Kalman filtering for drift correction in IR detectors

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Abstract: - In modern imaging systems that use thermal or infrared detectors, it is necessary to control the system temperature with accuracy, in order to avoid variations in the response of the detector. In this work, we propose a solution based on the implementation of the Kalman filter to correct the signal drift in a focal plane array (FPA), which consists of a mosaic of 16x16 of Lead Selenide (PbSe) infrared detectors. We show that the Kalman filter-based solution improves other existing design methods. The performance of the proposed method is demonstrated by simulated and real data.

Key-Words: - Detectors, IR, drift, Kalman filter.

1 Introduction

PbSe is a photoconductive detector sensitive to the radiation in the spectral range of 4.5 microns, MID-IR, [1], [2]. When this kind of detector is used, it is necessary to control its temperature with accuracy in order to avoid signal drift in the response of detector due to thermal instability. For this purpose, a thermoelectrical cooler based on Peltier effect, and a temperature controller (based on Proportional-Integral-Derivative (PID) regulator algorithm) are used. Nevertheless, in a low cost system, it is necessary to avoid the hardware associated to the temperature control, as well as to use low power circuitry.

The proposed technique in this work avoids the drift due mainly to the thermal instability in IR low cost systems, and is based on Kalman filtering [3]. The performance of the proposed algorithm is analyzed by using real infrared and simulated data. The block diagram of the system is presented in Figure 1:

Fig. 1. Block diagram

The rest of the paper is organized as follows. The problem formulation is given in Section 2. In Section 3, the Kalman filter is derived. In Section 4, the proposed technique is applied to simulated data. In Section 5, the technique is applied to real infrared data, and its performance is evaluated. The conclusions are given in Section 6.

2. Problem formulation

In IR detectors without temperature control, output signal of analog-digital converter in Figure 1 tends to drift temporally as a result of variations in the temperature of detector and the package. To observe this phenomenon, let us present an example: we have captured 64 sets of 1000 samples of real data, with a sample period of 1 ms, and the mean value of each set of data was calculated. Figure 2 depicts the drift of the signal.
One possible technique to correct the drift of the signal is by using a reference detector in order to cancel the drift of the rest of detectors in the FPA. However, the thermal variation does not affect in the same way to all detectors, so it is no possible to get a full satisfactory cancellation of the drift, just as is presented in Figure 3:

Therefore, a more sophisticated technique is needed when accurate correction is required. In this work, the proposed algorithm to cancel the drift of each detector is a Kalman filtering-based, and its performance is studied in section 3.

### 3. The Kalman filter

The solution to the problem of correcting the drift of the signal of a detector proposed in this work is based on the Kalman filter, as is presented in Figure 1. The Kalman filtering is used in applications with nonstationary processes, [3], [4], as in the case of the temporal randomly drift. This filter is derived following the general procedure given in ref. [3], [4]. The derivation can be outlined in four main steps: derivation of the predictor estimate of the state vector, derivation of the predictor estimate of the observation vector, derivation of the Kalman gain, and derivation of a recursive equations for the error covariance matrix.

Given the observations \( y[n] \) in presence of Johnson noise, \( v[n] \), i.e., \( y[n] = z[n] + v[n] \), the filter must estimate the signal drift, \( z[n] \), in order to cancel it.

#### 3.1 Filter equations

The output signal of ADC in Fig. 1 can be written as:

\[
y[n] = z[n] + v[n].
\]  

(1)

The drift signal to estimate is, in this case, \( z[n] \), and \( v[n] \) is zero mean Gaussian noise, i.e., the noise in the observations. Because \( z[n] \) is a signal with slow temporal variations, we adopt a model for this process follows:

\[
z[n] = z[n-1] + \alpha[n-1] + w[n],
\]  

(2)

where \( z[n-1] \) is a process that models the offset of \( y[n] \), i.e., a constant value obtained in the calibration process, and \( \alpha[n] \) is a process which we use to model the slow drift of the signal around the calibration point; in this way, we model the drift \( z[n] \) as the composition of two process, a constant \( z[n-1] \) and a ramp \( \alpha[n] \); besides, \( w[n] \) is a noise process to model the errors in the estimate of \( z[n] \). The covariance matrix of \( w[n] \) is:

\[
Q_w[n] = \begin{pmatrix} w_1 & w_2 \\ w_3 & w_4 \end{pmatrix},
\]  

(3)

where:

\[
w_1 = E(\sigma_z[n]^2), \ \text{covariance of } z[n]
\]  

(4)

\[
w_4 = E(\sigma_\alpha[n]^2), \ \text{covariance of } \alpha[n]
\]  

(5)

\[
w_2 = w_3 = E(z[n] \alpha[n]), \ \text{cross-correlation of } z[n] \text{ and } \alpha[n].
\]  

(6)

State equation:

\[
X[n] = \begin{pmatrix} z[n] \\ \alpha[n] \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z[n-1] \\ \alpha[n-1] \end{pmatrix} + w[n]
\]  

