DRAUGHT MODELING UNCERTAINTIES IN THE SOLAR-GRAVITY POWER PLANT

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Abstract. The SEATTLER solar power plant, consisting of a tall tower where the atmospheric air is heated by a heliostat array through a solar receiver and drives an air turbine by the new mechanism of cold air gravity draught, is modeled into a mathematical integral equivalent. The all-air working fluid principle is proposed as the result of an encouraging previous, small scale research sponsored by the CNCSIS Grant No. 27642 of 14 March 2005 in Romania. The presented numerical investigation on the model is targeted towards demonstrating the feasibility and efficiency of the all-air system at full scale. The natural peculiarities of the solar tower systems are connected to their small working pressures and, in the present case, to the medium working temperatures in particular. At the same time, air enthalpy capture efficiencies between 65% and 80% are known within the SOLAIR project in Spain, while the SEATTLER efficiency could go much further. The alternative here evoked is focused on preliminary temperature ranges of 100-300°C and over 1300°C. In either case simple and high efficiency solar radiation solid-air heat exchangers are to be yet developed. Due to the small pressure variations along the gasdynamic channel, the mathematical model here presented proves a high instability in regard to the assumptions introduced and to the limit conditions of the flow at entrance and exit of the tower. The quasi-resonance behavior is demonstrated through several independent paths. The characteristic of the turbine tower proves markedly different from the simple, warm air thermal draught in high stacks.

1 INTRODUCTION

SEATTLER’s scientific and technical objectives are to create a low-noise, ultra-low turbulence non-mechanical means of accelerating the flow of air into a channel with double application. First as an aero-acoustic wind tunnel for infra-turbulence studies and second, as a cheap direct solar radiation to air turbine action for an innovative electric power plant. The first application for powering a wind-tunnel is based on its revolutionary zero driving noise, a result of the total absence of any moving fans and drivers, means that the noise of industrial fans and especially of aircraft engines would be ideally studied and means of suppression found, at a yet unattained level of acoustic accuracy. The second application for electricity production claims a far much smaller ground area alternative to the famous SB GmbH Solar Tower power plant, that uses an embarrassing huge greenhouse solar collector. With its reduced area solar collector by two orders of magnitude and a high radiation temperature, the SEATTLER solar energy tower ends in a cheaper and really environment friendly electricity production system. Regarding the SEATTLER tunnel, the crucial progress is the complete removal of all moving parts and consequently of all driving noise sources. Named SEATTLER from Solar Energy Actuator for Tall Tower Low-cost Electricity Research, it allows the aerodynamic noise of the flowing air be definitely perceived. The new tool will address the area of both fundamental and industrial research for noise protection of the environment and especially noise reduction in aeronautics, as desired by the ACARE-2020 European integrated project. The avenue towards environment-friendly aircraft engines will thus be cleaned. Besides the obvious acoustical quality of the rig, its construction is by far much simpler, less expensive and really safe than for any other existing noise test facility. It has a vertical, small space consuming, lightweight and attractive architectural aspect (Fig. 1 next page).
2 AIR ACCELERATION BY GRAVITATIONAL DRAUGHT

The source of the gravitational draught is the ascending effect of lower density volumes of internal gases in contrast to the higher density surrounding air as a result of their different weights, quantitatively described by the famous Archimedes' law. As far as we keep standing in the frames of this effect of gravitation the problem remains entirely immobile. The accompanying buoyant force supplies the maximal estimate for the draught effect. However, when the upward motion of the rarefied gas from the interior is considered, the more elaborated dynamic equilibrium must be approached by the rules of gasdynamics, although its roots remain in the realm of a crude Archimedes' effect.

The draft in fig. 3 shows a vertical stack circulated by the air that enters the lower intake, passes through the heat exchanger where it is warmed and accelerated by gravitation and, after exiting its top opening, defuses into the outer atmosphere. It is surrounded by higher density atmospheric air and the effect of gravity must be accounted in computing the different inner and outer pressures that act on stack’s walls and the virtual hood. The entire tube is supposed as behaving with constant internal cross area, except for the local contraction in the test chamber. To simplify the treatment, the zones of air intake (0-1) and heating (1-2) by the heat exchanger (a solar receiver in particular) are considered as superposed in a single point on the axis. For the sake of simplicity also, the air density is considered as independent of altitude [11], while different inside and outside. More elaborated atmospheric models may be used, without changing the main conclusions however.

The air acceleration by thermal-gravitational draught is subjected to a dynamic equilibrium of pressure losses on one side and of the buoyancy force on the other, ending in a flow characteristic that naturally depends on the amount of heat introduced. We expect the balance be very tight because the pressure differences between the main stations of a draught tower are fractions of the atmospheric one. Even very small effects may considerably alter the mathematical model, when a simplified one-dimensional one is considered.

The basic effect is the aerostatic influence of the gravitation. This is given by the equation of the pressure gradient vs. altitude, written both inside (density $\rho$) and (density $\rho_0$) outside the stack,

$$\frac{d\rho}{dz} = -g \rho, \quad \frac{d\rho_0}{dz} = -g \rho_0.$$  \hspace{1cm} (1)

The right hand term in these equations is nothing but the slope of the curves, measured to the left of the vertical in each pressure diagram from figure 3. It shows that the inner pressure in the stack (left, orange) is decreasing less steeply and remains closer to the vertical than the outer air pressure (right, dotted blue). The dynamic equilibrium is established when, following a series of transforms, the pressures inside and outside, before and after the flow, become equal (fig. 3). While the air outside the stack preserves immobile and due to the effect of gravitation its pressure decreases with altitude from $p_o(0)=p_0$ at the stack's pad to $p_a(f)$- at the tip of the stack "4", the inner air is flowing and consequently its pressure $p_{in}$ varies not only due to gravitation but also due to acceleration and braking along the 0-1-2-3-4 cycle and in other inner parts of the stack (turbine).

Starting from rest, the air is accelerated by suction at tower inlet between 0-1, with very slight modifications of temperature or density. Bernoulli's (energy) equation may be used to model this process and compute the pressure loss. Thereafter the air temperature is raised in the heater that simultaneously lowers its density, producing a thermal acceleration in the direction of the flow. The impulse conservation law may here be used to compute the pressure loss for thermal acceleration. Under the induced rarefaction the lower density air is now boosted upward by the Archimedes' effect up to the upper exit of the stack. Here a sort of pressure recovery takes place between the internal and the external air and the model greatly depends on the hypotheses used. While the inside air is flowing with a certain speed, the outer air preserves immobile but the two fluids must achieve equilibrium. A series of assumptions are to be checked to model this process.
2.1 Hypotheses for the intake and outlet of the air

The air heating in the constant area heat exchanger/solar receiver between 1-2, with the heat \( q \) per kg of air added, is governed by the impulse equation that gives, for a presumed value of the mass flow rate \( \dot{m} \), the over-pressure required upwind to accelerate the expanding mass of gas. Downwind the pressure will develop accordingly smaller and this way a pressure loss or “dilatation drag” \[5\] appears in the heater, even if the drag by friction \( \Delta p_R \) would first be ignored,

\[
p_2 + \frac{m_2^2}{\rho_2 A^2} = p_1 + \frac{m_1^2}{\rho_0 A^2} - \Delta p_R.
\]

At the air intake either an incompressible acceleration of the atmospheric air or a compressible process may be considered, with very small variation in the density in fact. The Bernoulli description of the compressible flow of a gas with the adiabatic exponent \( \kappa \) reads

\[
p_1 = p_0 - \frac{\Gamma}{2} \cdot \frac{m^2}{\rho_0 A^2} \quad (\Gamma = \frac{\kappa - 1}{\kappa}).
\]

with a mean, constant density \( \rho_0 \) all along for sake of simplicity. The incompressible process is directly rendered when considering \( \Gamma = 1 \). Although equivalent, the two assumptions provide slightly different flow characteristics of the stack that must be confronted with the experiment.

The reversed process seems to take place at the stack upper opening, where the low density air flows with velocity and further brakes to stagnation into the resting atmosphere, where the total pressure behaves equal to the atmospheric one. While this means that at the very tower exit a lower static pressure of the inner air manifests and considering that a flow may always occur from greater to lower pressures Unger \[5\] is claiming that a natural flow condition must be based on considering equal static pressures inside and outside at tower exit. These two different approaches render also different results over the stack characteristic. To simplify the description the relative rarefaction \( \gamma \) for the amount of heating instead of the heat quantity itself will be used,

\[
\gamma = \frac{\rho_0 - \rho_2}{\rho_0} = 1 - \beta, \quad \beta = \frac{\rho_2}{\rho_0} \lesssim 1.
\]

