Optimization of an Electric Network Operation By Reactive Power and Voltage
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Abstract: To solve a problem of a complex optimization for an electric system at simultaneous change of all regulated parameters is rather difficult. To change active and reactive power of a station, node voltages, and complex transformation coefficients of regulated transformers pose a problem of high dimension that requires a long calculation at an existing computers. Usually, the optimization problems are solved in two steps, first optimization of the active power distribution and second the optimization of the electric network operation by reactive power and voltage.

Key-words: electric network; optimization; reactive power; voltage; load; energy; operation; synchronous compensator.

Introduction
The problem of optimization of the electric network operation by reactive power and voltage can be formulated in the following way.

We have to minimize the total loss of the active power under the limitations in the form of equalities on the conditions of the power balance \( \Delta P_S \) at every node of the network except the balancing one

\[
\hat{S}_s - \bar{I}_s V_s = 0, \quad s = 1, 2, \ldots, S.
\] (1)

And limitations in the form of inequalities on the active power of the balancing node

\[
P_r^\text{min} \leq P_r \leq P_r^\text{max},
\] (2)

On the reactive power of the sources

\[
Q_j^\text{min} \leq Q_j \leq Q_j^\text{max}, \quad j = 1, 2, \ldots, j
\] (3)

On voltages at the load points of the network

\[
V_s^\text{min} \leq V_s \leq V_s^\text{max}, \quad s = 1, 2, \ldots, S
\] (4)

On active power at the power controlled parts of the network

\[
P_L^\text{min} \leq P_L \leq P_L^\text{max}, \quad l = 1, 2, \ldots, L
\] (5)

On currents at the current controlled parts of the network

\[
I_i^\text{min} \leq I_i \leq I_i^\text{max}, \quad i = 1, 2, \ldots, I
\] (6)

On regulating e.m.f. of the regulated transformers

\[
e_r^\text{min} \leq e_r \leq e_r^\text{max}, \quad r = 1, 2, \ldots, R
\] (7)

On the base points voltages of the electric network

\[
V_t^\text{min} \leq V_t \leq V_t^\text{max}, \quad t = 1, 2, \ldots, T
\] (8)

Limitations in the form of inequalities include limitations on independent variables of optimization-reactive power of the sources (3), base point voltages (8), regulating contacts of the regulated transformers.

Besides, this group of limitation includes functional limitations on the active power of the balancing node (2), active power at the power controlled parts of the network (5), currents at the current controlled parts of the network (6), voltage at the load points of the network.

We call regulating e.m.f. the modules of voltage increments on the windings of transformers or autotransformers received as the results of regulation means of the constructions.
Usually optimization of reactive power distribution is divided in two problems—exploitation one and design one. In the first case optimum load of the exciting sources of reactive power is defined, while the active power loss in the electric network $\Delta P \Sigma$ is used as the optimization criterion. In the second case the problem of installation of new control devices is solved together with the problem of optimal loading of exciting sources of reactive power. In this case one should take into account expenses connected with new device installation directly and indirectly through active power loss and resulting extra fuel at the power stations to compensate this loss. In general expenses necessary for compensation of active power loss in the network and exciting sources of reactive power, on installation and exploitation of additional control devices can be defined as follows:

$$ C = \Delta P \Sigma \tau + \sum_{j=j+1}^{N} \left( Q_j C_j \left( P_N + P_j \right) + \Theta \Delta \Theta_j \right) $$

$$ j = j + 1, j + 2, \ldots, N $$

(9)

Where $\Delta P \Sigma$ stands for total loss of active power in electric network; $\Theta$ - specific expenses on compensation of active power loss; $\tau$ - time of maximum loss; $j$–number of points, where additional control devices are installed; $Q_j$–reactive power of the control device installed in the $j$-th point; $C_j$ – specific price of the control device type, planned for installation in that point (for existing reactive power source); $P_N$- norm of the efficiency coefficient for capital investment; $P_j$ - coefficient for amortization and exploitation expenses; $\Delta \Theta_j$ - energy loss in control devices. Generally for an optimization problem of an electric network operation the following function can be used as an optimum criterion

$$ \phi = \Delta P \Sigma + \sum_{j=j+1}^{N} \left( Q_j C_j \left( P_N + P_j \right) + \Theta \Delta \Theta_j \right) $$

(10)

The function is a result of division of (9) over $\Theta \tau$. Naturally, extremism of (9) coincides with that of (10). When solving a design problem of optimization function (10) is to be optimized, while in an exploitation problem the second term is equal to zero and the active power loss in the electric network is to be optimized. The problem of reactive power and voltage optimization in the electric network is formulated in that case as a problem of minimization of (10) with the limitations in the form of equalities (1) and inequalities (2)…(8).

Synchronous generators of power stations, synchronous compensators and batteries of static capacitors are used as regulated reactive power sources in regional electric networks.

