Compression of color image using nonlinear multiresolutions

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Abstract: A comparison between several linear and nonlinear multiresolution algorithms for image color compression is presented. Some nonlinear techniques adapted to the presence of edges emerge as good alternatives to classical linear schemes. The numerical stability of the schemes is analyzed.

Key Words: Multiresolution, interpolation, nonlinear schemes, color, compression, edges.

1 Introduction

In the last years, several attempts to improve the classical linear multiresolutions of wavelet type have led to nonlinear multiresolutions.

In [3]-[4], a new one dimensional nonlinear multiresolution called PPH multiresolution was presented.

The goal of the present paper is to perform a comparison of this new multiresolution with other classical as Interpolatory fourth order (LIN4) [6]-[7] or ENO fourth order (ENO4) multiresolutions [5]-[8]-[9]-[1]-[2] in the compression of color images [10].

In next section several experiments are analyzed.

2 Experiments with color images

In this section we test the PPH multiresolution scheme with two color images. The behavior of the algorithm, when we use color images, is compared with ENO4 and LIN4. We will see that ENO4 obtains worse results, but that PPH can improve LIN4..

The first image we consider, a girl, is a real image and the second a geometric.

Using a threshold $\epsilon = 5$ we obtain the table 1. In this table we can see the result of each algorithm, for each one of the bands of the RGB image.

It is possible to see in table 1, that the compression rate obtained, if we only look

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$\epsilon = 5$			
LIN4	Red band	Green band	Blue band
nnz	96144	90228	91878
r_c	0.0918	0.0862	0.0878
l_1	2.0453	1.930	2.113
l_∞	17.739	18.269	17.942
l_2	7.377	7.143	7.729
$PSNR$	39.451	39.592	39.249
ENO	Red band	Green band	Blue band
nnz	124048	114606	116857
r_c	0.1185	0.1095	0.1116
l_1	2.452	2.286	2.497
l_∞	166.67	231.11	111.00
l_2	15.279	13.773	14.854
$PSNR$	39.851	39.991	39.625
PPH	Red band	Green band	Blue band
nnz	101626	93459	95293
r_c	0.0971	0.0893	0.0910
l_1	1.945	1.827	2.021
l_∞	16.256	16.318	17.052
l_2	6.729	6.514	7.087
$PSNR$	39.851	39.991	39.625

Table 1: Girl Image : Number of non zero coefficients, compression ratio, l_∞, l_1 and l_2 reconstruction error norm, $PSNR$ quality index Peak Signal to Noise Ratio, multiresolution levels $L = 4$.

at the quality of the reconstructed image, is nearly the same for the three algorithms. Even though the quality reached by all the algorithms is the same, the table shows that the PPH method is the one that obtains a better relationship between the quality of the reconstructed image, measured through the PSNR [11], and the norms of the errors obtained after de reconstruction process. To emphasize this results, we can decrease the threshold.

$\epsilon = 25$			
LIN4	Red band	Green band	Blue band
nnz	10764	10126	9564
r_c	0.0102	0.0096	0.0091
l_1	6.200	5.674	5.656
l_∞	75.160	75.896	73.609
l_2	91.286	77.010	74.478
$PSNR$	28.526	29.265	29.410
ENO	Red band	Green band	Blue band
nnz	13603	12088	11422
r_c	0.013	0.0115	0.0109
l_1	6.206	5.456	5.483
l_∞	166.67	224.25	209.91
l_2	103.58	87.034	81.937
$PSNR$	27.977	28.733	28.996
PPH	Red band	Green band	Blue band
nnz	10690	10130	9491
r_c	0.0102	0.0096	0.0090
l_1	5,728	4,937	5,047
l_∞	77,044	71,702	77,715
l_2	81,867	64,311	63,654
$PSNR$	28,999	30,047	30,092

Table 2: Girl Image : Number of non zero coefficients, compression ratio, l_∞, l_1 and l_2 reconstruction error norm, $PSNR$ quality index Peak Signal to Noise Ratio, multiresolution levels $L = 4$.

Using the new threshold $\epsilon = 25$ we obtain the table 2. PPH obtain a better quality (PSNR) for all the bands, than LIN4. Furthermore, PPH obtains less non zero coefficients, i.e., a better compression rate. Looking at this table, we can also remark that PPH shows the best relationship between r_c, (the compression radius), and the quality of the reconstructed image. As before, ENO4 algorithm shows worse results than the other two algorithms. Looking at both tables 2, 1, it is easy to notice that different compression rates are obtained for each band, reaching a maximum for the Blue one. (This could be due to the color composition of the image that we are using).

In what follows we proceed doing the same numerical experiments on a color geometric image. Using a threshold of $\epsilon = 5$ we obtain the table 3. Looking at this table, it is easy to find out that the best result reached for this kind of images, is obtained using the PPH algorithm. It achieves a better compression ratio, translated in a less number of non zero coefficients. It is also the one that reaches the best quality level, measured through the PSNR. This observations are valid for all the three bands, Red, Green and Blue.

