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Pseudo-Invariant Diffusive Robust Control and application for a DC Motor

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Abstract:- This article presents a new approach so-called 'diffusive pseudo invariant' with application on the robust control of a DC motor with uncertain parameters. The fundamental property of this approach is to preserve as much as possible, in the uncertainty domain, the dynamic characteristics of the system. The controllers obtained are of fractional nature and can be achieved in a non-hereditary way by diffusive model.

Keys-Words: - Diffusive representation, Pseudo invariant control, Robustness, DC motor, Uncertain parameters, Fractional operator.

1 Introduction

The uncertain dynamic control systems with respect to unknown parameters, requires the implementation of control laws that are able to ensure a compromise between performances and robustness. The present work is a progress of different works [1] [5] [7] where the problem of the robust control of an uncertain linear system was treated.

Our objective is to improve the performances obtained in previous works [2] [5]. In this study, we suppose that the uncertain parameters of a dc motor are the moment of inertia and load J and the constant time of the internal current loop T_{bc}. The concept of the robustness in the sense of the pseudo invariance under group transformation (invariance under frequency scaling) is based on the property of invariance of the fractional differential equation. Generally, the strict invariance is inaccessible, and the design of the pseudo invariant control, for the non strict invariance, consists in the minimization of an adequate cost functional. The optimal controllers are pseudo –differential operators.

The diffusive realizations of these operators lead to a non-hereditary system and simplify the analysis and the numerical approximation. The step responses of the closed-loop controlled system are similar in the entire uncertainty domain with time-scaling. The pseudo invariance warrants dynamic stability of the controlled system (by analogy to the nominal system). This article is organized as follows. Section 2 presents the classic controller of a dc motor. In section 3 we describe the principle of the invariant pseudo control. Section 4 presents the diffusive pseudo-invariance formulation via diffusive model, as well as the numerical approximation. Finally, the results and commentaries are given in section 5

2 Classical Control DC Motor

2.1 Model

In This study, we will use the linear model of the dc motor presented by the figure 1, where K_c is the torque constant, L is the inductance and r is the coil resistance.

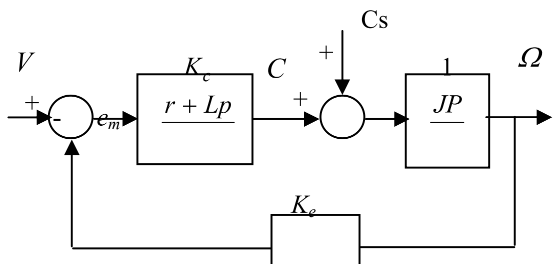

Figure 1: DC motor model

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We consider the following uncertain parameters: the moment of inertia, the load, and the time constant T_{bc} associated to an internal loop.

2.2 Classical controller

The classical controller structure widely used constitutes of two overlapped loops. The first is an internal current loop (see figure 2), that allows to control the torque and reject the influence of the electromotive force. Therefore, the use of a PI controller is sufficiently and lead to a first order transfer of the current loop of time constant T_{bc}. The second is an external speed loop of the controlled system, we uses generally a PI controller (see figure 3).

$$H_{PI} = A \cdot \frac{1+TP}{TP} \qquad (1)$$

The PI parameters may be determined by a system reduction to a 2nd order system neglecting the current loop $i=i_c$.

The tracking transfer function is given by:

$$H(p) = \frac{\Omega(p)}{\Omega_c(p)} = \frac{1+TP}{1+TP+\frac{TJ}{AK_C}P^2} \qquad (2)$$

The proper pulsation and damping coefficient are respectively:

$$\omega_n = \sqrt{\frac{AK_C}{JT}} \quad \text{and} \quad \zeta = \frac{1}{2}\sqrt{\frac{AK_C T}{J}} \qquad (3)$$

We establishe that the damping coefficient is as much weakly (and the stability as much more fragile) that the moment of inertia is great.

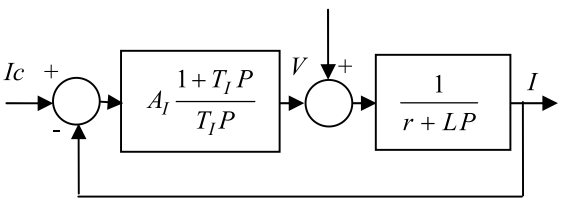

Figure 2 : Current loop

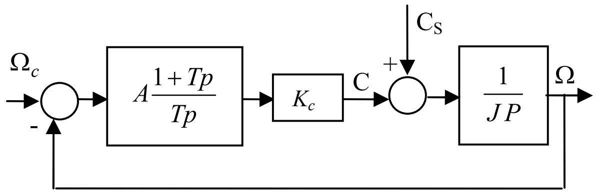

Figure 3 : Classical speed loop

3 Pseudo Invariant Control

3.1 Fractionnaire differential equation analysis

We consider the fractional differential equation:

$$K\frac{d^\beta}{dt^\beta}x(t) + x(t) = u(t) \qquad 1 \pi \beta \pi 2 \qquad (4)$$

where K is an uncertain parameter and □ is the order of the fractional equation.

