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Estimation Level Fusion in Multisensor Environment

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Abstract: **-** The integration and fusion of information, from a combination of different types of instruments (sensors), is often used in the design of control systems. Typical applications that can benefit the use of multiple sensors are industrial tasks, military command, mobile robot navigation, multi-target tracking, and aircraft navigation. In recent years, there has been growing interest to fuse multisensor data to increase the accuracy of estimation parameters and system states. This interest is motivated by the availability of different types of sensors having different spectrum characteristics. The observations, used in the estimation process, are assigned to a common target through association process. If it is decided that all local sensors observe the same target, then the next problem is how to combine (fuse) the corresponding local estimates?
A new algorithm, for estimation level fusion, is proposed that uses optimal mean square combination of arbitrary number of local estimates. Local estimates are produced by applying Kalman filter on individual sensor measurements.

Key-words: **-** Dynamic system, Kalman filter, Suboptimal filter, Data fusion

1. Introduction

A data fusion system is needed to estimate the state of system based on combined information from sensor measurements. Many of the techniques developed for data fusion attempt to emulate the ability of human to perform fusion [1]. Fused data from multiple sensors provides several advantages over data from single sensor. First, if several identical sensors are used, combining the observations will result in an improved estimate of the target position and velocity [2].
The ultimate target of well designed system is large gain of information with minimized and real-time processing. The integration and fusion of information is used in design of high-accuracy control systems. Multiple sensors in a system are used to attain the higher level of accuracy in real-time environment. Multiple sensors can be used in a system for different purpose like target tracking, guidance and surveillance, industrial and scientific applications [4], [5].

In [6], the fusion formula which represents an optimal mean square linear combination of local estimates with weights depending on cross-covariance of estimation errors was derived. In this paper we applied the fusion formula [6] in filtering problems for accurate state estimation, while the measurements are processed in parallel architecture for real time results.

This paper is organized as follows, in section 2, the filtering problem in multi-sensor environment is described with derivation of new filter from fusion formula. In section 3, the proposed filter is tested for state estimation of different dynamic systems in multisensor environment, section 4 contains concluding remarks.

2. Global Estimation Filtering Based On Fusion Formula

Consider a continuous-time linear dynamic system

$$\dot{x}_t = F_t x_t + G_t v_t, \quad t \ge 0, \qquad (1)$$

where $x_t \in \mathbf{R}^n$ is the state vector, $\mathbf{R}^n$ is an **n**-dimensional Euclidean space, $v_t \sim (0, Q_t)$ is the normal distributed white noise with zero mean and intensity Q_t. Suppose that the system has N sensors,

$$\begin{aligned} y_t^{(1)} &= H_t^{(1)} x_t + w_t^{(1)}, \quad y_t^{(1)} \in \mathbf{R}^{m_1}, \\ &\ldots \quad \ldots \\ y_t^{(N)} &= H_t^{(N)} x_t + w_t^{(N)}, \quad y_t^{(N)} \in \mathbf{R}^{m_N}, \end{aligned} \qquad (2)$$

where $w_t^{(i)} \sim (0, R_t^{(i)})$. We assume that the initial state $x_0 \sim N(\bar{x}_0, P_0)$, system noise v_t, and measurement noise $w_t^{(i)}$, $i = 1, \ldots, N$ are mutually uncorrelated.

State estimate is based on the *overall* measurements

$$Y_t = \left[y_t^{(1)^T} \quad \ldots \quad y_t^{(N)^T} \right]^T,$$

$$Y_t = \begin{bmatrix} H_t^{(1)} \\ \vdots \\ H_t^{(N)} \end{bmatrix} x_t + \begin{bmatrix} w_t^{(1)} \\ \vdots \\ w_t^{(N)} \end{bmatrix}. \qquad (3)$$

In case of limited computing and communication resources, the classical Kalman filter (KF) can not produce well-timed results as it requires observations, from all sensors, to computer state estimate, requiring higher communication bandwidth and computation. Here we show that the fusion formula [6] may serve as an alternative to solve this problem. The derivation of new filter is based on idea that the individual sensor measurements, $y_t^{(1)}, \ldots, y_t^{(N)}$, can be processed in decentralized or distribution fashion instead of centralized architecture.