(7)

\[
\]  

(8)
Observation equation:
\[
y[n] = (1 \ 0) \begin{pmatrix} z[n] \\ \alpha[n] \end{pmatrix} + v[n]
\] (9)

\[
y[n] = C[n] X[n] + v[n]
\] (10)

According to the theory of the Kalman filter in [3], the algorithm is implemented as follows:

Initial conditions of the algorithm:
\[
\hat{x}[0 | 0] = E\{x[0]\} = 0
\] (11)

\[
\hat{\alpha}[0 | 0] = 0
\] (12)

\[
P[0 | 0] = E\{X[0]X[0]^T\} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\] (13)

Iterations for \(n=1,2, \ldots\):
\[
\hat{x}[n | n-1] = A[n-1] \hat{x}[n-1 | n-1]
\] (14)

\[
\hat{z}[n | n-1] = \hat{z}[n-1 | n-1] + \hat{\alpha}[n-1 | n-1]
\] (15)

\[
\hat{\alpha}[n | n-1] = \hat{\alpha}[n-1 | n-1]
\] (16)

\[
\] (17)

\[
P[n | n] = (1 \ 0)P[n | n-1](1 \ 0)^T + \begin{pmatrix} w_1 & w_2 \\ w_3 & w_4 \end{pmatrix}
\] (18)

\[
\] (19)

Kalman gains:
\[
\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = P[n | n-1] \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha^T
\] (20)

\[
\hat{x}[n | n] = \hat{x}[n | n-1] + k_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}[y[n] - C[n] \hat{x}[n | n-1]]
\] (21)

\[
\hat{z}[n | n] = \hat{z}[n | n-1] + k_2 \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}[y[n] - \hat{z}[n | n-1]]
\] (22)

\[
\hat{\alpha}[n | n] = \hat{\alpha}[n | n-1] + k_3 \begin{pmatrix} w_3 \\ w_4 \end{pmatrix}[y[n] - \hat{z}[n | n-1]]
\] (23)

\[
P[n | n] = \begin{pmatrix} 1 - k_2 \alpha[n] \end{pmatrix} P[n | n-1]
\] (24)

We assume that noise \(w[n]\) in Eq. (2) is smaller than noise in the observations, \(v[n]\), in Eq. (1), so the estimates of \(z[n], \hat{z}[n]\), are reliable.

In this way, we impose the next condition:
\[
w_1 \ll \sigma_v^2, \ w_1 = w_2.
\] (25)

Besides, we assume that \(z[n]\) and \(\alpha[n]\) are uncorrelated, i.e. \(w_2 = w_3 = 0\). In simulation and implementation of the filter, \(w_1 = 10^{-6}\). In this way, it is allowed a small error in the model of variation of \(z[n]\) process, i.e., a few distrust in the estimates is assumed, in the case of the drift does not match to the model in Eq. (2).

### 4. Simulations

In order to show the performance of our approach, we have tested it in two different set of data. First, we have simulated the behavior of the Kalman filter by means of synthetic data. The generated input signal consists of four different signals: a constant signal of value 30, a signal with slow drift, modeled as 20000 samples of a sinusoidal with low frequency and amplitude 10, a ramp signal with slope 0.5, and finally, a zero mean Gaussian noise with standard deviation 0.5. In addition, we have modeled two point targets at different instants.

Figure 4 (a) depicts the original signal, Figure 4 (b) depicts the estimated signal, and Figure 4 (c) shows the corrected signal.

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As observed in Figure 4, results obtained in the filter performance are satisfactory, because drift is estimated and canceled in a reliable way, whereas the targets are perfectly recovered.
On the other hand, in Figure 5 the effect of variation of raised time of estimate $\hat{z}[n]$ is presented. In this case, the drift signal is modeled as a step function. We consider different values of the noise variance in the observations, $\sigma^2_n$.

In Figure 5, it is observed that the raised time is increased with the noise variance in the observations; this effect is due to the distrust of the observations when $\sigma^2_n$ increases, so the filter takes a long time to reach the final value. Also, an oscillation in the amplitude of the estimate is observed, because we suppose in the simulation that $w_1 = w_4 = 0$. Thus, we assume the model in Eq. (2) correct, but this model is unmatched to the step function. To avoid the oscillation, a value $w_1 = 10^{-8}$ or less is valid.

5. Experiments with real data

We have extended the experiments with synthetic data, thus the Kalman filter was implemented and applied to real data. Figure 6 shows the infrared camera used in our experiments. In the experiments, we have used a room temperature FPA of 16x16 detectors.

In order to show the performance of our approach, we have tested it in two different set of data. In the first set of data, we have tested the behavior of the Kalman filter with the signal of one detector of the FPA when no target is present. The input signal consists of 4253 samples, with a sample period of 1 ms.

In all experiments, we use $\hat{\text{var}}(v[n])$ as an estimate of the variance of $v[n]$, calculated after the calibration of the camera, from 1000 first samples of signal, because we assume that no drift is present and the signal $y[n]$ consists of the noise in the observations.

