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A Robot Cinematic Calibration Technique

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Abstract: - An algorithm for the cinematic calibration of a robot arm is presented. This technique uses the images of a couple of tele-cameras to obtain a stereoscopic vision. By a number of images, taken from the tele-cameras, and by measuring the joint parameters for each of the frame, it is possible to compute the other Denavit-Hartemberg parameters. It was found that by means of a suitable number of pictures, it is possible to obtain both the robot arm calibration and the camera system calibration.

Key words: Robot cinematic calibration, Vision system, Camera calibration

1 Introduction

Among the characteristics that define the performances of a robot the most important can be considered the repeatability and the accuracy. The repeatability represents the ability of the robot to bring its end-effector a number of time in a same assigned position, while the accuracy represents the ability to reach as exactly as possible an assigned position in the work space. Both these characteristics depend on fortuitous factors due to backlashes, load variability, positioning and zero putting errors, dimensional errors, limits of the transducers and so on. Some of these sources of error can be limited by means of the cinematic calibration. Basically, by the cinematic calibration it is assumed that if the error in the positioning of the robot's end effector is evaluated in some points of the working space, by means of these errors evaluation it is possible to predict the error in any other position thus offset it. In few words, the main aim is to obtain precise evaluations of the Denavit-Hartenberg parameters.

A calibration procedure can be divided in two main steps:

I. positioning and orientation error of the end-effector in a given number in the work space:

II. developing of a mathematic technique to predict and offset the errors.

The cinematic calibration techniques generally doesn't not consist in the direct measurement of the geometric parameters of the robot arm but needs the possibility to measure the end-effector position with a very high accuracy.

If is useful if the calibration technique can be applied to existing industrial robots and doesn't require to set up a complex device.

The calibration technique that we present is based on the employment of a vision system and uses a couple of tele-cameras.

2 The calibration technique

This calibration technique essentially consists in the following steps:

I. The end-effector is located in an even position in the work space;

II. A vision system acquires and records the robot's image and gives the coordinates of an assigned point of the end-effector, expressed in pixels on the image plane.

III. By means of a suitable software, from these coordinates expressed in pixels, the coordinates of the assigned point of the end-effector are computed in the word (Cartesian) frame.

IV. By means of the servomotor position transducers, the values of the joint position parameters are recorded for that end-effector position in the work space.

In this way, for each of the camera images, the following arrays are obtained:

$$\begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} , \begin{pmatrix} \theta_{1,i} \\ \theta_{2,i} \\ \theta_{3,i} \end{pmatrix} \qquad (1)$$

where: i = 1,…,N, and N is the number of acquired camera images (frames).

If the coordinates in the working space and the joint parameters are known, it is possible to write the direct kinematics equations in which the unknown are those Denavit-Hartenberg parameters that differ from the joint parameters. Thus these Denavit-Hartenberg parameters represent the unknown of the cinematic calibration problem.

The expression of these equations is obtained starting from the transform matrix (homogeneous coordinates) that allows to transform the coordinates in the frame i to the coordinates in the frame i-1:

$$^{i-1}A_i = \begin{bmatrix} C\vartheta_i & -C\alpha_i \cdot S\vartheta_i & S\alpha_i \cdot S\vartheta_i & a_i \cdot C\vartheta_i \\ S\vartheta_i & C\alpha_i \cdot C\vartheta_i & -S\alpha_i \cdot C\vartheta_i & a_i \cdot S\vartheta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

It is well known that by means of such matrixes it is possible to obtain the transform matrix that allows to obtain the coordinates in the frame 0 (the fixed one) from those in frame n (the one of the last link) : $^0T_n = {}^0A_1 \cdot {}^1A_2 \cdot \ldots \cdot {}^{n-1}A_n$.

As for an example, if we consider a generic 3 axes revolute (anthropomorphic) robot arm we will obtain an equation that contains 9 constant cinematic parameters and 3 variable parameters $(\theta_1, \theta_2, \theta_3)$.

So, the vector :

$$\pi_{DH} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ d_1 \\ d_2 \\ d_3 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \qquad (3)$$

represent the unknown of the cinematic calibration problem. Say:

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \qquad (4)$$

the direct kinematics equation for this manipulator can be written as :

$$w = t_4(\pi_{DH}, \Theta). \qquad (5)$$

where w is the position vector in the first frame and t_4 is the fourth row of the Denavit-Hartenberg transform matrix $^0T_3$. In eq.(5) it clearly appears that the position depends on the joint parameters and on the others Denavit-Hartenberg parameters. Again, eq.(5) can be seen as a system of 3 equations (in Cartesian coordinates) with 9 unknowns that are the elements of vector π_{DH}.

