A Novel Nearest Feature Space Classifier for Face Recognition

YAN CHEN¹, QIONG LI¹, XIAMU NIU¹, HONGZHI ZHANG², WANGMENG $ZUO²$, and SIMING $LIU³$ 1. Information Countermeasure Technique Institute Harbin Institute of Technology, Harbin 150001, China 2. School of Computer Science and Technology Harbin Institute of Technology, Harbin 150001, China 3. School of Mechanical Engineering and Automation Beijing University of Aeronautics and Astronautics, Beijing 100083, China cheny0451@gmail.com and zhanghz@hit.edu.cn

Abstract: - Nearest feature space (NFS) classifier had been proposed to enhance the limited prototypes' representation capacity. In this paper, we provide a formal definition and a theoretical analysis to Feature Space. Furthermore, a novel NFS classifier is proposed for face recognition. Experimental results using the ORL face database indicate that the proposed NFS obtains a better recognition performance than Chien's NFS, which means a larger prototype representation capacity.

Key-Words: - classifier design, nearest feature line (NFL), nearest feature space (NFS), face recognition

1 Introduction

As a widely applied identity authentication method, face recognition has received an increasingly investigations over the past decade. Due to the variations of face images in view point, illumination and expression, face recognition is facing great challenges, including both feature extraction and classification.

Classifier design is one of the most important issues in face recognition. Nearest neighbor classifier (NN) had been widely investigated because of its simplicity and effectiveness [1]. However, one serious limitation of NN is that its performance is deteriorated with the lack of available prototypes in each class. In literature [2], Li et al. proposed a nearest feature line (NFL) classifier to enhance the representation capacity of the prototypes. The NFL classifier extends the capacity of prototypes by exploiting a linear function to interpolate and extrapolate each feature pair belonging to the same class [3-5]. Therefore, in Li's method, any two prototype points belonging to the same class are generalized by the feature line (FL) passing through the two points [2].

Recently, Nearest Feature Space (NFS) and nearest feature plane (NFP) was introduced as a generalization of NFL by Chien's conclusion [6]. The NN, NFL, NFP, NFS are referred as the nearest feature classifiers with the nearest feature distance of point, line, plane, and space, respectively. As an extension of NFL, NFS extending the geometrical

concept from feature line to space accommodates a larger prototype's representation capacity.

Currently, there are still some problems in the classical NFS classifier. One disadvantage of the classical NFS is that the projection of a vector *x* into the feature space (FS) varies with the change of the coordinate origin. Besides, the FS generated by *n* prototypes should be an *n*-1 dimensional space, but for classical NFS the dimensionality is *n*.

In this paper, we present a correction of the classical NFS classifier and propose a modified NFS classifier with formal theoretical definition. Experimental results using the ORL face database show that the proposed NFS is superior to classical NFS in recognition performance.

2 Theoretical Analysis on NFS

Definition 1 [1]. Given two prototypes in one class x_1 and x_2 , an FL is defined as the straight line passing through x_1 and x_2 .

Theorem 1. Given an FL determined by two prototypes $\{x_1, x_2\}$ and a feature point x_n , x_n on $FL \Leftrightarrow x_n = \alpha x_1 + (1 - \alpha)x_2$, where $\alpha \in \mathcal{R}$.

Proof. Firstly, we prove that x_p on FL \Rightarrow $x_p = \alpha x_1 + (1 - \alpha)x_2$ where $\alpha \in \mathfrak{R}$. If x_p on FL, x_n can be represented as

$$
x_p = x_1 + \mu(x_2 - x_1), \tag{1}
$$

where $\mu \in \mathfrak{R}$. So,

$$
x_p = (1 - \mu)x_1 + \mu x_2.
$$
 (2)

Let $\alpha = 1 - \mu$, we can get x_n on FL \Rightarrow $x_n = \alpha x_1 + (1 - \alpha)x_2$.

Secondly, we prove that $x_n = \alpha x_1 + (1 - \alpha)x_2 \Rightarrow$ x_p on FL. If $x_p = \alpha x_1 + (1 - \alpha)x_2$, we can rewrite it as

$$
x_p = x_1 + (1 - \alpha)(x_2 - x_1) \tag{3}
$$

Let $\mu = 1 - \alpha$, we can obtain $x_p = x_1 + \mu (x_2 - x_1)$. So x_n is a feature point on FL.