Figure 7 (a) depicts the original signal and the estimated signal when the parameters of the filter were:

$$\sigma^2_v = 5 \hat{\text{var}}(v[n]), w_1 = w_4 = \text{var}(w[n]) = 10^{-6}, w_2 = w_3 = 0.$$  

Figure 7 (b) depicts the original signal and the estimated signal, but in this case the parameters of the filter were:

$$\sigma^2_v = 5 \hat{\text{var}}(v[n]), w_1 = w_4 = \text{var}(w[n]) = 10^{-8}$$  

Figure 7 (c) depicts the original signal and the estimated signal, when the parameters of the filter were:

$$\sigma^2_v = 5 \hat{\text{var}}(v[n]), w_1 = w_4 = \text{var}(w[n]) = 10^{-10}$$
In Figures 7 (a), (b), (c), it is observed that in the case of no target is present, results obtained in the performance of the filter are satisfactory, i.e., the drift is well estimated with parameters of the filter $w_i = w_d = 10^{-6}, 10^{-8}, 10^{-10}$. However, because of the drift does not match exactly to the model of constants and ramps in Eq. (2), we get the estimate of drift with more accuracy when big errors in the model are allowed, i.e., when $w_i = w_d = \text{var}(w[n]) = 10^{-6}$, as in the case of Fig. 7 (a). Nevertheless, we consider that the results are satisfactory with smaller values, $w_i = 10^{-8}$, as in the case of Fig. 7 (b).

Figure 8 depicts the signal $\hat{x}[n]$ that models ramps in Eq. (2), in two cases, when $w_i = 10^{-6}$ and $w_i = 10^{-8}$. As depicted in Figure 8, in the case of $w_i = 10^{-8}$, $\hat{x}[n]$ is smoother than when $w_i = 10^{-6}$; besides, we have calculated the mean value and the standard deviation of $\hat{x}[n]$ in both cases:

$$E(\alpha[n])_{w_i=10^{-6}} = 0.005447$$
$$\text{var}(\alpha[n])_{w_i=10^{-6}} = 1.3 \times 10^{-4}$$

In the second set of data, we have tested the behavior of the Kalman filter when a target is present. Now a target with a length of 100 samples is present.

Figure 9 (a) depicts the original signal and the estimated signal, and figure 9 (b) shows the corrected signal. The parameters of the filter were $\sigma_r^2 = 5 \text{var}(v[n]), w_i = w_d = 10^{-6}$.

As can be observed in Figure 9, when $w_i = 10^{-6}$ because of the errors in the model are high, a part of target is cancelled.

Figure 10 (a) depicts the original signal and the estimated signal, and figure 10 (b) shows the corrected signal, when the parameters of the filter were $\sigma_r^2 = 5 \text{var}(v[n]), w_i = w_d = 10^{-8}$. 

$$E(\alpha[n])_{w_i=10^{-8}} = 0.005941$$
$$\text{var}(\alpha[n])_{w_i=10^{-8}} = 1.42 \times 10^{-5}$$
And, finally, Figure 11 (a) depicts the original signal and the estimated signal, and figure 11 (b) shows the corrected signal, when parameters of the filter were $\sigma^2_V = 5 \text{var}(v[n])$, $w_1 = w_4 = 10^{-10}$.

As depicted in Figure 10 and Figure 11, when a target is present, good results are achieved, i.e., the target is not canceled, if the parameters $\omega_1$ and $\omega_4$ are $10^{-8}$ or smaller, $10^{-10}$; thus, we assume that the errors in the model in Eq. (2) are small, and the filter only estimates a slow drift, so the target is no cancelled; moreover, the parameter of the filter, $\sigma^2_V$, calculated as $\sigma^2_V = 5 \text{var}(v[n])$ is big enough to avoid transitory oscillations.

6. Conclusions

In this work, the performance of an algorithm for the drift correction in IR detectors by using real infrared and simulated data is analyzed. In a more complex and sophisticated camera, the matrix of detectors will be cooled and the temperature accuracy controlled. In this way, thermal unstability is not significant; nevertheless, the Kalman filtering is very useful to correct another serious problem of modern imaging systems which use a mosaic of detectors: the spatial nonuniformity. Each detector in the FPA has a photoresponse slightly different from that of its neighbors, called Fixed Pattern Noise (FPN). Moreover, what makes the nonuniformity problem more challenging is the fact that spatial nonuniformity drifts slowly in time; thus a one-time factory calibration will not provide a permanent remedy to the problem. Extern conditions, such as the surrounding temperature, variation of amplifier gain, and the time-dependence nature of the object irradiance, can cause the gain and the offset of each detector to drift slowly and randomly in time. The task of any nonuniformity correction algorithm is to compensate as needed to account for the temporal variation in the detectors’ response. The Kalman filtering is a powerful tool for the adaptive correction of any slow drift in the signal of an infrared and thermal imaging system.

References: