Abstract: - The paper extends the procedures of wastewater biodegradation process (WBP) identification, as have been first presented by the authors in [11], [12]. These procedures allow identification of all process parameters in both cases they are time constant or time variant. The identification problem is formulated as a condition to vanish the existence relation of the system. This relation is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. As the WBP expresses rational dependences between parameters and some measurable variables, the main idea of these procedures is to use a hierarchical multi layer structure of identification, which allows obtaining string of linear algebraic systems of equations in the unknown parameters. The coefficients of these algebraic systems are functionals depending on the input and output variables evaluated through some testing functions from distribution theory. According to the proposed procedure, in the first layer, only some state equations are evaluated throughout testing functions to obtain a set of linear equations in some parameters. The results of this first layer of identification are utilized for expressing other parameters by linear equations in the next layer. This process is repeated until all parameters are identified. The time variant laws are expressed as finite degree time polynomials whose parameters are included in the set of parameters to be identified. Applications for parameter identification of waste water biodegradation processes are presented. By examples, the potential of the method is revealed.

Key-Words: - Identification; Bioprocesses, Wastewater biodegradation, Distribution theory; Functionals.

1 Introduction
As presented in [11], progresses have been made in the area of continuous-time system identification. Many discussions, methods and results on continuous-time identification are presented in, [2]; [4], [8], [9]; [14], [15], [16].
A novel approach for continuous-time system identification is that based on distribution theory, using deterministic distributions [10] or random distributions [13]. Identification of the non-linear continuous-time systems is far away more complicated. The traditional procedures are based on the Volterra functional series, expressed in time domain [3] or frequency domain [6]. The parameter identification of deterministic nonlinear continuous-time systems (NCTS), modelled by polynomial type differential equation, has been considered by numerous authors, [15], [16]. In [11], it is presented a method for identification of nonlinear continuous time systems (NCTS) considering that the unknown parameters can appear in rational relations with measured variables. Using techniques utilized in distribution approach [7], [8], [9], the measurable functions and their derivatives are represented by functionals on a fundamental space of testing functions. Such systems are common in biotechnology [1], [17], [18].
The main idea from [11] is to use a hierarchical multi layer structure of identification. First, some state equations are utilized to obtain a set of linear equations in some parameters. The results of this first stage of identification are utilized for expressing other parameters by linear equations in the next layer. This process is repeated until all parameters are identified. The time variant laws are expressed as finite degree time polynomials whose parameters are included in the set of parameters to be identified. Applications for parameter identification of waste water biodegradation processes are presented. By examples, the potential of the method is revealed.
model of wastewater biodegradation process is given in Section 2. Section 3, presents some aspects regarding distribution approach of identification. The hierarchical structure of identification and estimation equations takes the space of Section 4. Some experimental results are presented in Section 5, and conclusions in Section 6.

2 Mathematical model of wastewater biodegradation process

We consider a biomethanation process - wastewater biodegradation with production of methane gas that takes place inside a Continuous Stirred Tank Bioreactor whose reduced model is presented in [18]. It is a two phases process. In the first phase, the glucose from the wastewater is decomposed in fat volatile acids (acetates, propionic acid), hydrogen and inorganic carbon under action of the acidogenic bacteria. In the second phase, the ionised hydrogen decomposes the propionic acid CH3CH2COOH in acetates, H2 and carbon dioxide CO2. In the first methanogenic phase, the acetate is transformed into methane and CO2, and finally in the second methanogenic phase, the methane gas CH4 is obtained from H2 and CO2. [1], [17]. The following simplified reaction scheme is considered,

\[
\begin{align*}
S_1 & \rightarrow X_1 + S_2 \\
S_2 & \rightarrow X_2 + P_1 
\end{align*}
\]

(1)

where: \( S_1 \) represents the glucose substrate, \( S_2 \) the acetate substrate, \( X_1 \) is the acidogenic bacteria, \( X_2 \) the aceticlastic methanogenic bacteria and \( P_1 \) represents the product, i.e. the methane gas. The reaction rates are denoted by \( \phi, \phi_1 \).

