# An hybrid model of the MCDA for the GIS: Application to the localization of a site for the implantation of a dam

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### Abstract

The choice of the localization of a urban infrastructure is a complex problem of the multicriterion decision aid (MCDA). To deal with this situation, several authors have demonstrated the interest of the association of the geographical information systems (GIS) and the MCDA. This association permits to manage the information of spatial reference, and at the same time to apply new methods of MCDA to select the best solutions. However, these analysis methods not only take into account the quantitative criteria, but also the qualitative and imprecise ones, linked to the environment, the regional development, and the socioeconomic impacts. To interpret these qualitative criteria correctly, it is necessary to represent them by precise values rather than by classic intervals. We propose in this paper, a hybrid model associating the multi-representation GIS (MRGIS), and the MCDA methods. Then, to facilitate the insertion of the qualitative criteria in our model, we propose an approach of data modelling based on the fuzzy logic, which allows us to represent all the information necessary to the decision making adequately. Finally, we will illustrate our subjects by an application of the localization of a site for the implantation of a dam.

Key words: MCDA, GIS, MRGIS, Fuzzy logic, Qualitative criteria,

## **1. Introduction**

In the field of the territory management, one of the most difficult problems is the localization of the zones that answer precise spatial and non spatial criteria. It is a strategic choice whose repercussions are very significant on several levels. The choice of the method adopted to facilitate the decision-making is therefore primordial and must be as rigorous as possible.

Our interest is essentially carried, on the one hand, on the widening of the possibilities that a GIS can offer for this kind of problems through the support of the multiple representations. The new generation of the GIS conceived offers, thus more possibilities and guarantees more reliability compared to the classical GIS. On the other hand, the interest is also carried on the MCDA methods because of the diversity of the criteria and factors taken into account in the analysis.

The use of the MRGIS in the domain of the territorial management constitutes a revolution thanks to the power of spatial analysis that they offer. The MCDA methods have become the complementary processes that are based on the results provided by the MRGIS. The advantage offered by the MRGIS combined with the MCDA methods is to reduce the number of possible variants managed by the MRGIS and to evaluate them while considering the quantitative and qualitative criteria proposed by the decision-makers. However, the principal concern of the decision-makers is to convert these qualitative or linguistic data in the form of precise values as pertinent as possible. This makes it possible to facilitate their insertion in the MCDA models.

The aim of this research is to propose hybrid models of MCDA based on the use of the MRGIS and the MCDA methods of the AHP type. This model also contains a modelling approach of the qualitative data by the fuzzy set theory, which permits to interpret these qualitative values by precise values rather than by classic intervals. This will enable us to contribute to the development of new multicriterion analysis methods to help those in charge of making the most pertinent decisions for the localization of a zone in a urban space. We will present therefore in this paper, the MRGIS and the MCDA (section 2 and 3), in section 4, we will speak of the interest of the fuzzy logic for the MCDA. In section 5, we will describe our hybrid model of MCDA. Section 6, illustrates our subjects by an application of MCDA for the choice of a site of implantation of a dam in the west of Morocco. Finally, we finish by the conclusion and the perspectives of our work.

#### 2. The multiple representations geographical information systems

At present, the GIS are not regarded any more as spatial data management systems. They are from now on able to represent the spatial data according to various possible points of view [5]. Thus for each real data, the system will be able to maintain several different representations simultaneously. They are called multiple representations GIS or multi-representation GIS (MRGIS).

The fact that MRGIS takes into account these various factors is a very significant base to meet the requirements which the new function of the GIS imposes; that is to say to be a tool of the MCDA. In fact, the MRGIS have the possibility of aggregating all information necessary concerning a decisional project in a coherent and structured way. This aggregation of information is of primary importance for the problems of the localization of a site. Moreover, the integration of the powerful tools for analysis such as the MCDA methods will permit the MRGIS to propose relevant solutions to help the decision makers to make the best choice.