The correspondent transfer function is :

$$H_\beta(P) = \frac{1}{KP^\beta + 1} = \frac{1}{(\tau p)^\beta + 1} \qquad (5)$$

where $\tau = K^{\frac{1}{\beta}}$

With the frequency scaling $\tilde{p} = \tau p$, we obtained the new transfer function $H(P) = \frac{1}{\tilde{p}^\beta + 1}$, that posses the invariance property.

The transfer is close (in the sense H_2) to damping second order transfer. The damping is linked directly to β [2][7]. The frequency responses (or step) of the equation (5) are similar up to a frequency (time) scaling for any value of K.

3.2 Strict invariance

Let us suppose that the time constant T_{bc} is known. The uncertainty affecting only the moment of inertia and the load noted J. we can simply proof the following result:

The controller defined by:

$$K(p) = A_f \frac{1+T_f}{p^\alpha} \qquad 0 < \alpha < 1 \qquad (6)$$

where $T_f = T_{bc}$

Lead to the closed loop transfer function:

$$H_K(p)=\frac{1}{1+\frac{J}{K_c}p^{1+\alpha}} \qquad (7)$$

The transfer function (7) is invariant under the frequency scaling transformation T_{σ_J} , where:

$$\sigma_J=\frac{J}{J_0}^{\frac{1}{1+\alpha}} \qquad (8)$$

The figure 4 show the new functional diagram of the control system

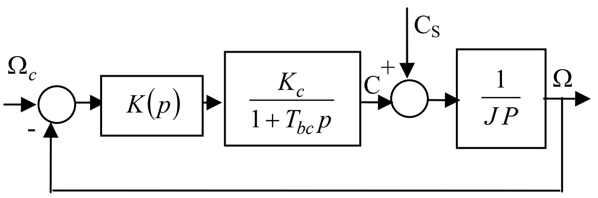

Figure 4: New speed loop

3.2 Non strict invariance

Now, we suppose that the uncertain parameters are the moment of inertia, load and the time constant T_{bc} associated to internal current loop. In this case, it is not possible to obtain in the closed loop the transfer defines by the equation 5. So does not exist a corrector that lead to the strict invariance.

The robustness problem consists in minimizing a functional constructed in order to translate the gap between the real closed loop response and the ideal response. The goal is construct a compensators that allows to obtain a closed loop behavior close the most possible to *H(p)* define by (5).

We define the transformation group $T_\varsigma = \{T_\sigma, \sigma \in C(\Lambda, R^+)\}$,where :

$$(T_\sigma H)(\lambda,p)=H(\lambda,\sigma(\lambda)p) \qquad (9)$$

The pseudo-invariant control under transformation group can be formulated in Hilbert space by the following optimization problem [4]:

$$\min_{K,T}\left\{(1-q)\left\|T_\sigma.H_\lambda(K)-H_{\lambda_0}(K_0)\right\|^2 + q\left\|T_\sigma.H_\lambda(K)-H_{\lambda_0}(K)\right\|^2\right\} \qquad (10)$$

Where:

$H_\lambda(K)$ is the uncertain transfer system controlled by K, Λ is the uncertain parameters set ($\lambda\in\Lambda$), λ_0 is the nominal parameter, $H_{\lambda_0}(K_0)$ is the reference transfer system controlled by the classical controller K_0 (nominal model), and $q\in[0, 1]$.