The corresponding dynamical model is

\[
\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ S_1 \\ S_2 \\ P_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -k_1 & 0 & 0 \\ -k_2 & 0 & D & S_m & 0 \\ 0 & 1 & 0 & \phi_1 & -D \\ 0 & -k_3 & 0 & P_1 & 0 \\ 0 & k_4 & 0 & 0 & -Q_1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ S_1 \\ S_2 \\ P_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
\]

(2)

where the state vector of the model is

\[
\xi = [X_1, S_1, X_2, S_2, P_1]^T = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5]^T
\]

(3)

whose components are concentrations (g/l).

The reaction rates are nonlinear functions of the state components, expressed as

\[
\phi = \phi(\xi) = [\phi_1(\xi) \ \phi_2(\xi)]^T
\]

(4)

The vector of feed rates and of rates of removal of components is denoted

\[
F = [0 \ \mathbf{D} \cdot S_m \ 0 \ 0 \ -Q_1]^T
\]

(5)

where, \( D \) is the dilution rate, a scalar in this particular case, \( S_m \) represents the concentration of the externally influent substrate–glucose, \( Q_1 \) is the methane gas outflow rate.

The dynamical model (2) can be compactly written

\[
d\xi/dt = K \cdot \phi(\xi) - D \cdot \xi + F.
\]

(6)

In fact, this model describes the behavior of an entire class of biotechnological processes. It referees as the general dynamical state-space model of this class of bioprocesses [1]. In (6), \( K \) is the so-called matrix of the yield coefficients \( k_{ij} \)

\[
K = \begin{bmatrix} 1 & -k_1 & 0 & k_2 & 0 \\ 0 & 0 & 1 & -k_1 & k_4 \end{bmatrix}
\]

(7)

The reaction rates for this process are given by the Monod law

\[
\phi_1(\xi) = \mu_1 \cdot \frac{S_1 \cdot X_1}{K_{M_1} + S_1},
\]

(8)

and the Haldane kinetic model

\[
\phi_2(\xi) = \mu_2 \cdot \frac{S_2 \cdot X_2}{K_{M_1} + S_1 + S_2 / K_{i}},
\]

(9)

where \( K_{M_1}, K_{M_2} \) are Michaelis-Menten constants; \( \mu_1, \mu_2 \) represent specific growth rates coefficients and \( K_i \) is the inhibition constant. For simplicity, shall we denote the plant parameters by the vector

\[
\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T
\]

(10)

where

\[
\begin{align*}
\theta_1 &= k_1; \theta_2 = k_2; \theta_3 = k_3; \theta_4 = k_4 \\
\theta_5 &= \mu_1; \theta_6 = \mu_2 \\
\theta_7 &= K_{M_1}; \theta_8 = K_{M_2}; \theta_9 = K_i
\end{align*}
\]

Because the dilution rate \( D \) can be externally modified, it will be considered the third component of the input vector \( u = [u_1, u_2, u_3]^T \). The other two components of \( u \) are the concentration \( S_m \) and the methane gas outflow rate \( Q_1 \), so,

\[
u_1 = S_m; \nu_2 = Q_1; \nu_3 = D;
\]

(14)

Usually \( Q_1 \) depends on state variables, \( Q_1 = \Psi(\xi) \), determining a feedback to the input \( u_3 \). Written explicitly by components, the state equations (2) or (6), within the above notations, takes the form

\[
\begin{align*}
\xi_1 &= \phi_1 - u_1 \cdot \xi_1 \\
\phi_1 &= \theta_1 \cdot \frac{\xi_1 \cdot \xi_2}{\theta_0 + \xi_2} \\
\xi_2 &= -\theta_0 \cdot \phi_1 - u_1 \cdot \xi_2 + u_1 \cdot u_3 \\
\phi_2 &= \theta_1 \cdot \frac{\xi_3 \cdot \xi_4}{\theta_0 + \xi_4 + \theta_0 \cdot \xi_4^2} \\
\xi_3 &= \phi_2 - u_3 \cdot \xi_3 \\
\phi_3 &= \theta_3 \cdot \frac{\xi_5 \cdot \xi_6}{\theta_0 + \xi_6 + \theta_0 \cdot \xi_6^2}, \theta_0 = 1 \\
\xi_4 &= \theta_4 \cdot \phi_1 - \theta_5 \cdot \phi_2 + u_1 \cdot \xi_4 + u_3 \cdot \xi_4 \\
\phi_4 &= \theta_4 \cdot \frac{\xi_7 \cdot \xi_8}{\theta_0 + \xi_8 + \theta_0 \cdot \xi_8^2}, \phi_5 = 1 \\
\xi_5 &= \theta_5 \cdot \phi_2 - \theta_6 \cdot \phi_3 + u_3 \cdot \xi_5 \\
\phi_6 &= \theta_5 \cdot \frac{\xi_9 \cdot \xi_{10}}{\theta_0 + \xi_{10} + \theta_0 \cdot \xi_{10}^2}
\end{align*}
\]

(15-21)
3 Distribution approach of identification

Let us denote by \( \Phi_{\phi} \) the fundamental space from distribution theory [5], of the real fundamental functions,

\[
\phi : \mathbb{R} \to \mathbb{R}, t \to \phi(t),
\]

with compact support \( T \), having continuous derivatives at least up to the order \( n \). A distribution is a linear, continuous (in the above topology) real functional on \( \Phi_{\phi}, F : \Phi_{\phi} \to \mathbb{R}, \phi \to F(\phi) \in \mathbb{R} \). Let

\[
q : \mathbb{R} \to \mathbb{R}, t \to q(t)
\]

be a function which admits a Riemann integral on any compact interval \( T \) from \( \mathbb{R} \). Using this function, a unique distribution \( F_q : \Phi_{\phi} \to \mathbb{R}, \phi \to F_q(\phi) \in \mathbb{R} \) can be build by the relation \( F_q(\phi) = \int q(t) \cdot \phi(t) \, dt, \forall \phi \in \Phi_{\phi} \).

In distribution theory, the notion of distribution \( k \)-order derivative, \( k = 0, n \), \([5]\), is,

\[
F_q^{\phi_k}(\phi) = (-1)^k \cdot F_q (\phi^{k}), \forall \phi \in \Phi_{\phi},
\]

\[
\phi \to F_q^{\phi_k}(\phi) = (-1)^k \cdot \int q(t) \cdot \phi^{k}(t) \, dt \in \mathbb{R}
\]

Let now consider a dynamical continuous time system with \( n_i \) inputs, \( u : \mathbb{R} \to \mathbb{R}^n, t \to u(t), u \in \Omega \) and \( n_i \) outputs, \( y : \mathbb{R} \to \mathbb{R}^n, t \to y(t), y \in \Gamma \), where \( \Omega \) represents the set of admissible inputs and \( \Gamma \) is the set of possible outputs. It can be expressed by a differential operator,

\[
q_{\theta(u,y)} = Q(u,y, \theta) = 0
\]

whose expression depends on a vector of parameters \( \theta = [\theta_1, ..., \theta_i, ..., \theta_p]^\top \). The operator (25), whose class can be determined, represents a family of models with a given structure in constant parameters. A special case is the model (25) expressing a linear relation in the parameters

\[
q_{\theta(u,y)} = Q(u,y, \theta) = \sum_{i=1}^{n_i} w_i \cdot \phi - \nu = w^\top \cdot \phi - \nu,
\]

where \( w_i \) and \( \nu \) represent a sum of the derivatives of some known, possible nonlinear, functions \( \psi_i, \psi_i' \), with respect to the input and output variables,

\[
w_i = \sum_{j=1}^{n_j} [\psi_i'(u,y)]^{(n_i)}, i = 1:p, \nu = \sum_{j=1}^{n_j} [\psi_i'(u,y)]^{(n_i)}
\]

Parameters \( p_i, n_i, p_n, n_n \) are given integer numbers. The identification problem, into condition (26), has a unique solution. An identification problem means to determine the parameter \( \theta = \theta^* \), given the priori information on the model structure \( Q \), (25), and the observed input-output pair \((u_i, y_i), \theta = \theta^*(u_i, y_i, Q)\), in such a way that, \( q_{\theta^*(u_i,y_i)}(t) = 0, \forall t \in \mathbb{R} \).

Now let us consider known the set of continuous time scalar functions (27),

\[
w_i(t) = \sum_{j=1}^{n_j} [\psi_j'(u(t), y(t))]^{(n_j)}, i = 1:p
\]

\[
y(t) = \sum_{j=1}^{n_j} [\psi_j'(u(t), y(t))]^{(n_j)}\cdot t = \sum_{j=1}^{n_j} [\psi_j'(u(t))]^{(n_j)} \cdot t
\]

Based on these functions, the regular distributions \( F_{w_i}, i = 1:p \), are generated by relations,

\[
F_{w_i} = F_{w_i}^{(n_i)} : \Phi_{\phi} \to \mathbb{R}, \phi \to F_{w_i}(\phi)
\]

They constitute the row vector,

\[
F_l(\phi) = [F_{w_1}(\phi), ..., F_{w_p}(\phi)] \in \mathbb{R}^p
\]

Also, the regular distribution \( F_{\nu} \), is

\[
F_{\nu} = F_{\nu}^{(n_\nu)} : \Phi_{\phi} \to \mathbb{R}, \phi \to F_{\nu}(\phi)
\]

Into this conditions, any input-output pair \((u, y)\) observed from the system (25) is described by a pair of regular distribution \((F_u, F_y)\) for any \( \phi \in \Phi_{\phi} \).

In such a way, the problem of identification regarding the parameters of the real system (25) can be represented by distributions. For example, the regular distribution generated by the continuous function \( q_{\theta(u,y)} \) from (25), into the specific case of (26) is related to the parameter vector \( \theta \) as

\[
F_{q_{\theta(u,y)}}(\phi) = \sum_{i=1}^{n_i} F_{w_i}(\phi) \cdot \theta_i - F_{\nu}(\phi), \forall \phi \in \Phi_{\phi}\]

If a triple \((u^*, y^*, \theta^*)\) is a realization of the model (25), then the identity (34) takes place,

\[
F_{q_{\theta(u,y)}}(\phi) = F_{q_{\theta^*(u,y)}}(\phi) = 0, \forall \phi \in \Phi_{\phi}
\]

and vice versa, if an input-output pair \((u^*, y^*)\) of the family of models (25), with unknown parameter \( \theta \), generates a distribution

\[
F_{q_{\theta(u,y)}}(\phi) = \sum_{i=1}^{n_i} F_{w_i}(\phi) \cdot \theta_i - F_{\nu}(\phi)
\]

which satisfies \( F_{q_{\theta(u,y)}}(\phi) = F_{q_{\theta^*(u,y)}}(\phi) = 0, \forall \phi \in \Phi_{\phi} \), then \( \theta = \theta^* \). As \( \theta \) has \( p \) components it is enough a choice (utilize) a finite number \( N \geq p \) of fundamental function \( \phi_i, i = 1:N \) and to build an algebraic equation,

\[
F_{w_i} \cdot \theta = F_{w_i}
\]

where \( F_{w_i} \) is an \((N \times p)\) matrix of real numbers

\[
F_{w_i} = [F_{w_1}(\phi_1); ..., F_{w_i}(\phi_i); ..., F_{w_p}(\phi_p)]^\top
\]

where \( i \)-th row \( F_{w_i}(\phi_i) \) is given by (31). The symbol \( F \) denotes an \( N \)-column real vector built from (32),
\[ F_i = [F_1(\varphi, \ldots, F_n(\varphi)]^T. \]  

(38)  

When only the restriction \((u, y)\) of the pair \((u, y)\) on the time interval \(T\), is available, then one must choose \(\varphi\), such that \(\text{supp}(\varphi) \subset T \subset \mathbb{R}\), \(i = 1 : N\). If \(r = \text{rank}(F_i) = p\), then a unique solution is obtained.

\[ \vartheta = (F^*_u \cdot F_u)^{-1} \cdot F_u \cdot \vartheta = \vartheta^* \]  

(39)

4 The hierarchical structure of identification and estimation equations

Consider all state variables accessible for measurements so \(y = \xi\). The dynamical system \((15) + (21)\) contains rational dependences between parameters and measured variables. To obtain linear equations in unknown parameters, the identification problem is split in several simpler interlinked identification problems called identification layers.

Based on the specific structure of this system, it is possible to group the state equations, in such way to determine five interconnected identification problems of the type \((39)\), labelled Layer_*, \(* = a, b, c, d, e\). They are organized in a hierarchical structure. First, in Layer_a, some state equations are utilized to obtain a set of linear equations in some parameters. The results of this first stage of identification are utilized for expressing other parameters by linear equations in Layer_b. This process is repeated in the other layers until all parameters are identified. For each identification layer, the same type of procedures and numerical algorithms are applied.

Layer_a: Identification of \(\theta\).

Substituting expression \(\phi\) from \((15)\) into \((17)\) we obtain, the Layer_a model \((25)\)

\[ q_{\theta(a,\gamma)} = (\xi_2 + u_1 \cdot \xi_2 \cdot \gamma - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \]  

(40)

characterized by \(\theta^* = [\theta^*_1 \theta^*_2]=[\theta^*_1 \theta^*_2]\), \(p^* = 1\)

\[ F_n^a(\varphi) = \frac{1}{\xi} \{[\xi_2(t) - \xi_1(t)] \cdot \varphi(t) \} \cdot dt + \{[u_1(t) \cdot \xi_2(t)] \cdot \varphi(t) \} \cdot dt \]  

(42)

Layer_b: Identification of \(\theta, \theta\).

Considering known \(\theta^* = \hat{\theta}^*\) from the Layer_a, and substituting \((16)\), equation \((17)\) becomes,

\[ \dot{\xi}_2 = -\theta_1 \cdot \theta_2 \cdot \frac{\xi_1}{\theta_1 + \xi_2} - u_3 \cdot u_1 + u_3 \]

The Layer_b model \((25)\) is now,

\[ q_{\theta(b,\gamma)} = (\xi_2 + u_1 \cdot \xi_2 \cdot \gamma - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_1 - \)  

\[ -(\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_2 \]  

(41)

characterized by \(\theta^* = [\theta^*_1 \theta^*_2 \theta^*_3] = [\theta^*_1 \theta^*_2 \theta^*_3]\), \(p^* = 2\)

\[ F_n^b(\varphi) = \frac{1}{\xi} \{[\xi_2(t) - \xi_1(t)] \cdot \varphi(t) \} \cdot dt + \{[u_1(t) \cdot \xi_2(t)] \cdot \varphi(t) \} \cdot dt \]  

(44)

Layer_c: Identification of \(\theta, \theta\).

Considering known \(\theta^* = \hat{\theta}^*\) from the Layer_b, the estimated expression \(\phi\), of the rational \(\phi\), is

\[ \hat{\phi} = \hat{\theta}_1 \cdot \frac{\xi_1 + \xi_2}{\theta_1 + \xi_2} \]  

(42)

whose time expression is \(\hat{\phi}(t) = \frac{\xi_1(\xi_2(t) - \xi_2(t))}{\theta_1 + \xi_2(t)}\).

Substituting expression \(\phi\) from \((18)\) and \((42)\) instead of \(\phi\) into \((20)\) we obtain,

\[ \hat{\xi}_2 = \hat{\theta}_1 \cdot \hat{\phi}_2 + \frac{\xi_1}{\theta_1 + \xi_2} - u_3 \cdot \xi_4 \]

which determines the Layer_c model \((25)\)

\[ q_{\theta(c,\gamma)} = (\xi_2 + u_1 \cdot \xi_2 \cdot \gamma - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_1 - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_2 \]  

(43)

characterized by \(\theta^* = [\theta^*_1 \theta^*_2 \theta^*_3] = [\theta^*_1 \theta^*_2 \theta^*_3], p^* = 2\)

\[ F_n^c(\varphi) = \frac{1}{\xi} \{[\xi_2(t) - \xi_1(t)] \cdot \varphi(t) \} \cdot dt + \{[u_1(t) \cdot \xi_2(t)] \cdot \varphi(t) \} \cdot dt \]  

(45)

Layer_d: Identification of \(\theta, \theta, \theta\).

Considering known \(\theta^* = \hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3\) from the Layer_c, and substituting \((19)\) in equation \((20)\) where \(\phi\) is replaced by \(\hat{\phi}\) we obtain,

\[ \hat{\xi}_2 = \hat{\theta}_2 \cdot \hat{\phi}_2 - \frac{\xi_1 + \xi_2}{\theta_1 + \xi_2} - u_3 \cdot \xi_4 \]

The Layer_d model \((25)\) is now,

\[ q_{\theta(d,\gamma)} = (\xi_2 + u_1 \cdot \xi_2 \cdot \gamma - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_1 + + (\xi_2 + u_1 \cdot \xi_2 \cdot \gamma - (\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_2 - \)  

\[ -(\xi_2 - u_1 \cdot \xi_2 + u_1 \cdot u_3) \cdot \theta_3 \]  

(44)

characterized by \(\theta^* = [\theta^*_1 \theta^*_2 \theta^*_3] = [\theta^*_1 \theta^*_2 \theta^*_3], p^* = 3\)

\[ F_n^d(\varphi) = \frac{1}{\xi} \{[\xi_2(t) - \xi_1(t)] \cdot \varphi(t) \} \cdot dt \]  

(45)
\[ F^e_w(\varphi) = \int \left[ -\xi(t) \cdot \varphi^0(t) \right] dt + \int \left[ u_i(t) \cdot \xi(t) \right] \cdot \varphi^0(t) \cdot dt + \int \left[ -\tilde{\xi}(t) \cdot \tilde{\varphi}(t) \cdot \varphi^0(t) \right] dt ; F^e_w(\varphi) = \frac{F^e_w(\varphi)}{F^C_w(\varphi)} \]

\[ F^c_w(\varphi) = \frac{1}{2} \left[ \xi(t) \cdot \varphi^0(t) \right] dt + \int \left[ -\xi(t) \cdot \xi(t) \right] \cdot \varphi^0(t) \cdot dt + \int \left[ \tilde{\xi}(t) \cdot \tilde{\varphi}(t) \cdot \varphi^0(t) \right] dt \]

**Layer e: Identification of \( \theta_4 \)**

Considering known \( \theta_3 = \hat{\theta}_3 ; \alpha = \hat{\alpha} ; \beta_k = \hat{\beta}_k \), from the Layer d identification, the estimated expression \( \hat{\varphi}_2 \), of the nonlinear function \( \varphi_2 \), is

\[ \hat{\varphi}_2(\varphi) = \hat{\theta}_h \cdot \frac{\xi(t) \cdot \xi(t)}{\theta_3 + \xi(t) + \theta_4 \cdot \tilde{\xi}(t)} \]

where time expression is

\[ \tilde{\xi}(t) = -u_i \cdot \xi(t) + \tilde{\varphi}_2 - u_2 \]

which determines the Layer e model (25)

\[ q_{\theta_4 \theta_5} = (\hat{\varphi}_2) \cdot \theta_4 - (\hat{\xi}_s + \hat{\xi}_d + u_2) \]

characterized by \( \theta^* = [\theta^*] = [\theta_4] \), \( \theta^* = \beta \)

\[ F^e_w(\varphi) = \int \left[ \hat{\varphi}_2(t) \cdot \varphi^0(t) \right] dt ; F^e_w(\varphi) = \frac{F^e_w(\varphi)}{F^C_w(\varphi)} \]

Also,

\[ F^c_w(\varphi) = \int \left[ -\xi(t) \cdot \varphi^0(t) \right] dt + \int \left[ u_i(t) \cdot \xi(t) \cdot \varphi^0(t) \right] dt + \int \left[ u_i(t) \cdot \xi(t) \cdot \varphi^0(t) \right] dt \]

**7 Experimental results**

The model given by (15) - (21) and the hierarchical identification procedure developed in this paper has been implemented in Matlab. Three types of experiments were performed. 1. Noise free; 2. Constant parameters but output measurements are noise contaminated; 3. Some process parameters have random variations around constant values. Twelve types of testing functions \( \varphi(t) \), characterized by a bounded support \( T = [t_a, t_b], t_a < t_b \) are considered. All of these accomplish the condition \( \varphi(t)=0, t \notin [t_a, t_b] \), The nonzero restriction, is of the form \( \varphi(t) = \alpha \cdot \beta(t_a, t_b) \cdot \Psi(t, t_a, t_b) \in \Phi_d \), where, for \( p \geq n + 1 \)

\[ \Psi(\varphi) = \Psi(t, t_a, t_b) \in C^r([t_a, t_b]) \]

is one of the four types, 1. Exponential: \( \Psi(t) = \exp \left[ \frac{t_a \cdot t_b}{|t(t_a) - t(t_b)|} \right] \)

2. Sinusoidal: \( \Psi(t) = \sin \left[ \pi \cdot \left( t(t_a) - t(t_b) \right) / \left( t(t_a) - t(t_b) \right) \right] \)

3. Polynomial: \( \Psi(t) = \left( t(t_a) - t(t_b) \right)^n \cdot \left( t(t_a) - t(t_b) \right)^n \)

4. Product: \( \Psi(t) = f_g(t) \cdot f_b(t) \), where

\[ f_g \in C([t_a, t_b]) \], \( f_b \in C^r([t_a, t_b]) \), \( p_g \cdot p_b \geq n + 1 \)

For each of the four types, three variants can be implemented with respect to the coefficient \( \beta = \beta(t_a, t_b) \). Here, \( \alpha \) is a scaling factor.

a. Free amplitude: \( \beta(t_a, t_b) = 1 \), \( \forall t_a, t_b \)

b. Normalized peak: \( \beta(t_a, t_b) = 1 / \max_{\varphi(t)} |\Psi(t, t_a, t_b)|, \forall t_a, t_b \)

c. Normalized area:

\[ \beta(t_a, t_b) = 1 / \int_{t_a}^{t_b} |\Psi(t, t_a, t_b)|, \forall t_a, t_b \]

**Fig.1** shows the noise free system time response.

**Fig.1** Time response for noise free system

<table>
<thead>
<tr>
<th>Noise free identification results</th>
<th>Real values</th>
<th>Identified values</th>
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<tr>
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<td>0.75000000000000000000</td>
<td></td>
</tr>
<tr>
<td>1.00000000000000000000</td>
<td>1.00000000000000000000</td>
<td></td>
</tr>
<tr>
<td>20.99999362172800000000</td>
<td>20.99999362172800000000</td>
<td></td>
</tr>
</tbody>
</table>

For this identification, 3 testing functions \( \varphi_1, \varphi_2, \varphi_3 \) of the type \( p_c = 2c \) (sinusoidal-normalized area), of the degree \( p = 4 \), on the intervals \( T_1 = (0, 5) \), \( T_2 = (5, 10) \), \( T_3 = (10, 15) \) has been utilized. Fig.2 shows the noise contaminated measured variables utilized in identification.

**Fig.2** Noise contaminated measured variables

<table>
<thead>
<tr>
<th>Noise contaminated identification</th>
<th>Real values</th>
<th>Identified values</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.75000000000000000000</td>
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<tr>
<td>20.99999362172800000000</td>
<td>22.94696065372222</td>
<td></td>
</tr>
</tbody>
</table>
8 Conclusion

Through this research has been proved that it is possible to identify all parameters of continuous time nonlinear systems even if they are related to measured variables by rational expressions. This is possible if the identification problem is formulated as a set of interconnected identification problems with linear dependences between parameters and measured variables. The problem of functionals based identification consistency has to be analyzed for a broader class of nonlinear systems.

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References:

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