Considering also \( A=\text{const} \) for the cross-area of the heating zone the continuity condition shows that the variation of the speed is given simply by

\[
c_2 = c_1 / \beta.
\]

Using a reduced mass flow rate (RMF) expression \( R^2 \) for the equilibrium flow \[5\], the following results are obtained when the different assumptions from above are considered:

1°-Compressible intake acceleration with a mean density \( \rho_0 \) and dynamic equilibrium re-establishment at the exit level when equal static pressures occur for the inner air and the outer air \[16\],

\[
D^2 = \frac{m^2}{2g / \rho_0 A^2} = \frac{\gamma \cdot (1 - \gamma)}{\gamma (2 - \Gamma) + \Gamma}.
\]

2°-Incompressible intake acceleration and dynamic equilibrium again re-established at the exit level in the condition of equal static pressures inside and outside \[5\],

\[
D^2 = \frac{\gamma \cdot (1 - \gamma)}{\gamma + 1}.
\]

3°-Incompressible intake acceleration with dynamic equilibrium re-establishment at the exit level by full recovery of the total air pressure, considered through a compressible exiting process with a mean density for the inner air \( \rho_2 = \rho_3 = \rho_4 \) \[6\],

\[
D^2 = \frac{\gamma (1 - \gamma)}{1 + \gamma - \Gamma}.
\]

4°-Compressible intake acceleration with a mean density \( \rho_0 \) and dynamic equilibrium re-establishment at the exit level by full recovery of the total air pressure, considered through an incompressible exiting process with a constant density for the inner air \( \rho_2 = \rho_3 = \rho_4 \),

\[
D^2 = \frac{\gamma (1 - \gamma)}{\gamma (2 - \Gamma) - (1 - \Gamma)}.
\]
5°-Incompressible intake acceleration with dynamic equilibrium re-establishment at the exit level by full recovery of the total air pressure through an incompressible exit, model that derives directly from the previous one when setting \( \Gamma = 1 \),

\[
D^2 = 1 - \gamma .
\]  

(10)

6°-Compressible intake acceleration with a mean density \( \rho_0 \) and dynamic equilibrium re-establishment at the exit level by full recovery of the total air pressure, considered through a compressible exiting process with a mean density for the inner air \( \rho_2 = \rho_3 = \rho_4 \),

\[
D^2 = \frac{1 - \gamma}{2 - \Gamma} .
\]  

(11)

While the first 1° to 4° modeling schemes give physically reasonable RMF characteristics the latest 5°-6° schemes in fact destroy the true RMF characteristic of the stack. They would suggest that at zero heating the outflow is maximal, which is nonsense. This observation shows the extent of modeling instabilities of the simple thermal draught, due to the very slight variations of the parameters in the stack.

Useful to note that these say incomplete characteristics are induced by the heat exchanger itself. Considering thus a heat exchanger alone, working in a vertical pipe with no other intake-outlet pressure losses, the following RMF characteristic appears:

\[
R^2 = \frac{1 - \gamma}{2} .
\]  

(12)

Neglecting the dilatation drag and thus considering an isobaric heating process with entrance acceleration losses only and an isobaric exit condition a continuously increasing RMF characteristic of the stack appears

\[
R^2 = \frac{\gamma}{\Gamma} .
\]  

(13)

These two separate characteristics explain the parabolic aspect of the complete RMF when acceptable limit conditions are introduced. The physically acceptable diagrams of variation of the stack RMF versus the heating rate \( \gamma \) are drafted for comparison in figure 4.

![Figure 4. Stack RMF discharge \( D^2 \) versus air heating intensity \( \gamma \).](image)

The accelerating potential and the expense of heat to perform this acceleration at optimal conditions result from equations (6)-(9). In a practical manner, the velocity \( c_e \) results in regard to the equivalent free-fall velocity (Torricelli) \( c_t \).

\[
c_{2H} = \sqrt{\frac{\gamma \cdot 2gL}{(1 - \gamma)[(2 - \Gamma) + \Gamma]}}, \quad c_{2L} = \sqrt{\frac{\gamma \cdot 2gL}{1 - \gamma^2}}
\]  

(14)  

(15)

In fact these formulae render identical results for the optimal \( \gamma \) values of each (Table 2). Important to mention that the drag losses that could be considerably high depending on the construction of the heat exchanger were up to this point neglected. Some recent, yet unpublished experiments show that the measured velocity may come down to around 50% of the theoretical value when simple obstacles intervene in the flowing channel. Experimental measurements are further required.

For a contraction area ratio of 10 the maximal airflow velocities in the test chamber \( c_e \) of the aeroacoustic tunnel versus the tower height are given in Table 2.
Table 2. Draught vs. tower height for a contraction ratio 10.

<table>
<thead>
<tr>
<th>ℓ</th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>cₑ</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m/s</td>
<td>m/s</td>
<td>m/s</td>
<td>m/s</td>
</tr>
<tr>
<td>7</td>
<td>11.72</td>
<td>4.85</td>
<td>8.28</td>
<td>82.8</td>
</tr>
<tr>
<td>14</td>
<td>16.57</td>
<td>6.86</td>
<td>11.72</td>
<td>117.2</td>
</tr>
<tr>
<td>30</td>
<td>24.26</td>
<td>10.05</td>
<td>17.15</td>
<td>171.5</td>
</tr>
<tr>
<td>70</td>
<td>37.05</td>
<td>15.35</td>
<td>26.20</td>
<td>262.0</td>
</tr>
<tr>
<td>140</td>
<td>52.40</td>
<td>21.71</td>
<td>37.05</td>
<td>370.5</td>
</tr>
</tbody>
</table>

The value of \( cₑ \) was computed according to the simple, incompressible assumption, which renders a minimal estimate for the air velocity in the contracted area. Compressibility whatsoever will tend to increase the actual velocity in the test area, while drag losses, especially in the heat exchanger, will decrease that speed.

3 TURBINE EFFECTS

According to the design in Fig. 16, a turbine is introduced in the SEATTLER facility next to the solar receiver, with the role to extract at least a part of the energy recovered from the sun radiation and transmit it to the electric generator, where it is converted to electricity. The heat from the flowing air is thus transformed into mechanical energy with the payoff of a supplementary air rarefaction and cooling in the turbine. The best energy extraction will take place when the air recovers entirely the ambient temperature before the solar heating, although this desire remains for the moment rather hypothetical. To search for the possible amount of energy extraction, the quotient \( ω \) is introduced, as further defined. Some differences appear in the theoretical model of the turbine system as compared to the simple gravity draught wind tunnel previously described.

The previous analysis of the simple draught shown how easily the hypothesis of isobaric heating leads to an incomplete result, by eliminating the drag produced by the thermal dilatation and the acceleration throw heating, thus reducing the problem to a linear one, without physical anchorage. Considering a non-isobaric relation complicates drastically the model, which becomes completely nonlinear, and the mass flow-rate cannot be expressed by a direct relation. It remains to be analyzed whether such an inconvenient model leads to physically acceptable results for the values of mass flow-rate in the turbine tower.

The heat introduced produces a temperature increase given by,

\[
Δh_{23} = c_pT_2\left(1 - \frac{T_3}{T_2}\right) = ωq(γ),
\]

where the heat received in non isobaric heat exchanger is expressed, through the equation of energy, in the complete form:

\[
q(γ) = c_pT_0\frac{γ}{1−γ}\left(1 - \frac{Γ}{2}\right) \cdot \frac{γ}{Γ(1−γ)} \cdot \frac{m^2}{ρ_0^2A^2} \cdot \frac{Δp_R}{Γ(1−γ)p_0},
\]

(70)

to take also into account the possible pressure losses due to friction in the solar receiver \( Δp_R \).

The absorbed heat (70) will also be used in its complete form in the relation that supplies the pressure at stator exit:

\[
\frac{p_3}{p_2} = \left(1 - \frac{q}{c_pT_2}\right)^{\frac{κ}{κ−1}},
\]

(71)

fact that obviously induces another level of non-linearity. Using also the equation of state, the pressure from the stator exits writes from (71),

\[
p_3 = p_2\left[p_2 - ω\left(γp_0 - \frac{2 - Γ}{2} \cdot \frac{γ}{1−γ} \cdot \frac{m^2}{ρ_0^2A^2} - Δp_R\right)\right]^{\frac{κ}{κ−1}},
\]

(72)

and for the value \( p_2 \), the pressure losses from the entrance through Bernoulli acceleration and in the heater will be now respectively inferred,

\[
p_2 = p_0 - \frac{m^2}{ρ_0^2A^2} \left(\frac{Γ}{2} + \frac{γ}{1−γ}\right) - Δp_R.
\]

(73)
Taking into account the draught from the tower (63) and the fluid brake at exit (64), the equilibrium of static pressure reads

\[
(p_2 - \omega \beta \chi \ p_0) \frac{2}{k-1} \left[ \frac{(p_2 - \omega \beta \chi \ p_0)^k}{\rho \rho_0 \ell} - \beta \right] + \frac{\Gamma}{2} \frac{\dot{m}^2}{\rho_0^2 A^2 g \ell} \ p_0^2 \frac{2}{k-1} - \pi \left[ (1 - \pi) \left( p_2 - \omega \beta \chi \ p_0 \right) \right]^{1/2} = 0
\]

(74)

Here the notation was used:

\[
\pi = \frac{p_0}{\rho_0 \ell} >> 1
\]

(75)

In the followings the undimensionalised flow-rate \( D^2 \) will be considered as the solving variable of the problem, a variable that naturally appears from the previous equation (74), under the form of the ratio

\[
D^2 = \frac{D^2}{\pi} = \frac{\dot{m}^2}{2 \rho_0^2 A^2 g \ell} \ p_0^{2/3} \equiv \frac{c_1^2}{c_0^2} \equiv \left( \frac{c_1}{c_0} \right)^2
\]

(76)

where also naturally the characteristic velocity \( c_0 \) of the air \( c_0 \) appears, namely

\[
c_0 = \sqrt{2RT_0} \approx 415,5 \ m/s
\]

(77)

The characteristic velocity \( c_0 \) is actually related to the local sound velocity in the air \( a_0 \), manifesting proportional to it, so that the relative mass flow-rate can be written in the absolutely equivalent form,

\[
a_0 = \sqrt{\kappa RT_0} \approx 348,2 \ m/s
\]

(78)

in connection with which the relative flow-rate couls also be expressed, in the form

\[
D^2 = \frac{D^2}{\pi} = \frac{\kappa}{2} \frac{c_1^2}{a_0^2} \equiv \frac{\kappa}{2} M_1^2
\]

(79)

in other words this flow-rate is proportional to the squared local Mach number.

From (74) the equation of the flow-rate \( D^2 \) is obtained as a function of the working conditions \( \omega \) and \( \gamma \),

\[
(a \cdot c - b \ D^2)^5[(c - b \ D^2)^{1.4} - c^{2.4}] + d \cdot c^{0.4} \ D^2 (c - e \ D^2)^{2.5} + f \cdot c^{1.4} (c - e \ D^2)^{2.5} = 0
\]

(80)

where the constant coefficients are reproducing the working conditions,

\[
a = \frac{1}{1 - \alpha \gamma}, \quad e = 2 \gamma + \Gamma (1 - \gamma),
\]

\[
c = 1 - \gamma, \quad d = \Gamma \pi,
\]

\[
f = \frac{1}{1 - \pi}, \quad b = e - \alpha \gamma (2 - \Gamma)
\]

(81)

The algebraic non linear equation (80) will be solved using a standard numerical method to obtain the mass flow-rate solutions depending on the different working conditions concerning the heating level applied in the solar receiver \( \gamma \) and respectively the degree of recovery of the heat introduced through the receiver \( \omega \). For a complete recovery of energy \( (\omega=1) \), the numerical solutions are the following (Table 1):

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( D^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.50</td>
</tr>
<tr>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>0.2</td>
<td>(1.280)</td>
</tr>
<tr>
<td>0.3</td>
<td>(0.875)</td>
</tr>
<tr>
<td>0.4</td>
<td>0.61133100</td>
</tr>
<tr>
<td>0.5</td>
<td>0.428961326</td>
</tr>
<tr>
<td>0.6</td>
<td>0.29897500</td>
</tr>
<tr>
<td>0.7</td>
<td>0.199248700</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1098315 and 0.012898027</td>
</tr>
<tr>
<td>0.9</td>
<td>0.0634500 and 0.055130000</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2. The equilibrium flow-rate as a function of the rarefaction \( \gamma \) for \( \omega=1 \)

It proves however that the above given model is not properly reproducing the Stack-Turbine (S-T) characteristic at low heating rates \( (\gamma \to 0) \), while at the upper end \( (\gamma \to 1) \) it acceptably does this.
The resulting discharge characteristic of the tunnel is drawn in dark red. A very slight change in the assumptions could therefore deeply affect the result of the simulation modeling, due to the small overall magnitudes of pressure and density gradients along the S-T channel.

REFERENCES