Obviously, it is impossible to solve this system of equations, but it is possible to use more camera images taken for different end-effector positions:

$$\begin{cases} t_4(\pi_{DH}, \Theta^1) = w_1 \\ t_4(\pi_{DH}, \Theta^2) = w_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ t_4(\pi_{DH}, \Theta^N) = w_N \end{cases} \qquad (6)$$

With $N \geq 9$.

As, for each of the camera images the unknown Denavit-Hartemberg parameters are the same, equations (6) represent a system of N non linear equations in 9 unknowns. This system can be numerically solved by means of a minimum square technique.

It is known at a minimum square problem can be formulated as follows:

Given the equation (5), find the solutions π_{DH} that minimize the expression:

$$\int_{D_\Theta} \left| t_4(\pi_{DH}, \Theta) - w \right|^2 \cdot d\Theta \qquad (7)$$

This method can be simplified by substituting the integrals with summations, thus it must be computed the π_{DH} vector that minimize the expression:

$$\sum_{i=1}^{N} \left| t_4(\pi_{DH}, \Theta^i) - w_i \right|^2 \qquad (8)$$

If we formulate the problem in this way, the higher is the number of images that have been taken (hence the more are the known Θ_i parameters) the more accurate will be the solution. So it is necessary to take a number of pictures.

Finally, the proposed calibration technique can be summarized as follows:

1. Locate a light led on the end-effector and take number N of pictures, each one of them in a different end-effector (hence joint) position.
2. Record, for each position, the values measured from the joint transducers.
3. Consider the generic matrix ^0T_n for a manipulator having that cinematic architecture.
4. Write the direct kinematics equation for each of the picture, considering as unknown the parameters that differ from the joint parameters.
5. Solve the system of equations by a minimum square technique and obtain a number of solutions.

6. Screen the solution by discarding those solution that appear to be incongruent with the given manipulator's structure.

3.1 The mechanical calibration and the telecamera calibration.

In a previous investigation on telecamera system modelling [1, 2] each one of a couple of tele-cameras was modelled by means of a matrix that was obtained from the model reported in fig 1; this puts in relation the coordinates of a generic point in spatial frame with its coordinates in a camera system image plane.
These [3x4] matrixes have the following expression, [2]:

$$M = \begin{bmatrix} \mu_1^T & \mu_{14} \\ \mu_2^T & \mu_{24} \\ -D^T/f & 0 \end{bmatrix},$$

(9)

$$M' = \begin{bmatrix} \mu'^T_1 & \mu'_{14} \\ \mu'^T_2 & \mu'_{24} \\ -D'^T/f' & 0 \end{bmatrix}$$

From the matrixes reported above, the problem of the stereoscopic vision was solved by the following system:

$$\begin{bmatrix} (u \cdot D + f \cdot \mu_1)^T \\ (v \cdot D + f \cdot \mu_2)^T \\ (u' \cdot D' + f' \cdot \mu'_1)^T \\ (v' \cdot D' + f' \cdot \mu'_2)^T \end{bmatrix} \cdot w = \begin{bmatrix} \mu_{14} \\ \mu_{24} \\ \mu'_{14} \\ \mu'_{24} \end{bmatrix} \qquad (10)$$

where:

- u, and v are the coordinates (expressed in pixel) of generic point in the image plane and have been already defined in [1, 2] as:

$$\begin{cases} u = -\dfrac{f}{D^T w}\left[\left(\dfrac{1}{\delta_u}\cdot\hat{\xi} - \dfrac{u_o}{f}\cdot D\right)^T w + \dfrac{1}{\delta_u}\cdot t_\xi\right] \\ v = -\dfrac{f}{D^T w}\left[\left(\dfrac{1}{\delta_v}\cdot\hat{\eta} - \dfrac{v_o}{f}\cdot D\right)^T w + \dfrac{1}{\delta_v}\cdot t_\eta\right] \end{cases};(11)$$

- w is the vector that represents the spatial frame homogenous coordinates of generic point;
- f and f' are the focal lengths of two cameras;
- D and D' are two vectors, defined in [1], by means of which is possible to describe the perspective transformation.