With the purpose of testing the behavior of the algorithm we increase the threshold parameter ϵ. With $\epsilon = 25$ we obtain table 4, where it is possible to see that the PPH exhibits again the best relationship between r_c and the quality of the reconstructed image. This is clearly seen if we look at PSNR ratio, that remains above the results obtained by the other methods, and at error norms, that reach the lower values among the methods we are comparing.

The threshold could be increased even more, to check the resulting effects on the final image. With the threshold parameter $\epsilon = 35$ we obtain table 5, where we can see that even though the threshold is high, the quality level of the resulting image is quite good, and the errors remain small.

$\epsilon = 5$			
LIN4	Red band	Green band	Blue band
nnz	14448	19075	16725
r_c	0.0138	0.0182	0.0159
l_1	0.1146	0.0828	0.1450
l_∞	14.956	12.515	13.332
l_2	0.2899	0.1755	0.3314
$PSNR$	53.507	55.687	52.926
ENO	Red band	Green band	Blue band
nnz	9245	11097	11169
r_c	0.0088	0.0106	0.0106
l_1	0.1202	0.2032	0.1524
l_∞	265.91	265.91	102.52
l_2	3.7336	9.6025	2.6969
$PSNR$	42.409	38.306	43.822
PPH	Red band	Green band	Blue band
nnz	8659	10788	10713
r_c	0.0082	0.0103	0.0102
l_1	0.0500	0.0285	0.0788
l_∞	14.384	16.607	14.265
l_2	0.1739	0.0984	0.2966
$PSNR$	55.727	58.197	53.408

Table 3: Geometric Image : Number of non zero coefficients, compression ratio, l_∞, l_1 and l_2 reconstruction error norm, $PSNR$ quality index Peak Signal to Noise Ratio, multiresolution levels $L = 4$.

3 Conclusion

Using tensor product strategies, it has been shown the possible applications of the PPH multiresolution transform in color image compression. It has been explored the resolution in the edges of the images, the robustness in the presence of noise and texture, and the compression ratio. It has been also

$\epsilon = 25$			
LIN4	Red band	Green band	Blue band
nnz	3793	6810	3259
r_c	0.0036	0.0065	0.0031
l_1	1.5414	2.1262	1.5052
l_∞	79.686	68.256	84.598
l_2	23.565	36.826	19.604
$PSNR$	34.408	32.469	35.207
ENO	Red band	Green band	Blue band
nnz	4831	7458	5251
r_c	0.0046	0.0071	0.0050
l_1	0.5251	0.5167	0.5605
l_∞	261.13	261.13	120.78
l_2	15.448	20.670	11.337
$PSNR$	36.241	34.977	37.585
PPH	Red band	Green band	Blue band
nnz	4324	6849	4470
r_c	0.0041	0.0065	0.0042
l_1	0.4263	0.2083	0.5096
l_∞	83.093	80.928	80.937
l_2	11.045	3.2072	10.076
$PSNR$	37.699	43.069	38.097

Table 4: Geometric Image : Number of non zero coefficients, compression ratio, l_∞, l_1 and l_2 reconstruction error norm, $PSNR$ quality index Peak Signal to Noise Ratio, multiresolution levels $L = 4$.

shown that the PPH algorithm obtains better results with geometric images than LIN4 or ENO4 do, reaching a better relationship between the compression ratio and the quality of the reconstructed image, when the geometric nature of the original image increases. Furthermore, instabilities at image edges are avoided, allowing better adaptation to discontinuities and avoiding also Gibbs effects

$\epsilon = 35$			
PPH	Red band	Green band	Blue band
nnz	2537	5209	2442
r_c	0.0024	0.0049	0.0023
l_1	0.5893	0.4030	0.7611
l_∞	106.360	122.570	84.222
l_2	15.834	10.926	16.974
$PSNR$	36.134	37.746	35.832

Table 5: Geometric Image : Number of non zero coefficients, compression ratio, l_∞, l_1 and l_2 reconstruction error norm, $PSNR$ quality index Peak Signal to Noise Ratio, multiresolution levels $L = 4$.

as well as other undesirable effects. Diffusion in the edges of the images, present in the results obtained using linear schemes, are also avoided. For real images we have obtained results similar to those of linear schemes.

The numerical experiments confirm that, as it is proven in [4], the PPH multiresolution transform is a nonlinear stable algorithm, that can be used for image compression purposes. This result should be compared with ENO multiresolution transform, which stability can only be assured using error control strategies.

Using the reconstruction via primitive function procedure [9], PPH interpolatory reconstruction can be adapted to the cell average framework, more suitable to image processing.

The results obtained with the RGB color model, can be adapted to other color spaces, to try to narrow the application field of this kind of algorithms. Other possible ways of investigation are opened such as video compression, image denoising or image zoom in the context of the subdivision theory.

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