we can see, the algorithm are useful for any kind of filter, order, approximation, structure, type of quantization...

	Exhaustive	**Fixed Alg.**		**Prop.Alg.**	
		Time	**States**	**Time**	**States**
Filter 1	37	<1	416	<1	49
Filter 2	2021	2	1280	<1	130
Filter 3	>9469	8	3872	<1	197
Filter 4	>237770	34	11648	1	387
Filter 5	>83334	116	34976	2	777

Table 1: time in seconds and number of state vectors tested in fixed algorithm[6] and proposed. It has been used a Pentium IV 2,4 Ghz processor.

Table 1 shows the time required to perform the analysis and as we can see, the proposed algorithm time is significantly shorter than the corresponding analysis by exhaustive search and fixed algorithm [6]. Also we can see the guided character of the proposed algorithm by the different number of tested states depending on the actual performance of the filter instead the fixed algorithm that performs the analysis of a fixed set of vectors without taking account the actual behavior of the filter.

Also the table 1 shows that such exhaustive search lead to a loss of time because testing the convergence of all possible vectors up to a theoretical bound, usually very conservative. However, as these algorithms show, is sufficient to test only some strategic state vectors around an area. But as in general this confined zone is unknown, this algorithm is guided to detect it. As we can see the time in exhaustive search required in filter 4 and 5 seems some odd, but it is due to the algorithm used to detect each limit cycle from a state vector. In this case all analysis are realized by the algorithm proposed in [5], where the stability of the filter force the process speed.

Table 2 shows the good performance of the algorithm since no errors in these general proposed filters occur (besides, the amplitude in each inner register is presented). For simplicity all the values at the inner registers are expressed as integers multiples of the quantization step. In all cases 16 bits and saturation are used.

Besides it can be tested that the theoretical upper bound (obtained from the shorter of [2,3,4]) is quite greater than actual bound, so the actual maximum amplitude lead to the behavior of the filter.

	Theoretical Bound	**Exhaustive Alg.**	**Fix. (M)**	**Prop. (M)**
Filter 1 Two's C. Truncation Ladder WDF(Fetweis) [6] Butterworth	$\begin{bmatrix}16\\203\\16\end{bmatrix}$	$\begin{bmatrix}1\\49\\1\end{bmatrix}$	$\begin{bmatrix}1\\49\\1\end{bmatrix}$	$\begin{bmatrix}1\\49\\1\end{bmatrix}$
Filter 2 Rounding WDF CGIG Butterworth	$\begin{bmatrix}98\\9\\197\\11\end{bmatrix}$	$\begin{bmatrix}27\\0\\29\\0\end{bmatrix}$	$\begin{bmatrix}26\\0\\29\\0\end{bmatrix}$	$\begin{bmatrix}27\\0\\29\\0\end{bmatrix}$
Filter 3 Rounding Ladder WDF(Lawson) [5] Butterworth	$\begin{bmatrix}23\\24\\24\\24\\24\end{bmatrix}$	$\begin{bmatrix}3\\3\\3\\3\\3\end{bmatrix}$	$\begin{bmatrix}3\\3\\3\\3\\3\end{bmatrix}$	$\begin{bmatrix}3\\3\\3\\3\\3\end{bmatrix}$
Filter 4 Rounding WDF CGIG Direct Chebyhev	$\begin{bmatrix}21\\11\\51\\17\\72\\10\end{bmatrix}$	$\begin{bmatrix}4\\2\\8\\2\\6\\1\end{bmatrix}$	$\begin{bmatrix}4\\2\\8\\2\\6\\1\end{bmatrix}$	$\begin{bmatrix}4\\2\\8\\2\\6\\1\end{bmatrix}$
Filter 5 Two's C. Truncation Direct 2 Form Direct Chebyhev	$\begin{bmatrix}23\\24\\24\\24\\24\\24\\24\end{bmatrix}$	$\begin{bmatrix}4\\4\\4\\4\\4\\4\\4\end{bmatrix}$	$\begin{bmatrix}4\\4\\4\\4\\4\\4\\4\end{bmatrix}$	$\begin{bmatrix}4\\4\\4\\4\\4\\4\\4\end{bmatrix}$

Table 2: Comparative of Maximum bound obtained by theoretical calculations, exhaustive methods, algorithm in [6] and the proposed algorithm.

4 CONCLUSIONS.

The present algorithm is valid for the analysis of limit cycles in any digital filters under any type of quantization conditions. Besides, they are valid not only to detect the maximum actual bound of limit cycles but also, and given the normal configuration of these parasitic oscillations, to characterize the behavior of the filter.

The low time required to complete the process make it useful to take part of other algorithms to optimize the performance of the filters.

Besides, the guided character of the algorithms make unnecessary initial considerations of the filters, as in other exhaustive algorithms [1],[2]. This guided character implies that the time and number or vectors tested only are representative to compare algorithms with the same filter.

For the reasons above, is evident that exhaustive search algorithms up to certain conservative theoretical bound are inefficient for most filters.

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