The equation (10) represents the stereoscopic problem and consist in a system of 4 equation in 3 unknown (w_x,w_y,w_z); this system can be solved by a least square algorithm.
The elements of the matrixes M and M' depend on several parameters. If the model of thin lenses is adopted [1, 2] these parameters are:

- The versors of the frames of the cameras.
- The vectors of the translation from one of the frames above to the other one.
- The optical parameters of the cameras (with a thin lens model, just the focal length).
- The pixel dimensions (image resolution)

The camera calibration consists, hence, in the evaluation of the parameters above; once they are known, the camera model is applicable.
In order to calibrate the tele-cameras a technique similar to the one for the mechanical calibration could be used; in this case the unknown are the camera parameters and the known terms are the vectors and w_i the images of them m_i. So, it is evident that it is necessary to know the vectors w_i.
This problem can be solved by gathering both cameras and cinematic calibrations in a single calibration problem.

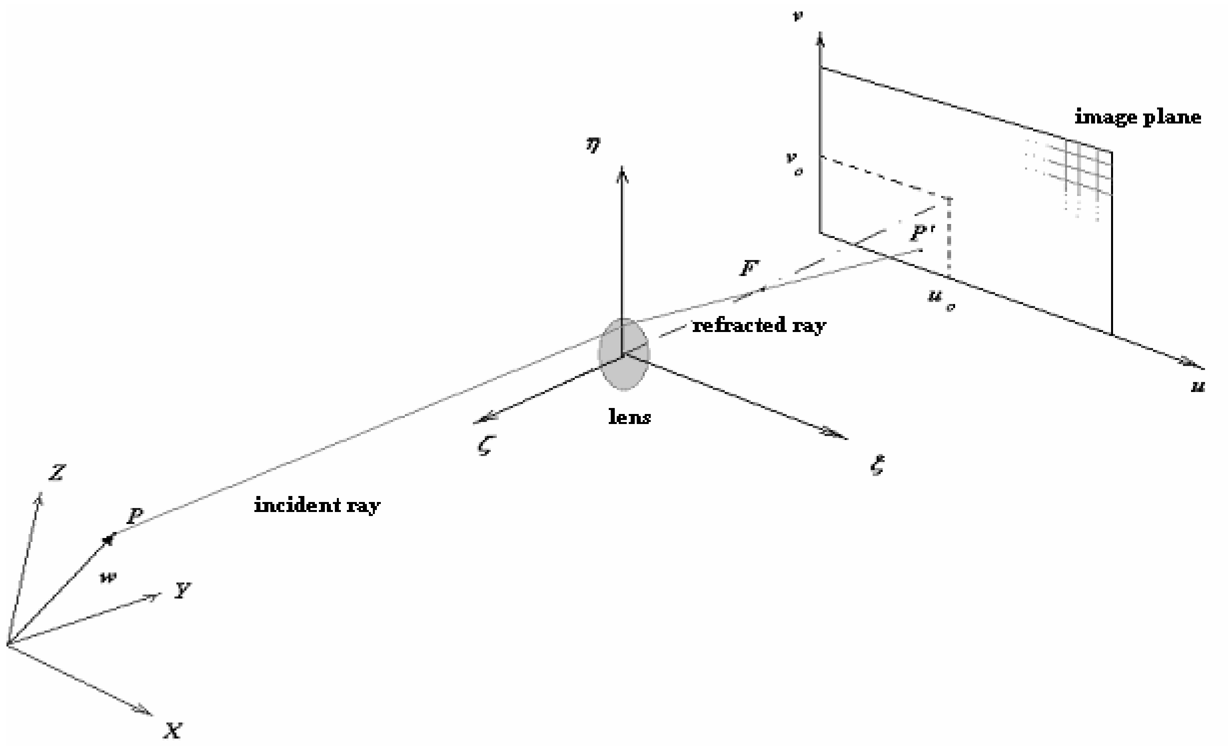

fig.1

Say π_c, π'_c the parameters of the tele-cameras, equation (10) can be briefly written as follows:

$$Q(\tilde{m}, \tilde{m}', \pi_c, \pi'_c) \cdot w = q(\tilde{m}, \tilde{m}', \pi_c, \pi'_c) \quad (12)$$

where Q and q are, respectively, the matrix and the vector in eq.(10).
If equation (5) is substituted in eq.(12) we have:

$$Q(m, m', \pi_c, \pi'_c) \cdot t_4(\pi_{DH}, \Theta) - q(m, m', \pi_c, \pi'_c) = [0]$$
(13)

It can be easily observed that:

1. the vectors *m* and *m'* are known as they can be directly observed on the image.
2. the vector is known as it is obtained from the joint transducers.