Synchronous generators of power stations are the main sources of reactive power in electric networks. Specific loss of active power for turbo generators is equal to 0.003-0.0055 kW/Kvar, and for hydro generators- 0.0064-0.01 kW/Kvar. Synchronous compensators can be used in power systems as for generation as for consummation of the reactive power. Specific active power loss for reactive power generation in modern synchronous compensators changes from 0.0109 to 0.025 kW/Kvar. Batteries of static capacitors are used nowadays for voltages from 6 to 110kV. One of the main advantages of capacitors batteries is a small specific active power loss (about 0.3%).

Active power loss in synchronous generators of power stations are proportional to the square of load and are calculated by introducing the corresponding active resistors in the scheme of calculation for the network. In that case one does not need to take into account open circuit loss in (10), as they have no influence on the optimization result at given working machines.

If the capacitor batteries are used as reactive power sources then energy loss $\Delta \Theta_j$ is calculated using the following expression

$$ \Delta \Theta_j = 0.003 Q_j T_j $$

(11)
Where 0.003 is the specific active power loss in the battery; \( T_j \) – time of the battery operation, equal to 7000 hours for non-regulated batteries and 5000…6000 hours for regulated ones.

Energy loss in additional synchronous compensators is the most difficult point in calculation. In general active power losses are defined according to the formula

\[
\Delta P_j = K_a K_f \Delta P_{NJ} + (1 - K_a) \Delta P_{NJ} \left( \frac{Q_j}{Q_N} \right)^2, \quad (12)
\]

Where \( \Delta P_{NJ} \) - power loss in synchronous compensators at nominal reactive power; \( K_f \) - coefficient comparing maximum loss time and working time of the synchronous compensator; \( K_a \) - coefficient defining the part of the open circuit loss in total loss. Usually it is equal to 0.3…0.5. Knowing \( \Delta P_j \), we define energy loss.

\[
\Delta \tau_{sc} = \Delta P_j \tau_{sc} \quad (13)
\]

Where maximum loss time \( \tau_{sc} \) in the zone of most probable operation time of the compensator \( (T_j = 4000-8000 \text{ hours}) \) is equal 0.2 \( T_j \).

Direct application of the expression (12) at the optimization of the reactive power distribution makes the algorithm of the solution very complicated, while for gradient methods of optimization it causes significant difficulties because of the singularities in the first derivatives of the functions \( \Delta P_j(Q_j) \) in the transition points from one to two synchronous compensators, from two to three compensators and so on. Thus it is reasonable to approximate the functions as a polynomial type.

\[
\Delta P_j(Q_j) = a_{0j} + a_{1j} Q_j, \quad (14)
\]

Where \( a_{0j} \) and \( a_{1j} \) are the approximation coefficients.

The least square method can be used for approximation. Using the function table \( \{X_j, y_i\} \quad i=1, 2, 000, N \) one can define a polynomial power \( M<N \)

\[
Y(x) = a_0 + a_1 x + \ldots + a_k x^k + \ldots + a_M x^M \quad (15)
\]

Sufficiently close to the function. The method gives the possibility to choose a polynomial, which minimizes the function

\[
\sum_{i=2}^{N} \left[ y_i - y(x_i) \right]^2 = f(a_0, a_1, \ldots, a_M) \quad (16)
\]

To define the system of equations, describing the polynomial coefficients \( a_0, a_1, \ldots, a_M \), one should differentiate (16) over every coefficient \( a_k \) and set the obtained derivatives equal to zero. As a result we have the system of equations

\[
\sum_{j=0}^{M} a_j \sum_k + j = T_k, \quad K = 0,1,\ldots, M \quad (17)
\]

In Eq. (17) \( \sum_{i=1}^{N} x_i^k = S_k; \quad \sum_{i=1}^{N} y_i x_i^k = T_k \).

In our case function \( \Delta P_j(Q_j) \) is Approximated by a straight line, that is \( M=1 \). So the system (17) is transformed to the form

\[
a_0 S_0 + a_1 S_1 = T_0; \quad a_0 S_1 + a_1 S_2 = T_1 \quad (18)
\]

Where

\[
S_0 = N; \quad S_1 = \sum_{i=1}^{N} x_i; \quad S_2 = \sum_{i=1}^{N} x_i^2 \quad T_0 = \sum_{i=1}^{N} y_i; \quad T_1 = \sum_{i=1}^{N} x_i y_i \quad (19)
\]
Substituting (19) in (18) we get

\begin{align*}
    a_0 N + a_1 \sum_{i=1}^{N} x_i^2 &= \sum_{i=1}^{N} y_i; \\
    a_0 \sum_{i=1}^{N} x_i + a_1 \sum_{i=1}^{N} x_i^2 &= \sum_{i=1}^{N} x_i y_i 
\end{align*}

(20)

As a result of the system (20) solution the coefficients \(a_0\) and \(a_1\) are defined.

**Conclusion**

The considered method is used to approximate active power loss in synchronous compensators as a function of the generated reactive power. The approximation coefficients (12) are defined for the applied nowadays synchronous compensators in the cases of one, two and three compensators installed in one point.

So, when solving the optimization problem of the reactive power distribution in power systems, the electric energy loss in synchronous generators is taken into account through introducing active resistors in the scheme of calculation, loss in synchronous compensators is taken into account using approximation polynomials (14), and the loss in capacitor batteries is taken into account using expression (11).

**References**