Noted that the strict invariance transfers are accessible for q=1

4 Diffusive Controller

4.1 Diffusive Model

The diffusive realization noted $\mu(\zeta)$of the pseudo differential operator K_μ: $u(t)\rightarrow y(t)=K_\mu(\partial_t)u(t)$ is defined by the dynamic input-output system [3][6]:

$$\begin{cases}\partial_t\psi(\zeta,t)=-\zeta\psi(\zeta,t)+u(t)\\ y(t)=\int_0^\infty -\mu(\zeta)\psi(\zeta,t)d\zeta\\ \psi(\zeta,0)=0,\quad \zeta \phi 0\end{cases} \qquad (11)$$

The diffusive realization of the controller K_μ defined by (6) is :

$$\mu(\zeta)=K_f T_{bc}\frac{\sin\pi\alpha}{\pi}\left(\frac{1}{T_{bc}} fp \frac{1}{\zeta^{1+\alpha}}+\frac{1}{\zeta^\alpha}\right) \qquad (12)$$

4.2 Numerical approximation of diffusive model

We consider a finite network on the ξ variable, with the convenient hypothesis [3].

$$N=\{\zeta_1,\zeta_2,...,\zeta_i,\zeta_{i+1}\},\ 0<\zeta_i<\xi_{i+1} \qquad (13)$$

The finite-dimensional differential systems obtained from (14) on N :

$$\begin{cases}\partial_t\psi_i(t)=-\zeta_i\psi_i(t)+u(t)\\ y(t)=\sum_{i=1}^N \mu_i\psi_i(t) \qquad i=1,...,N\\ \psi_i(0)=0\end{cases} \qquad (14)$$

With $\mu_i=\int_\zeta \mu(\zeta)\Lambda_i(\zeta)d\zeta$

Where: Λ_i are convenient piecewise affine functions with bounded support.

The approximated transfer function of the pseudo-differential operator is :

$$K_\mu(p)=\sum_{i=1}^{N}\frac{\mu_i}{p+\zeta_i} \tag{15}$$

4.3 Diffusive Formulation of pseudo invariant control

Under diffusive formulation, the pseudo invariant control defined by (10) can be rewritten [5]:

$$\min_{\sigma,\mu}\Big[(1-q)\big\|T_\sigma H_\lambda(K_\mu(p))-H_{\lambda_0}(K_0(p))\big\|+ q\big\|T_\sigma H_\lambda(K_\mu(p))-H_{\lambda_0}(K_\mu(p))\big\|^2\Big] \tag{16}$$

Where $K_\mu(p)$ is the diffusive controller whose transfer function is given by (15). The optimal solution noted $\mu_\cdot^*$ is the diffusive realization of the optimal controller that lead to an invariant transfer function.

It is clearly that the equation (16) is more simple then equation (10) either in analysis or in numerical approximation.

5 Results and Comments

Figure 5 presents the step responses of the uncertain system controlled by the PI controller. This figure shows the influence of the uncertain parameter on the behavior of the controlled system (when J decreases the overshoot increase).

Figure 6 presents the step responses of the uncertain system controlled by the controller defined by equation (6) under diffusive realization, when the strict invariance is accessible. The invariance up to time scaling, is clearly shown and the overshoot remains constant.

In figures 7, 8 and 9, we can see the magnitudes and phases of the pseudo-differential operators. Figure 7 shows clearly that the approximation of half integrator is exact in utile band width.

The bode diagram of the controller defined by equation (6) approached by the diffusive model (strict invariance), is shown in figure 8.

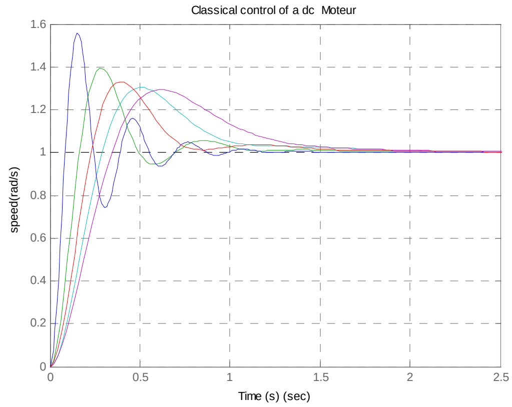

Figure 5: step responses of uncertain system controlled by classical compensator **(PI)**

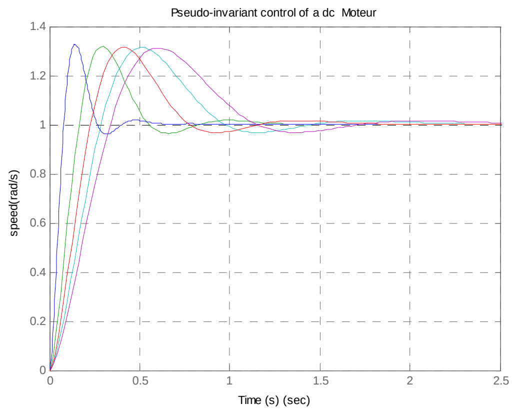

Figure 6: Step responses of uncertain system controlled by pseudo-differential compensator K_μ defined by (6)