Hence, in eq.(12) the unknown are the tele-cameras parameters and the Denavit-Hartenberg parameters.
As already told, for each of the robot arm position, it is possible to write 4 equations (eqs.(13)); so if a suitable number N of images is taken, it is possible to obtain a system of 4N equations and solve it by a least square algorithm.
Acting in this way, the cameras calibration and the cinematic calibration are gathered in a single calibration problem that allows the evaluation of all the system's parameters.

3.2 The number of unknown parameters

As N images allow to obtain 4N equations, it is necessary to determine the minimum number of parameters that are necessary to define the problem, that is to say: the number of unknown.

3.2.1 The parameters of the camera

The camera orientation in the robot's word frame can be defined in a simple way by the Eulerian angles (φ, θ, ψ). If is useful to locate the frame of the camera so that the versor that is normal to the image plane is parallel to the optical axis; in this way the component of this versor are the cosines of the Eulerian angles.
The translation of the robot's word frame ad the camera' frame can be defined by 2 parameters, because (see eq.(11) [1, 2]) the component ζ is not necessary.
The focal lenght is a known parameter.
As for the pixel dimensions, it has be observed wat follows:
If we consider the robot arm in the reset position, that is to say: all the joint parameters are equal to zero, the direct cinematic equation can be written as follows:

$$t_4^0(\pi_{DH}) = t_4(\pi_{DH}, 0) \qquad (14)$$

in fact, being equal to zero the vector , the end-effctor position depends only from the Denavit-Hartenberg parameters.
By using eq.(11) it comes:

$$\begin{cases} u_d = \dfrac{-f}{\delta_u} \cdot \left(\dfrac{\xi^T t_4^0(\pi_{DH}) + t_\xi}{D^T t_4^0(\pi_{DH})} \right) \\ v_d = \dfrac{-f}{\delta_v} \cdot \left(\dfrac{\eta^T t_4^0(\pi_{DH}) + t_\eta}{D^T t_4^0(\pi_{DH})} \right) \end{cases} \qquad (15)$$

Where u_d e v_d are the coordinates in pixel reset position. So it is possible to obtain the pixel dimensions :

$$\begin{cases} \delta_u = \dfrac{-f}{u_d} \cdot \left[\dfrac{\xi^T t_4^0(\pi_{DH}) + t_\xi}{D^T t_4^0(\pi_{DH})} \right] \\ \delta_v = \dfrac{-f}{v_d} \cdot \left[\dfrac{\eta^T t_4^0(\pi_{DH}) + t_\eta}{D^T t_4^0(\pi_{DH})} \right] \end{cases} \qquad (16)$$

From eqs.(16) it can be deduced that the pixel dimensions can be expressed by means of the Denavit-Hartenberg. In this way, the camera's parameters aren't independent and, hence, they don't represent unknowns for the problem.
Finally, the camera's parameters are five:

- 3 Euler's angles
- 2 translation components

As 2 cameras are utilised, we need 10 parameters.

3.2.2 The parameters of the robot

As for the cinematic calibration, if, for instance, a 3 axes revolute robot is considered, it is necessary to determine 9 Denavit-Hartenberg parameters. The unknown parameters will be 19 (10 for the cameras and 9 for the robot arm). In order to obtain a numerical solution, generally, it is opportune to consider a number of equation that is twice the number of the unknown; as for each position of the robot arm the pictures taken by the cameras allow to obtain 4 equations, if pictures are taken for 10 different robot arm's positions, 40 equations can be written.

4 Conclusions

An algorithm for the cinematic calibration of a robot arm has been formulated. The technique uses the images of a couple of tele-cameras in order to obtain a stereoscopic vision. A number of images is taken from a couple of tele-cameras while, for each of the frame, the joint parameters are measured by means of the servomotors encoders; from these measures, it is possible to compute the other Denavit-Hartemberg parameters.

It has been also found that by means of a suitable number of pictures, it is possible to obtain both the robot arm and the camera system calibration.

Next step of this investigation will consist in the experimental tests of the technique on a robot arm prototype designed and built in our department.

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