#### 3. Multicriterion decision aid

The MCDA, often called multicriterion analysis, is a domain that has known a lightning development these last years. The encouraging aspect of this development is that it does not exist in an isolated manner, but concerns all chapters of the operational research. In other words, practitioners are increasingly aware of the presence of the multiple criteria in the concrete problems of management and decision whatever their nature is.

The multicriterion analysis, as its name indicates, aims at providing the decision-makers with tools that permit them to progress in the resolution of the decision problems where several criteria, often contradictory, must be taken into account. But in general, these kinds of problems do not have a best decision that go simultaneously with all points of view. Therefore, the word " optimization " no longer has sense in such a context. This is the reason why the MCDA intends to look for a solution of "compromise". Its main goal is to help decision-makers to organize and to synthesize their information so that they feel at ease with their decision-making.

The MCDA analysis generally refers to a set of methods permitting to aggregate several criteria with the objective to select one or several actions, options or solutions [1]. These methods are used a lot in case of the decision of the localization in general, and the industrial localization in particular.

Among these methods, we can mention, the utility multi-attribute methods (MAUT, SMART, UTA, TOPSIS [8] and [10]), the methods of outranking (ÉLECTRE [11] and [12], PROMETHEE [2]), and the AHP method [14].

It is far from using up the list of the multicriterion methods found in the literature. Other methods permit to treat various imperfections of information contained in the appreciations and the assessments of the decision-makers, either of probabilistic, fuzzy or mixed nature [10].

However, most problems the MCDA deal with, take into account not only the qualitative criteria but also the qualitative and imprecise ones. The main concern for the decision-makers is to convert these qualitative or linguistic data into the form of precise values in the most possible relevant manner. In fact, the decision-makers need to study the sensitivity of the different actions in relation to certain aspects such as the socioeconomic impact and the diversification of the culture taken into account in our application example (see section 6.2). These aspects that are considered as the linguistic or qualitative appreciation criteria, are often represented by a verbal scale of the type: very little important, little important, fairly important, important, very important. However, the values of this classic interval are often incomplete or imprecise. Therefore, It is necessary to find the means to make these values objective and precise in order to allow the decision-makers to take a good decision.

## 4. The fuzzy logic

We think that the use of the fuzzy set theory [15] is the suitable means to solve the problem related to qualitative criteria appreciations. In fact, the fuzzy set theory is interested in the subjectivity in the human judgment. As the human beings are implied strongly in all processes of analysis and decision-making, it is therefore reasonable to take into consideration the human subjectivity in the decisions. This need of modelling of qualitative data related to the human judgment has led to the development of the fuzzy set theory. In the academic and technological world, the word "fuzzy" is a technical term which represents the ambiguity or the vague character of the human intuitions. It refers to the situation in which, there are not exactly definite boundary-marks of the observations set for which the descriptions apply. As for the fuzzy set, it is a type of objects, which a function that assigns a degree of membership to each of these objects is associated to.

Zadeh has positively defined a fuzzy set A as a subset of a set of objects (or x elements) called U, such as,  $U = \{x\}$  and,  $A = \{(x), \mu_A(x)\}, \forall x \in U$  where  $\mu_A(x)$  is the degree of adherence of x to A taking its values in the interval [0, 1].

If  $\mu(x) = 0$ , x is not an element of the set A, but if,  $\mu(x) = 1$ , x is surely an element of A. However, a precise value of  $\mu$  does not exist but it is rather assigned subjectively by the individuals. The fuzzy numbers constitute a particular case of the fuzzy set and are used to model the imprecise numeric quantities as "big", "small", "much bigger", "a lot smaller, etc. In the case of the fuzzy numbers, the membership function denotes the degree of truth that the fuzzy number takes a given specific real number.

The fuzzy set theory that is characterized by its flexibility in the modelling of the qualitative data as well as the hardiness of its results seems, thus, very suitable to solve the problems related to the qualitative and ambiguous assessments that characterize our problem of the choice of a site for the localization of the dam. In fact, the technique of the fuzzy set will allow us to convert the qualitative values into precise ones. This conversion is made in two stages. The first stage, consist in transforming the qualitative appreciations in trapezoidal or triangular fuzzy numbers [8] and [9]. This transformation is made according to the conversion scales proposed by [4].