From Theorem 1, we can give another equivalent definition to FL:

Definition 2. Given two prototypes x_1 and x_2 in one class, an FL determined by these two prototypes is the set of all feature point $x_p = \alpha x_1 + (1 - \alpha)x_2$ with the constraint $\alpha \in \mathfrak{R}$.

As a generalization of Definition 2, a formal definition of FS is given as follows:

Definition 3. Given *n* prototypes $\{x_1, x_2, \dots, x_n\}$ in one class, an FS determined by these *n* prototypes is the set of all feature points 1 *n* $_p - \sum_{i=1}^{\infty} \alpha_i \alpha_i$ $x_{n} = \sum_{i} \alpha_{i} x_{i}$ $=\sum_{i=1}^n \alpha_i x_i$ with the constraint $\sum_{i=1}^{n} \alpha_i = 1$ 1 $\sum_{i=1}^{n} \alpha_i = 1$ and $\alpha_i \in \mathfrak{R}$. *i* =

Although we have presented a formal definition of FS, it is still not easy to calculate the distance of a query feature point *x* to a FS according to Definition 3. So we continue our work by presenting a theorem on FS to make the calculation easier.

Theorem 2. Given an FS determined by *n* prototypes $\{x_1, x_2, \dots, x_n\}$, FS is equivalent to the space spanned by $\{x_1 - x_1, x_2 - x_4, \cdots, x_{n-1} - x_n\}$ with the origin x_μ , where 1 $(\sum x_i)$ *n i i* $x_{\mu} = (\sum x_i)/n$ $= (\sum_{i=1}^n x_i)/n$.

Proof. Firstly, we prove that $x_p \in FS \implies x_p$ is in the space spanned by $\{x_1 - x_1, x_2 - x_2, \cdots, x_n\}$ $x_{n-1} - x_{\mu}$ with the origin x_{μ} .

If
$$
x_p \in FS
$$
, x_p can be represented as
\n
$$
x_p = \sum_{i=1}^n \alpha_i x_i \text{ with } \sum_{i=1}^n \alpha_i = 1. \text{ Then,}
$$
\n
$$
x_p - x_\mu = \sum_{i=1}^n \alpha_i (x_i - x_\mu). \tag{4}
$$

Because
$$
x_{\mu} = \left(\sum_{i=1}^{n} x_i\right) / n
$$
,

$$
x_n - x_{\mu} = -\sum_{i=1}^{n-1} (x_i - x_{\mu}).
$$
 (5)

So we can obtain,

$$
x_p - x_\mu = \sum_{\substack{i=1 \ i=1}}^{n-1} \alpha_i (x_i - x_\mu) - \alpha_n \sum_{i=1}^{n-1} (x_i - x_\mu)
$$

=
$$
\sum_{i=1}^{n-1} (\alpha_i - \alpha_n) (x_i - x_\mu)
$$
 (6)

Thus x_p is in the space spanned by ${x_1 - x_{\mu}, x_2 - x_{\mu}, \cdots, x_{n-1} - x_{\mu}}$ with the origin x_{μ} .

Secondly, we prove that, x_n in the space spanned by $\{x_1 - x_{\mu}, x_2 - x_{\mu}, \dots, x_{n-1} - x_{\mu}\}\$ with the origin $x_u \Rightarrow x_v \in FS$. If x_p is in the space spanned by ${x_1 - x_{\mu}, x_2 - x_{\mu}, \cdots, x_{n-1} - x_{\mu}}$ with the origin x_{μ} , $x_p - x_u$ can be represented as

$$
x_p - x_\mu = \sum_{i=1}^{n-1} \beta_i (x_i - x_\mu) \tag{7}
$$

Let
$$
\alpha_n = (1 - \sum_{i=1}^{n-1} \beta_i) / n
$$
, then
\n
$$
x_p - x_\mu = \sum_{i=1}^{n-1} \beta_i (x_i - x_\mu) + \alpha_n \sum_{i=1}^{n-1} (x_i - x_\mu) - \alpha_n \sum_{i=1}^{n-1} (x_i - x_\mu)
$$
\n(8)