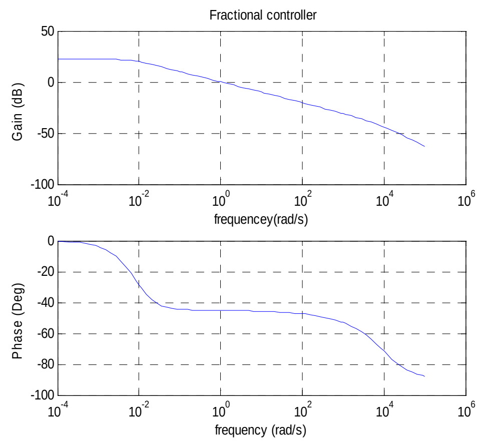

Figure 7: Bode diagram of half integrator

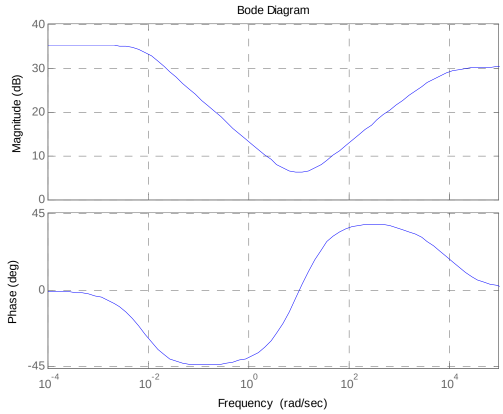

Figure 8: Bode diagram of the compensator K_μ defined by (6)

Figure 9, show a comparison between the controller obtained by the strict invariance with the one obtained by non-strict invariance.

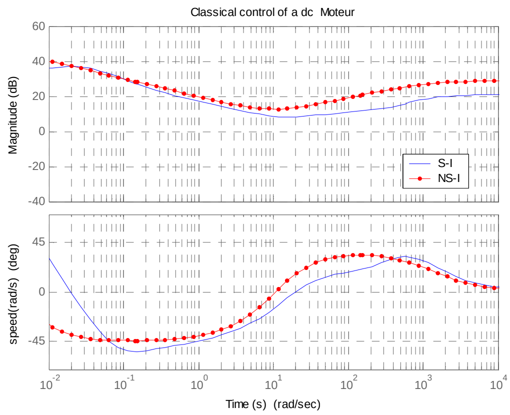

Figure 9: Comparison between the controller K_μ obtained with the strict invariance(S-I) and non-strict invariance (NS-I)

Figure 10 shows the step responses of the uncertain system that obtained in the case of the non-strict invariance. The performances are less acceptable than those with the strict invariance.

The step responses of the nominal system controlled by the classical controller (PI) and the one of the uncertain system controlled by the fractional controller are shown in figure 11. The coincidence of the two responses is clearly shown in this figure.

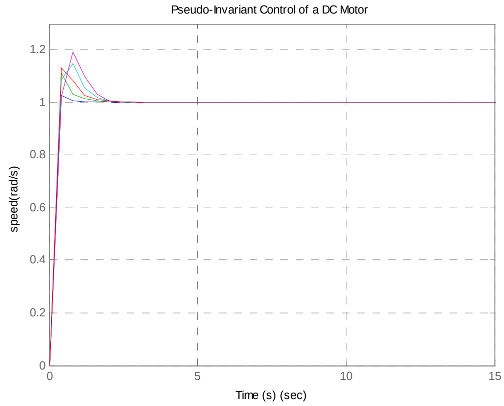

Figure 10: Step responses of uncertain system controlled by pseudo-differential compensator K_μ defined by (13)

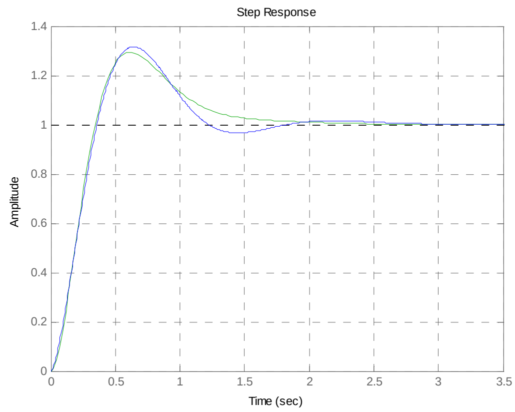

Figure 11: Step responses of uncertain system controlled by pseudo-differential compensator and the one of nominal system by the PI controller

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