Either M = (l, m, u), or m is the most probable value of M, l and u respectively the smallest and biggest possible value of M (l, m and u being any real numbers such as  $l \le m \le u$ ). M is a fuzzy number to which a membership function is associated, taking its values between 0 and 1 and is defined as follows:

$$\mu_{M}(x) = \begin{cases} 0, x \leq l \\ (x - l)/(m - l), l < x \leq m \\ (u - x)/(u - m), m < x \leq u \\ 0, x > u \end{cases}$$

If M possesses only one peak (the value m is unique and  $\mu_M$  (m)=1), it is called triangular fuzzy number whereas if it possesses 2 peaks (M = (l, m1, m2, u) such as m1 \le m\_2 et  $\mu_M$  (m<sub>1</sub>) =  $\mu_M$  (m<sub>2</sub>) =1), it is called trapezoidal fuzzy number. The fuzzy and triangular numbers constitute a particular case of the trapezoidal fuzzy numbers (m1 = m2).

The second stage consists in converting the fuzzy numbers into precise values. This method of conversion proposed by Chen and Hwang is an adaptation of the ranging approaches of Jain (1976, 1977) and of Chen (1985). Let's consider two sets, maximization (max) and minimization (min),

$$\mu_{\max}(x) = \begin{cases} x, 0 \le x \le 1 \\ 0 \end{cases} \quad \text{et} \quad \mu_{\max}(x) = \begin{cases} 1 - x, 0 \le x \le 1 \\ 0 \end{cases}$$

The right score of M is calculated like this:  $\mu_R(M) = \sup_x \{\mu_M(x) \land \mu_{max}(x)\}$ 

The left score of M, is the following:  $\mu_L(M) = \sup_{x} \{\mu_M(x) \land \mu_{\min}(x)\}$ 

Considering the left and right scores of M, the total value of M is determined in the following manner:  $\mu_T(M) = (\mu_R(M) + 1 - \mu_L(M))/2$ .

#### 5. Proposed model

The association of the MRGIS and the MCDA methods constitutes a privileged way to make, on the one hand, the MRGIS evolve toward real decision aid systems and permit, on the other hand, with the MCDA methods to widen their capacities of analysis while acquiring the transparency that they often lack [11]. For more details and to show the interest of this association, the reader can refer to the work of Laaribi [9]. Our thought process concerns the search for a site for the localization of a dam. It can be about a surface assigned for the construction of a residential district, as well as the construction of a factory, or any other project requiring the attribution of a contiguous surface. The research of the most favourable site has two dimensions; interregional and intra regional. This has led us to propose a hierarchical thought process made up of three processes (Figure 1). Proceedings of the 5th WSEAS Int. Conf. on Artificial Intelligence, Knowledge Engineering and Data Bases, Madrid, Spain, February 15-17, 2006 (pp171-182)



Figure 1: Hybrid model of the MCDA

- The first process is a spatial analysis one. With the help of the MRGIS, it is responsible not only for describing the perimeter of study, but also of building a map for the homogeneous zones according to spatial criteria fixed by the decision makers. The map is built according to several representations each of which reflects a criterion. Then, by using the layer superposition technique, the obtained homogeneous zones will be subdivided in sub homogeneous zones each in relation to each of the criteria. Finally, we get a map of the sub homogeneous zones in relation to each of the retained spatial criteria.
- The second process is a multicritera analysis one. It is responsible, through the use of the multicritera analysis method AHP, for the assessment of the different zones. This assessment is made while taking into account the generated spatial and non spatial criteria by the process of spatial analysis. The result is the choice of the best sub zone.
- The third process is assigned to evaluate the candidate sites of the best sub zone. It uses the fuzzy logic to convert the qualitative criteria into precise values, then, the AHP method evaluates the candidate sites in relation to the qualitative and quantitative criteria proposed by the decision-makers. The result is the choice of the most favourable site for the localization of the dam.

