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Error-control vs classical multiresolution algorithms for image compression

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Abstract: Multiresolution schemes within Harten's framework corresponding to interpolatory techniques are used for image compression. The numerical behavior of the error-control and classical algorithms are compared. Compression properties are demonstrated on various tests.

Key Words: Interpolatory multiresolution, nonlinear scheme, error-control, stability, compression.

1 Introduction

Given f^L a data where L stands for a resolution level, a multiresolution representation of f^L is any sequence of type $\{f^0, d^1, \ldots, d^L\}$ where f^j is an approximation of f^L at resolution $j < L$ and d^{j+1} stands for the details required to get f^{j+1} from f^j. Linear multiresolution representations of data, like the wavelet decompositions, are multiresolutions involving inter-resolution linear operators.

The efficiency of linear multiresolution decompositions is generally limited by the presence of edges. The numerically significant coefficients d_j^k are mainly those for which the wavelet support is intersected by such discontinuities.

In order to incorporate a *specific adaptive treatment of singularities*, the framework of Harten's multiresolution [11]-[12] has been developed. The advantage of this general framework lies in its flexibility, where the reconstruction operator plays a fundamental part. This framework makes possible to consider data-dependent reconstruction techniques, which are needed to obtain near to optimal data-denoising rates. Different types of settings can be considered depending on the linear discretization operator that produces the data, we refer to [5]-[6]-

[7] for more details. In this paper we consider the point value setting, because it is in this setting where the reconstructions are easily constructed. Using primitive functions we can obtain the associated reconstructions for other settings.

In the point-value framework, reconstruction means interpolation. Usually, the interpolation is performed data independent using polynomials (interpolatory wavelets). One option to obtain adaptation near singularities is to consider nonlinear (data dependent) interpolation. Theoretically, using Essentially Non-Oscillatory (ENO) schemes [13]-[14], we can obtain high order accuracy on all intervals without any singularity. The ENO interpolation uses piecewise polynomial reconstructions based on a stencil selection procedure that moves away from the singularities. The numerical results, in image compression, [3], reveal that these nonlinear reconstructions strongly outperform the more classical linear reconstructions in the case of piecewise smooth geometric images, but that they do not bring improvements for real images which contain additional texture.

Other the nonlinear multiresolution transform results from considering Piecewise Polynomial Harmonic (PPH) reconstruction techniques. Its “locality” (with centered stencil) leads to improvements specially when texture is present [4].

The stability analysis for linear prediction processes can be carried out using tools coming from wavelet theory, subdivision schemes and functional analysis (see [12], [6]), however none of these techniques is applicable in general when the prediction process is nonlinear.

In the nonlinear case, stability can be ensured by modifying the encoding algorithm. The idea of a modified-encoding to deal with nonlinear multiresolution schemes is due to Harten; one dimensional algorithms in several settings can be found in [11], [7], [5]. The goal of a modified-encoding procedure is to keep track of the accumulation error in processing the values in the multi-scale representation.

The aim of this paper is to analyze the compression properties of these error-control algorithms in comparison with the classical ones.

The paper is organized as follows: We recall in section 2 the discrete pointvalue framework for multiresolution introduced by Harten [12]. In next section, we review the 2-D interpolatory error-control algorithms. Finally, we present some numerical results in section 4.

2 The Interpolatory Multiresolution Setting

Let us consider a set of nested grids in $[0,1]$:

$$X^k = \{x_j^k\}_{j=0}^{J_k}, \quad x_j^k = jh_k, \quad h_k = 2^{-k}/J_0,$$

where $J_k = 2^k J_0$, J_0 some fixed integer. The point-value discretization

$$D_k : \begin{cases} C([0,1]) & \to V^k \\ f & \mapsto f^k = (f_j^k)_{j=0}^{J_k} = (f(x_j^k))_{j=0}^{J_k} \end{cases} \tag{1}$$

where V^k is the space of real sequences of length J_k+1. A reconstruction procedure for this discretization operator is any operator R_k such that

$$R_k : V^k \to C([0,1]); \quad \text{satisfying} \quad D_k R_k f^k = f^k, \tag{2}$$

which means that

$$(R_k f^k)(x_j^k) = f_j^k = f(x_j^k). \tag{3}$$

In other words, $(R_k f^k)(x)$ is a continuous function that interpolates the data f^k on X^k.

The most usual interpolatory techniques are polynomials. We can consider linear reconstruction techniques as data independent Lagrange interpolation but also data dependent interpolation as ENO reconstruction [5] and even full nonlinear reconstruction technique as the PPH reconstruction [4]. ENO and PPH schemes are designe to obtain good resolution near the edges of the image where linear schemes lose accuracy.