Suppose we have N local estimates

$$\hat{x}^{(1)}, \ldots, \hat{x}^{(N)}, \qquad (4)$$

of a state vector x_t, obtained by applying Kalman filter on individual sensor measurements $y_t^{(1)}, \ldots, y_t^{(N)}$, respectively. Let associated local error covariances are

$$P_t^{(ij)} = \mathrm{cov}\left(e_t^{(i)}, e_t^{(j)}\right), \quad e_t^{(i)} = x_t - \hat{x}_t^{(i)}, \quad i, j = 1, \ldots, N. \qquad (5)$$

The global estimate representing the linear combination of local state estimates is calculated as

$$\hat{x}_t^{ge} = \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)}, \qquad \sum_{i=1}^{N} c_t^{(i)} = I_n, \qquad (6)$$

where I_n is the $n \times n$ unit matrix, and $c_t^{(1)}, \ldots, c_t^{(N)}$ are $n \times n$ constant weighting matrices determined from the mean square criterion

$$E\left\| x_t - \hat{x}_t^{(i)} \right\|^2 = E\left(\left\| x_t - \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)} \right\|^2 \right) \to \min_{c_t^{(i)}} \qquad (7)$$

The following theorem completely defines global estimate $\hat{x}_t^{ge}$ and its error covariance

$$P_t^{ge} = \mathrm{cov}\left(e_t^{ge}, e_t^{ge}\right), \; e_t^{ge} = x_t - \hat{x}_t^{ge}.$$

Theorem [6]. Let $\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}$ be the local Kalman estimates (LKE) (4) of an unknown state x_t. The weighting matrices $c_t^{(1)}, \ldots, c_t^{(N)}$ are given by

$$\begin{aligned} &\sum_{i=1}^{N} c_t^{(i)} \left[P_t^{(ij)} - P_t^{(iN)} \right] = 0, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n, \\ &j = 1, \ldots, N-1, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n. \end{aligned} \qquad (8)$$

Corollary 1. If $\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}$ are unbiased local estimates then the global estimate $\hat{x}_t^{ge}$ in (6) is unbiased too.

Corollary 2. The global estimate error covariance P_t^{ge} is given by

$$P_t^{ge} = \sum_{i,j=1}^{N} c_t^{(i)} P_t^{(ij)} \left(c_t^{(j)}\right)^T . \qquad (9)$$

According to (1) and (2), we have **N** dynamic subsystems ($i = 1, \ldots, N$) with the state vector $x_t \in \mathbf{R}^{\mathbf{n}}$ and the individual sensor $y_t^{(i)} \in \mathbf{R}^{\mathbf{m_i}}$:

$$\dot{x}_t = F_t x_t + G_t v_t ,$$
$$y_t^{(i)} = H_t^{(i)} x_t + w_t^{(i)}; \; i = 1, \ldots, N \quad (10)$$

where **i** (the number of subsystem) is fixed.

To find local state estimate $\hat{x}_t^{(i)}$ we can apply the optimal KF to the subsystem (10) [5], [7]. We have

$$\begin{aligned} \dot{\hat{x}}_t^{(i)} &= F_t \hat{x}_t^{(i)} + P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} \left[y_t^{(i)} - H_t^{(i)} \hat{x}_t^{(i)} \right], \\ \dot{P}_t^{(ii)} &= F_t P_t^{(ii)} + P_t^{(ii)} F_t^T - P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} H_t^{(i)} P_t^{(ii)} \\ &\quad + G_t Q_t G_t^T . \end{aligned} \qquad (11)$$

The global estimate, $\hat{x}_t^{ge}$, of the state x_t based on all LKEs (11) is computed by applying fusion formula (6), i.e.,

$$\hat{x}_t^{ge} = \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)} , \qquad \sum_{i=1}^{N} c_t^{(i)} = I_n . \quad (12)$$

The cross-covariance, $P_t^{(ij)}$, where $i \neq j$, for computation of time-varying weighting matrices $c_t^{(1)}, \ldots, c_t^{(N)}$, (8), is given by following differential equation:

$$\begin{aligned} \dot{P}_t^{(ij)} &= \left(F_t - P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} H_t^{(i)}\right) P_t^{(ij)} \\ &\quad + P_t^{(ij)} \left(F_t - P_t^{(jj)} H_t^{(j)^T} R_t^{(j)^T} H_t^{(j)}\right)^T \\ &\quad + G_t Q_t G_t^T , \quad i, j = 1, \ldots, N; \quad i \neq j. \end{aligned} \qquad (13)$$

Eqs. (8),(11)-(13) completely define *global estimation filtering based on fusion formula.*.