## 6. Application

We are going to illustrate our model of MCDA by an example of the localization of a site for the implantation of a dam.

### 6.1. Choices of the best sub-zone

In this section, we will expose our thought process, while describing the different elements that intervene in our analysis. These elements are: the criteria at the level of the big spaces, the maps of the homogeneous zones, the map of the sub homogeneous zones and its multiple representations, as well as the synthesis and the choice of the best sub zone.

### • Retained criteria at the level of the big spaces

To divide up the territory in homogeneous zones, the responsible have proposed the criteria that permit on the one hand, to judge the interest of the project for the population. It is about constructing a family of criteria that can represent the objectives of the population. And on the other hand, to be precise to really discriminate between the sub zones and not to be redundant.

So on the basis of the consultations of several interlocutors concerned by the project, the decisionmakers have kept three aspects (Criteria) that appear to intervene in a consequent manner in the process of decision:

- Criterion C1: Presence of other dam; this criterion takes into account the presence of other dams in the zones to value.

- Criterion C2: Popular density; this criterion represents the popular agglomeration in the zones to value.

- Criterion C3: Tourist attraction; the choice of a localization must be at the level of the zones classified non touristy areas. This criterion takes into account the tourist attraction of the zones to value.

#### • Maps of the homogeneous zones in relation to each of the criteria

The use of the spatial analysis process has allowed us to obtain the maps of the homogeneous zones (figure 2) each reflecting a spatial criterion.



Figure 2: Maps of homogenous zones

These zones bring together the most similar spatial variants in relation to thresholds of indifference fixed by the decision-makers. The grouping is made in a continuous manner in the space; that is to say that two spatial variants can be similar, but considered in two different zones.

#### • Map of the sub homogeneous zones in relation to all criteria

According to the procedure of grouping, of the spatial variants that we adopt, we can say that every sub zone of a homogeneous zone is a homogeneous zone in relation to the same thresholds of indifference fixed by the decision-makers.

On the basis of this report and by superposition of the different layers of representation of the homogeneous zones, one can say that the sub zones of intersection (figure 3) are homogeneous zones in relation to all criteria.



Figure 3: Map of homogenous sub zones

The sub intersection zones are composed of the groupings of the spatial variants belonging to homogeneous zones at the level of all layers.

### • The multi-representation of the map of the homogeneous sub zones

The process of spatial analysis explores the same map by giving it several representations (figure 4). Each representation reflects a qualification of the importance of a criterion on its homogeneous zones. These qualifications are: weak, average, high, very high and extreme.



Figure 4 Evaluation layers in relation to criteria of localisation

## • Choice of the best sub zones

The process of the multicriterion analysis considers the sub zones, obtained by the superposition of the layers as actions (figure 5). At the time of this analysis, the decision-makers use the multi-representation map to have all necessary information to take the possible most pertinent decision. This process uses the fuzzy analysis and the AHP method to get the final assessments and to determine the best sub zone.



Figure 5: Superposition of evaluation

The figure 5 represents the five candidate sub zones above. The appreciations of these sub zones in relation to the criteria of decision are qualitative as the following Table shows:

Criteria /Zones	1	2	3	4	5
1	Weak	Weak	Very high	Weak	High
2	Extreme	High	Very high	Weak	Weak
3	Weak	Average	Average	High	High

Table 1 : Qualitative appreciations of sub zones

It is therefore necessary to quantify these qualitative appreciations, to have the desirable precision by the decision-makers. This quantification is achieved by the conversion technique based on the fuzzy logic [4], the obtained result is presented in the following Table:

Criteria /Zones	1	2	3	4	5
1	0.11	0.11	0.41	0.11	0.31
2	0.48	0.31	0.4	0.12	0.12
3	0.10	0.21	0.21	0.29	0.29

 Table 2 : Quantitative appreciations

Then, the AHP analysis method, allows us to determine the matrix of comparison, and the vector weight of the criteria presented in the following Table:

Criteria	1	2	3	Sums	Weights
1	1	5	7	13	0.69
2	1/5	1	3	4.2	0.23
3	1/7	1/3	1	1.47	0.08
Total :				18.67	1

Table 3: Comparison matrix of the criteria

Finally, by multiplying the sums of appreciations by the corresponding weights of the vectors, we gets the final assessment vector of the sub zones (see details of the calculations in section 6.2):

Sub zones	1	2	3	4	5
Evaluation	0.144	0.132	0.354	0.107	0.238

Table 5: Final evaluation vector of the sub zones

So the sub zone number 3 represents the best site for the implantation of the dam.

## 6.2. Choice of the best site at the level of the chosen sub zones

After having chosen the best sub zone, the third process that also combines the fuzzy analysis and the AHP method, must value the candidate sites of the best chosen sub zone. That's why, we construct a family of criteria that represents the objectives of the project, and we establish the table of assessment (Table 6). This table contains the actions (sites) and the appreciations of the criteria proposed by the decision-makers.

### a) <u>Choice of Criterion</u>

The criteria kept by the responsible to compare the different sites present two conditions:

- To permit to judge the interest of the project for the population. It is about constructing a family of criteria that can represent the objectives of the population.

- To be precise to distinguish between sites and to be redundant to avoid raising the importance assigned to any dimension. So on the basis of several interlocutor consultations concerned by the project, decision-makers have kept six aspects (Criteria) that appear to intervene in a consequent manner in the process of decision:

- ▶ Criterion 1: Capacity to supply drinking water in  $10^6 \text{ m}^3$ . This criterion essentially takes into account the point of view of the population that sees in the dam a source of drinking water supply.
- Criterion 2: Surface irrigated in <u>hectare</u>. This criterion conveys the farmers' point of view who see in the dam a means of irrigation.
- ➤ Criterion 3: The neat present value of the project in  $10^6$  dhs. This criterion takes into consideration the global interest for the community of the project. It aggregates all the project costs and advantages to the social updating rate according to the formula: VAN = I + Sum (annual Advantages annual costs) converted to current value.
- Criterion 4: The period of time of the dam realization in month. The necessity and the emergency of the project require knowing the delay of realization for every site susceptible to be finished in a period in conformity with the needs of the region.
- Criterion 5: Diversification of cultures. This criterion is qualitative. It takes into account the consequences of the project on the diversification of cultures (agriculture, forestry) and both the setting and the quality of life (lands cape, forest management, etc.)
- Criterion 6: Socio-economic impact. This criterion is qualitative. It takes into account the contribution of the project to the development of the activity of the region as well as its interest with regard to the national policy of the territory management.

#### b) Decision makers' choice of actions and appreciation

In the present study, thanks to multicriterion methodology, the decision makers have imagined five sites known as Ai, = 1, 2, 3, 4,5. Their evaluation in relation to each criterion is represented in the following table:

<b>Criterion /Sites</b>	Action 1	Action 2	Action 3	Action 4	Action 5
Criterion 1	560	636	820	873	780
Criterion 2	1340	1200	1400	1420	1390
Criterion 3	146	174	136	135	130
Criterion 4	34	40	36	30	38
Criterion 5	Little important	Important	Fairly important	Very little important	Little important
Criterion 6	Very little important	Very important	Important	Very little important	Fairly important

#### Table 6: Evaluation Table

We notice that the two criteria 5 and 6 are qualitative; it is therefore necessary to convert them in precise values [4]. We then get the following result:

Critère/Sites	Action 1	Action 2	Action 3	Action 4	Action 5
Critère 5	0.23	0.41	0.32	0.13	0.23
Critère 6	0.11	0.47	0.39	0.11	0.30

 Table 7: Precise values of criteria 5 and 6 in relation to actions

We then construct the final assessment table. This table contains the values of the quantitative criteria and the values of the qualitative criteria converted (see Table 8).