From Formula (5), Formula (8) can be rewritten as,

$$
x_{p} - x_{\mu} = \sum_{i=1}^{n-1} (\beta_{i} + \alpha_{n})(x_{i} - x_{\mu}) + \alpha_{n}(x_{n} - x_{\mu}).
$$
\n(9)

Let $\alpha_i = (\beta_i + \alpha_n)(i = 1,2,\dots, n-1)$, then

$$
\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n-1} \beta_i + n\alpha_n = \sum_{i=1}^{n-1} \beta_i + (1 - \sum_{i=1}^{n-1} \beta_i) = 1.
$$
\n(10)

So
$$
x_p = \sum_{i=1}^n \alpha_i x_i
$$
 and $\sum_{i=1}^n \alpha_i = 1$. Thus $x_p \in FS$.

3 The Proposed NFS

Based on Theorem 2, a novel NFS classifier is proposed in this section. Given *n* prototypes ${x_1, x_2, \dots, x_n}$, we establish an $m \times (n-1)$ matrix $P = [x_1 - x_\mu \quad x_2 - x_\mu \quad \cdots \quad x_{n-1} - x_\mu]$, where 1 $(\sum x_i)$ *n i i* $x_{\mu} = (\sum x_i)/n$ $=(\sum_{i=1}^n x_i)/n$ and *m* denotes the feature

dimension. Then the FS distance to a query feature point *x* is,

$$
d(x, FS) = ||(x - x_{\mu}) - x_{\mu}||_2, \qquad (11)
$$

where $\left\| \cdot \right\|_2$ is the Euclidean norm, and the projection point *xp* can be computed as

$$
x_p = P(P^T P)^{-1} P^T (x - x_\mu).
$$
 (12)

The proposed NFS approach classifies the query feature point *x* to class \hat{c} which has the minimum FS distance to *x*,

$$
d(x, FS_c) = \min_{1 \le c \le C_c} d(x, FS_c) \Rightarrow x \in \hat{c}, \quad (13)
$$

where FS_c is the FS distance to *x* of class *c*, and *C* is the number of classes.

Finally, we prove that NFL is just a specialization of the proposed NFS with the number of prototypes *n*=2, as follows.

Theorem 3 NFL is equivalent to the proposed NFS when the number of prototypes is 2.

Proof. Given two prototypes x_1 and x_2 and a query feature point x , the projection point x_{FL} can be computed by

$$
x_{FL} = x_1 + \frac{(x - x_1) \cdot (x_2 - x_1)}{(x_2 - x_1) \cdot (x_2 - x_1)} (x_2 - x_1),
$$
\n(14)

and the projection point x_{FS} can be calculated by

$$
x_{FS} = P(P^T P)^{-1} P^T (x - x_\mu), \qquad (15)
$$

where $P = x_1 - x_\mu$ and $x_\mu = (x_1 + x_2)/2$. So x_{FS} can be rewritten as,

$$
x_{FS} = x_1 + \frac{(x - x_1) \cdot (x_2 - x_1)}{(x_2 - x_1) \cdot (x_2 - x_1)} (x_2 - x_1) + x_{\mu}.
$$
\n(16)

Then it is easy to find

$$
d(x, FS) = ||(x - x_{\mu}) - x_{p}|| = ||x - x_{FL}|| = d(x, FL).
$$
\n(17)

So NFL is equivalent to the proposed NFS when *n*=2. Therefore, the proposed NFS is a generalization of NFL.

4 Experimental Results and Discussion

In this section, we use the ORL face database to evaluate the effectiveness of the proposed NFS classifier. We compared the recognition rate obtained by using proposed NFS with that obtained by Chien's NFS.

4.1 Data Set

The ORL database contains 400 facial images with 10 images per individual. Ten images of one person are shown in Fig.1.

The images vary in sampling time, light conditions, facial expressions, facial details (glasses/no glasses), scale and tilt angles. All the images are taken against a dark homogeneous background, with the person in an upright frontal position, with a tolerance for some tilting and rotation of up to about 20°. The size of these gray images is 112×92 [7].

Fig. 1 Ten Images of one Individual in the ORL Database

4.2 Experimental Results

For feature extraction, many principal component analysis or PCA-based approaches, such as Eigenfaces [8], have been very successful in image recognition. In this paper, we use a recently proposed Bi-Directional PCA (BDPCA) [9] approach, which has some significant advantages. First, BDPCA is directly performed on image matrix, while classical PCA is required to map an image matrix to a 1D vector in advance. Second, BDPCA can circumvent classical PCA's over-fitting problem. Third, the feature dimension of BDPCA is much less than 2DPCA. We use this approach for robust decision in presence of wide facial variations, to map the original sample to a 20×20 dimensional feature matrix. Then the proposed NFS classifier is explored for face recognition.

We randomly select *n* (3≤n≤9) samples per person for training while the rest for testing. To reduce the variation of recognition results, the averaged recognition rate (ARR) is adopted by calculating the mean of recognition rates over 50 runs.

Fig. 2 shows ARR obtained by Chien's and the proposed NFS classifiers with different *n* values. The ARR obtained by the proposed NFS is higher than that obtained by Chien's NFS for all *n* values. Thus the proposed NFS approach has a more steady theoretical foundation and better recognition performance than Chien's NFS.

Fig. 2 Comparison of Averaged Recognition Rate Obtained by Face Recognition Using Chien's and Proposed NFS

5 Conclusion

In this paper, we make a theoretical study on feature space, and then illustrate that Chien's NFS is not a real generalization of NFL. Moreover, a formal definition for NFS is present and discussed. Furthermore, we proposed a novel NFS classifier which accommodates a more efficient prototype representation capacity than Chien's. Then face recognition experiments are carried out on the ORL database. The results show that the proposed NFS obtains a better recognition performance than Chien's NFS.

Acknowledgments

This work is supported by the Foundation for the Author of National Excellent Doctoral Dissertation of P. R. China (Project Number: FANEDD-200238), the National Natural Science Foundation of China (Project Number: 60372052), the Multidiscipline Scientific Research Foundation of Harbin Institute of Technology. P. R. China (Project Number: HIT.MD-2002.11), National Program for New Century Excellent Talents in University and the Foundation for the Excellent Youth of Heilongjiang Province.

References:

- [1] T.M. Cover and P.E. Hart, Nearest Neighbor Pattern Classification, *IEEE Trans. On Information Theory*, Vol.13, Jan. 1967 , pp.21-27.
- [2] S.Z. Li and J.W. Lu, Face Recognition Using the Nearest Feature Line Method, *IEEE Trans. On Neural Networks*, Vol.10, No.2, 1999, pp.439-443.
- [3] S. Z. Li, Face Recognition Based on Nearest Linear Combinations, *Proceedings of CVPR*, 1998, pp.839-844.
- [4] S.Z. Li, Content-Based Audio Classification and Retrieval Using the Nearest Feature Line Method, *IEEE Trans. On Speech and Audio Processing*, Vol.8, No.5, 2000, pp.619-625.
- [5] S.Z. Li, Performance Evaluation of the Nearest Feature Line Method in Image Classification and Retrieval, *IEEE Trans. On PAMI*, Vol.22, No.11, 2000, pp.1335-1339.
- [6] J.T. Chien, and C.C. Wu, Discriminant waveletfaces and nearest feature classifiers for face recognition, *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol.24, No.12, 2002, pp. 1644-1649.
- [7] The ORL face database.. AT&T (Olivetti) Research Laboratories, Cambridge, U.K.. [Online]. Available: http://www.uk.research.att.com/facedatabase.h tml.
- [8] M.A. Turk and A.P. Pentland, "Face Recognition Using Eigenfaces," *Proc. IEEE Conf. Computer Vision and Pattern Recognition*, 1991, pp. 586-591.
- [9] W.M. Zuo, K.Q. Wang and D. Zhang, Bi-Dierectional PCA with Assembled Matrix Distance Metric, *ICIP 2005*, Vol.2, No.11-14, Sept. 2005, pp. 958 – 961.