Remark 1. The LKEs for individual sensor measurements are calculated mutually independent in parallel processing scheme.

Remark 2. The proposed algorithm is robust i.e. eliminates the diverging LKE in computation of global estimate.

Remark 3. The proposed algorithm supports distributed, decentralized fusion systems.

3. Experiment

3.1. Damper Harmonic Oscillator with Multisensor Environment

Consider a two dimensional model of the harmonic oscillator [7]

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\alpha \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t , \quad t \geq 0 ,$$

where $x_t = \left[x_{1,t} \quad x_{2,t}\right]^T$ and $x_{1,t}$ is position, and $x_{2,t}$ is velocity, $v_t \sim (0, q)$, $x_0 \sim N(\bar{x}_0, P_0)$ represent normally distributed system noise and initial state.
The measurement model containing two sensors is given by

$$y_t^{(1)} = H^{(1)} x_t + w_t^{(1)} ,$$
$$y_t^{(2)} = H^{(2)} x_t + w_t^{(2)} ,$$

where $H^{(1)}$ and $H^{(2)}$ are 1×2 measurement matrices, $w_t^{(1)} \sim (0, r_1)$ and $w_t^{(2)} \sim (0, r_2)$ represent white noises; uncorrelated mutually and with v_t, and x_0.
The optimal KF is applied to measurement model for estimation

$$Y_t = H x_t + w_t ,$$

where

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix}, \quad H = \begin{bmatrix} H^{(1)} \\ H^{(2)} \end{bmatrix}, \quad w_t = \begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}$$

The global estimate is calculated based on

$$\hat{x}_t^{ge} = c_t^{(1)}\hat{x}_t^{(1)} + c_t^{(2)}\hat{x}_t^{(2)},$$

where $\hat{x}_t^{(1)}$ and $\hat{x}_t^{(2)}$ are local Kalman estimates based on the single sensor measurement models:

$$y_t^{(1)} = H^{(1)}x_t + w_t^{(1)},$$
$$y_t^{(2)} = H^{(2)}x_t + w_t^{(2)}.$$

This experiment deals with three programs of measurement:

Program 1. Position $x_{1,t}$ is measured by two sensors i.e.

$$H^{(1)} = [1 \quad 0], \quad H^{(2)} = [1 \quad 0];$$

Program 2. Velocity $x_{2,t}$ is measured by two sensors i.e.

$$H^{(1)} = [0 \quad 1], \quad H^{(2)} = [0 \quad 1];$$

Program 3. Position and velocity both are measured by sensor-1 and sensor-2 respectively i.e.

$$H^{(1)} = [1 \quad 0], \quad H^{(2)} = [0 \quad 1];$$

Comparison of estimation results and error covariance shows the behavior of optimal KF and the global estimation filter, let $\omega_n^2 = 0.64$, $\alpha = 0.16$, $q = 1$, $r_1 = 0.02$, $r_2 = 0.01$, and $P_0 = \mathrm{diag}[2 \quad 1]$. Mean square error (MSE) of optimal KF and global estimation filter is represented as,

$$P_t^{KF} = E\left[e_t^{KF}\left(e_t^{KF}\right)^T\right] = \begin{bmatrix} P_{11}^{KF} & P_{12}^{KF} \\ P_{12}^{KF} & P_{22}^{KF} \end{bmatrix}$$

where $e_t^{KF} = x_t - \hat{x}_t^{KF}$

$$P_t^{ge} = E\left[e_t^{ge}\left(e_t^{ge}\right)^T\right] = \begin{bmatrix} P_{11}^{ge} & P_{12}^{ge} \\ P_{12}^{ge} & P_{22}^{ge} \end{bmatrix}$$

where $e_t^{ge} = x_t - \hat{x}_t^{ge}$

Following figures show the robust nature of proposed filter, using measurement program 3. First sensor observing position while second is dedicated only for velocity observation.

In measurement program – 3, each sensor is observing different dynamic system component, accuracy of local estimate depends upon actual observance of respective sensor. Sensor – 1 originally observing position can make accurate local estimate of position rather than velocity as depicted in figure 1. The global estimate computed is not much affected because of robust nature of proposed filter i.e. weight of diverging estimate tends to zero causing less effect on global estimate. Individual local estimates are not reliable because of diverging behavior, spatial and temporal coverage limitation.

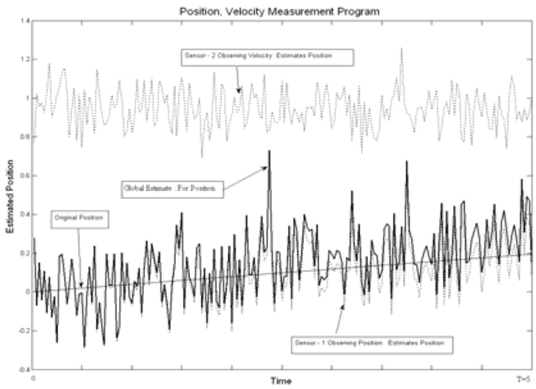

Figure 1: Comparison of Local, Global Position Estimation

Figure – 2 shows comparison of local estimates and global estimates for velocity. Its clear that sensor – 2, observing velocity, produces precise LKE while sensor -1 produces diverging LKE but in computation of global estimate diverging LKE is eliminated.

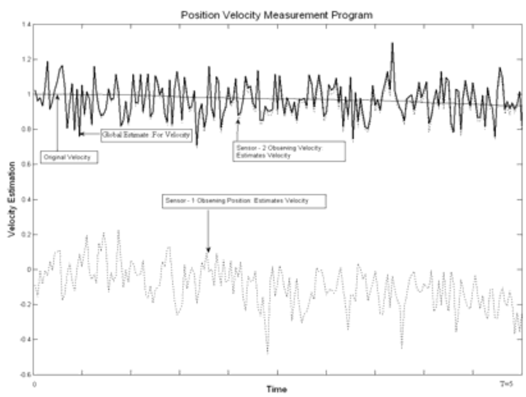

Figure 2 : Comparison of Local, Global Velocity Estimation

Figure – 3 represents MSE analysis of optimal KF and global estimation filter, we observe that differences between P_t^{KF} and P_t^{ge} are negligible especially for steady-state regime.

Figures 1-3 show that proposed filter exhibits efficiency i.e. quality of estimator increasing with number of measurements.

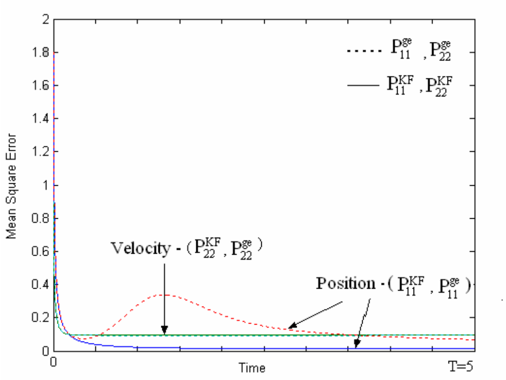

Figure 3: MSE Analysis of KF and GE Filter

3.2. Estimation of Constant Scalar Unknown with Different Number of Sensors

To estimate the value of unknown scalar "θ" using N- number of measuring devices while the continuous system model is

$$\dot{x}_t = 0, \quad t \geq 0, \quad x_0 = \theta$$

and measurement model (N- sensors) is

$$\begin{aligned} y_t^{(1)} &= x_t + w_t^{(1)}, \quad w_t^{(1)} \sim (0; r_1), \\ &\ldots \\ y_t^{(N)} &= x_t + w_t^{(N)}, \quad w_t^{(N)} \sim (0; r_N), \end{aligned}$$

where $w_t^{(i)} \sim (0, r_i)$ represent normally distributed white noise with zero mean and intensities $r_1, \ldots, r_N$. We also assume that unknown (initial state) $x_0 = \theta$ is normally distributed, i.e. $x_0 \sim N(\bar{x}_0; P_0)$ with mean $\bar{x}_0 = 0.5$, variance $P_0 = 1$.

According to the optimal KF equations, MSE - $P_t^{KF} = E\left[(x_t - \hat{x}_t^{KF})^2\right]$, is determined by

$$\dot{P}_t^{KF} = -\frac{1}{r} \cdot (P_t^{KF})^2, \quad P_0^{KF} = P_0, \quad 0 \leq t \leq T,$$

$$\frac{1}{r} = \frac{1}{r_1} + \ldots + \frac{1}{r_N}$$

And final solution becomes like

$$P_t^{KF} = \frac{r \cdot P_0}{r + t \cdot P_0}$$

The global estimate $\hat{x}_t^{ge}$ represents the weighted sum of the local Kalman estimates and MSE, $P_t^{ge} = E\left[(x_t - \hat{x}_t^{ge})^2\right]$, is determined by

$$P_t^{ge} = \sum_{i,j=1}^{N} c_t^{(i)} c_t^{(j)} P_t^{(ij)},$$

where the local error variances $P_t^{(11)}, \ldots, P_t^{(NN)}$, and the cross-covariance $P_t^{(ij)}$, $i \neq j$, are determined by the following differential equations:

$$\begin{aligned} &\dot{P}_t^{(ii)} = -\left(P_t^{(ii)}\right)^2 / r_i, \quad P_0^{(ii)} = P_0, \\ &\dot{P}_t^{(ij)} = -\left(P_t^{(ii)} / r_i + P_t^{(jj)} / r_j\right) P_t^{(ij)}, \quad P_t^{(ij)} = P_0, \\ &i, j = 1, \ldots, N; \quad i \neq j. \end{aligned}$$

The weights $c_t^{(1)}, \ldots, c_t^{(N)}$ satisfy linear algebraic equation

$$A_t c_t = b,$$

where the matrix $A_t = \left[(a_t)_{ij}\right]$, and the columns $\mathbf{c}_t$ and $\mathbf{b}$ take the form

$$(a_t)_{i,j} = \begin{cases} P_t^{(ij)} - P_t^{(jN)}, & i = 1, \ldots, N\text{-}1; \quad j = 1, \ldots, N \\ 1, & i = N; \quad j = 1, \ldots, N \end{cases}$$

$$\mathbf{c}_t = \left[c_t^{(1)} \quad \ldots \quad c_t^{(N)}\right]^T, \quad \mathbf{b} = [0 \quad \ldots \quad 0 \quad 1]^T.$$

Table - 1 shows the MSE analysis of two filters with different sensor combinations. The intensities of measurement noises are fixed as $r_1 = 0.2, r_2 = 0.1, r_3 = 0.06, r_4 = .004$ for simplicity. It is noticeable that difference between both MSEs decreasing not only with the increase in number of sensors but time too. Especially in steady-state regime difference in both MSEs becomes negligible. Some clever selection of types, number of sensors is recommended subject to availability of resources and accuracy needed.

Comparison Analysis MSE						
Time	2 Sensors		3 Sensors		4 Sensors	
	KF	GE	KF	GE	KF	GE
0.25	0.213	0.228	0.114	0.132	0.067	0.080
0.50	0.118	0.125	0.060	0.067	0.034	0.039
0.75	0.082	0.085	0.040	0.044	0.023	0.025
1.00	0.062	0.065	0.030	0.033	0.017	0.019
1.25	0.050	0.052	0.024	0.026	0.014	0.015
1.50	0.042	0.043	0.020	0.021	0.011	0.012
1.75	0.036	0.037	0.017	0.018	0.010	0.010
2.00	0.032	0.033	0.015	0.016	0.008	0.009
KF: Optimal Kalman Filter; **GE**: Global Estimate Filter						

Table 1 : MSE Analysis for Different Number of Sensors

4. Conclusion

In this paper, proposed algorithm is based on arbitrary number of LKEs fused by minimum mean square error criterion. The parallel structure of the filter can produce reliable results for real-time applications in fields like medical, industry, military, target tracking, guidance system etc. The proposed algorithm has support for decentralized, distributed sensor network architecture, with easy and economic future modification in network architectures.

The experiments show accuracy of proposed filter with increasing number of sensors and measurements.

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