Criteria/Sites	Action 1	Action 2	Action 3	Action 4	Action 5
Criterion 1	560	636	820	873	780
Criterion 2	1340	1200	1400	1420	1390
Criterion 3	146	174	136	135	130
Criterion 4	34	40	36	30	38
Criterion 5	0.23	0.41	0.32	0.13	0.23
Criterion 6	0.11	0.47	0.39	0.11	0.30

#### Table 8: The final evaluation table

The following phase of the third process consists in using the AHP method [14] to calculate the weight of every criterion. This weight is calculated in the following way:

$$P_{k} = \frac{\sum_{j=1}^{6} a_{kj}}{\sum_{i=1}^{6} \sum_{j=1}^{6} a_{ij}}$$

Pk is the weight of the criterion number k

The aij (with  $1 \le i, j \le 6$ ) represent the values of the comparison matrix,

Number 6 represents the number of the criteria taken into account.

For example the weight of criterion 1 is P1=(1+3+4+1+1+1)/53.68 = 0.2049

Criteria	C1	C2	C3	C4	C5	C6	Vector v	veights
<i>C1</i>	1	3	4	1	1	1	11	0.2049
C2	0.33	1	2	0.33	0.33	0.33	4.33	0.0806
С3	0.25	0.5	1	0.2	0.2	0.2	2.35	0.0437
C4	1	3	5	1	1	1	12	0.2235
C5	1	3	5	1	1	1	12	0.2235
C6	1	3	5	1	1	1	12	0.2235
Total:							53.68	1

#### **Table 9: Comparison Matrix of criteria**

The weights of the criteria being calculated, we then proceed to the normalization of the criteria values. (see Table 10):

Criterion/Sites	Action 1	Action 2	Action 3	Action 4	Action 5
Criterion 1	0.1938	0.2201	0.2838	0.3021	0.2699
Criterion 2	0.1985	0.1777	0.2074	0.2103	0.2059
Criterion 3	0.2024	0.2413	0.1886	0.1872	0.1803
1/Criterion 4	0.2074	0.1764	0.1954	0.235	0.1856
Criterion 5	0.1742	0.3106	0.2424	0.0984	0.1742
Criterion 6	0.0797	0.3405	0.2826	0.0797	0.2173

 Table 10: Normalisation Matrix of the criteria

This normalization is obtained by dividing the value of the criterion I corresponding to action J by the sum of the criterion value I of all actions 1,2,3,4,5. For example, the normalized value of the criterion 1 in relation to action 1 is calculated as follows: 0.1938 = 560 / (560+636+820+873+780).

The last phase of the process is the final assessment of actions. These assessments are calculated in the following way:



The synthesis of the results obtained has resulted in the following assessments:

Action 1	Action 2	Action 3	Action 4	Action 5
0.16	0.25	0.24	0.17	0.20

This assessment allows us to order the actions in the following way: action  $n^{\circ}2$  (0.25) proves to be the most interesting, followed action  $n^{\circ}3$  (0.24), action  $n^{\circ}5$  (0.20) is situated in the middle of the ordering and actions  $n^{\circ}4$  (0.17) and  $n^{\circ}1$  (0.16) are placed at the end of the ordering. This result shows well the precision raised from the proposed analysis as well as a strong significance of the values obtained.

#### 7. Conclusion

In this paper, we have explored the idea of the use of the new generation of GIS, called multiple representations GIS (MRGIS), capable of maintaining several representations simultaneously. Therefore, the user has a global and complete vision of all the working space according to the different possible representations. The research and the analysis according to the spatial criteria are becoming more efficient and particularly more pertinent.

The integration of the fuzzy logic and the AHP analysis method within MRGIS have created a new way of research that we have started and that seems very promising. We think that the use of the fuzzy set modelling tools coupled with the MCDA methods can be a pertinent decision aid tool for most choice and decision aid situations. Therefore, a complex problem can become simple by using the MCDA methods and the ambiguous judgments can become precise by using the fuzzy analysis. It is thus about exploring these advantages in order to propose thought processes used in a

simple way and conceived in a structured and coherent manner to help the decision makers to make the best choice

The works we are leading under way are directed on the one hand towards the implementation of a multiple representation GIS dedicated to the urban applications and on the other hand toward the test and the assessment of the different MCDA methods.

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