3 Multiresolution-based compression schemes with error-control

Multiresolution representations lead naturally to data-compression algorithms. The simplest data compression procedure is obtained by setting to zero all scale coefficients which fall below a prescribed tolerance. Let us denote

$$(\hat{e}^k)_{i,j} = \ \mathbf{tr}(e^k_{i,j}; \epsilon_k) = \begin{cases} 0 & |e^k_{i,j}| \leq \epsilon_k \\ e^k_{i,j} & \text{otherwise} \end{cases} \tag{4}$$

and refer to this operation as truncation. This type of data compression is used primarily to reduce the “dimensionality” of the data. A different strategy, which is used to reduce the digital representation of the data is ”quantization”, which can be modelled by

$$(\hat{e}^k)_{i,j} = \mathbf{qu}(e^k_{i,j}; \epsilon_k) = 2\epsilon_k \cdot \ \text{round} \ \left[\frac{e^k_{i,j}}{2\epsilon_k}\right], \tag{5}$$

where round $[\cdot]$ denotes the integer obtained by rounding. For example, if $|e^k_{i,j}| \leq 256$ and $\epsilon_k = 4$ then we can represent $e^k_{i,j}$ by an integer which is not larger than 32 and commit a maximal error of 4. Observe that if $|e^k_{i,j}| < \epsilon_k \Rightarrow \mathbf{qu}(e^k_{i,j}; \epsilon_k) = 0$ and that in both cases

$$|e^k_{i,j} - \hat{e}^k_{i,j}| \leq \epsilon_k. \tag{6}$$

By applying the inverse multiresolution transform to the compressed representation, we obtain $\hat{f}^L = M^{-1}\{\bar{f}^0, \hat{e}^1, \ldots, \hat{e}^L\}$, an approximation to the original signal $\bar{f}^L$. We expect the information contents of $\hat{f}^L$ to be very close to those of the original signal $\bar{f}^L$, and in order for this to be true, the stability of the multiresolution scheme with respect to perturbations is essential. Studying the effect of using $\hat{e}^k_{i,j}$ instead of $e^k_{i,j}$ in the input of M^{-1} is equivalent to studying the effect of a perturbation in the scale coefficients in the outcome of the inverse multiresolution transform.

Given a discrete sequence $\bar{f}^L$ and a tolerance level ϵ for accuracy, our task is to come up with a compressed representation

$$\{\bar{f}^0, \hat{e}^1, \ldots, \hat{e}^L\} \tag{7}$$

such that if $\hat{f}^L = M^{-1}\{\bar{f}^0, \hat{e}^1, \ldots, \hat{e}^L\}$, we have

$$\| \bar{f}^L - \hat{f}^L \| \leq C\epsilon \tag{8}$$

for an appropriate norm.

As observed by Harten [11], one possible way to accomplish this goal is to modify the encoding procedure in such a way that the modification allows us to keep track of the cumulative error and truncate accordingly.

In what follows we use a two-dimensional extension of the one dimensional algorithms in [11], [5] and the two dimensional tensor product in [2]. Given a tolerance level ϵ, the outcome of the modified encoding procedure is a compressed representation (7) satisfying (8). This enables us to specify the desired level of accuracy in the decompressed signal. A modified encoding procedure is designed keeping in mind the particular decoding procedure to be used.

We will consider truncation, but the algorithms are identical for another compression process.

For more details see [1].

4 Comparison of the compression properties

In this section we perform a comparative study using the PSNR (Peak Signal Noise Ratio) quality image indicator [16]. We recall that for an 8 bit image $(0-255)$,

$$PSNR = 20\log_{10}\left(\frac{255}{||f^L - \hat{f}^L||_{l_2}}\right)$$

We consider the photo 1.

Figure 1: ‘ *Cameraman,* 257×257 ’

We consider the separable case. In tables 1, 2 and 3, we consider Lagrange, ENO and PPH multiresolutions respectively. We compare the compression capabilities of error-control and classical algorithms. We can observe that for a given level of quality the compression attained by error-control schemes is higher than the classical ones. Moreover, the ENO results with E-C improve the result using the linear Lagrange reconstruction.

PSNR	Lagrange	Lagrange (E-C)
30	9101	6285
35	13918	11386
40	20386	16878
45	31213	24909

Table 1: Number non-zero details, $L = 4$, separable, Lagrange

PSNR	ENO	ENO (E-C)
30	12272	5593
35	19002	10455
40	38875	15934
45	49891	24180

Table 2: Number non-zero details, $L = 4$, separable, ENO

PSNR	PPH	PPH (E-C)
30	7068	5215
35	12117	9768
40	18435	15386
45	31739	23655

Table 3: Number non-zero details, $L = 4$, separable, PPH

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