Scattering of SH-waves and Ground Motion by a removable rigid cylindrical inclusion and a crack in half space

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Abstract: - In this paper, scattering of SH-waves by an elastic half space with a removable rigid cylindrical inclusion and a crack in any position and direction is studied with Green's function, complex function and multi-polar coordinate method. First, a suitable Green's function is constructed, which is the fundamental solution of the displacement field for a half space with a removable rigid cylindrical inclusion impacted by an out-plane harmonic line source loading at an arbitrary point in half space. Then a crack in any position and direction is constructed by means of crack-division in half space. Finally the displacement field and stress field are established in the case of coexistence of the inclusion and crack, and the expression of the ground motion of the horizontal surface for incident SH wave is given. According to numerical examples, the influences of different parameters on the ground motion are discussed.
Key-Words: - Scattering of SH-waves; Crack; inclusion; Green's function; Ground motion

1 Introduction

Scattering of elastic waves by the inclusion and crack has been a hot topic in recent years[1,2]. At present, most studies concerning scattering of SH-wave by the inclusion and crack focus on the radial crack, which originates from the boundary of inclusion and along the radius. In fact, when the forces are applied to the composite materials containing inclusions, the cracks are often found in the vicinity of the inclusions. The purpose of this paper is to study scattering of SH-waves by an interacting crack with any position and a removable rigid cylindrical inclusion in half space, which can supply some beneficial references to the strength designing and non-destructive inspection of composite materials.

Scattering of SH-waves by a crack in any position, any direction and limited lengths is studied in elastic half space with a removable rigid cylindrical inclusion by Green's function[3-5]. Firstly the scattered wave excitated by a removable rigid cylindrical inclusion in half space is constructed; next a suitable Green's function for the present problem is provided, which is the fundamental solution of the displacement field for an elastic half space containing a removable rigid cylindrical inclusion impacted by a time harmonic out-plane line source loading at an arbitrary point in half space. Then, a crack in any position and direction is constructed in half space by the method of crack-division. Finally the expressions of displacement field and stress field are established in the case of inclusion and the crack coexistent, and the ground motion of the horizontal surface is studied further.

2 Model and Governing equation

The model of the half space with the presence of a removable rigid cylindrical inclusion and a crack of any position is shown in Fig. 1. ρ and ρ_f are the mass densities of the media and the inclusion respectively.

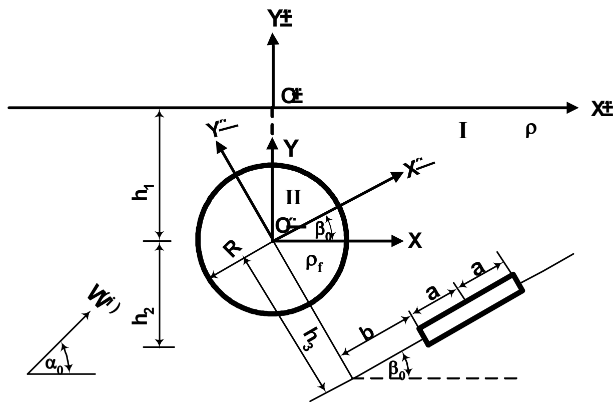

Fig. 1 Half space model with a removable rigid cylindrical inclusion and a crack

3 Green's function

3.1 The Governing Equation and the Boundary Condition

The Green's function in this paper is the solution of the displacement field for an elastic half space with a removable rigid cylindrical inclusion impacted by a time harmonic out-plane line source loading at an arbitrary point which is in the half space. The displacement is expressed as $We^{-i\omega t}$ and the displacement function W satisfies the following governing equation

$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}+k^2 G=0 \quad (1)$$

The stresses due to Eq. (1) are given by

$$\tau_{\theta z}=i\mu\left(\frac{\partial G}{\partial z}e^{i\theta}-\frac{\partial G}{\partial \bar{z}}e^{-i\theta}\right)$$
$$\tau_{rz}=\mu\left(\frac{\partial G}{\partial z}e^{i\theta}+\frac{\partial G}{\partial \bar{z}}e^{-i\theta}\right) \quad (2)$$

Boundary condition

$$G_{\mathrm{I}|r=R}=G_{\mathrm{II}} \quad (3)$$

For the removable rigid cylindrical inclusion which respond by antiplain motion , the boundary condition satisfies

$$m\times a=-\int_0^{2\pi}\tau_{rz}^{\mathrm{I}}\Big|_{r=R}Rd\theta \quad (4)$$

3.2 Derivation of Green's function

In a full elastic space, the wave field caused by the line source loading $\delta(\vec{r}-\vec{r}_0)$ is $G^{(i)}$, which is known as a fundamental solution of the full elastic space and can be described by the incident wave in the form:

$$G^{(i)}=\frac{i}{4\mu}H_0^{(1)}\left(k\left|z-z_0\right|\right) \quad (5)$$

The reflected wave from the horizontal interface can be expressed as:

$$G^{(r)}=\frac{i}{4\mu}H_0^{(1)}\left(k\left|z-\bar{z}_0-2h_1 i\right|\right) \quad (6)$$

The scattering wave which is excited by the removable rigid cylindrical inclusion and satisfies the stress free condition on the horizontal interface automatically can be considered as follows:

$$G^{(s)}=\sum_{n=-\infty}^{\infty}A_n\left[H_n^{(1)}\left(k|z|\right)\left(\frac{z}{|z|}\right)^n+H_n^{(1)}\left(k\left|z-2ih_1\right|\right)\left(\frac{z-2ih_1}{\left|z-2ih_1\right|}\right)^{-n}\right] \quad (7)$$

where A_n are unknown coefficients to be determined by the boundary condition of the removable rigid cylindrical inclusion.

In area I, the total wave field is:

$$G^{(t)}=G^{(i)}+G^{(r)}+G^{(s)} \quad (8)$$

Substitution of Eq. (8) into Eq. (4) gives:

$$k^2\mu\pi R\frac{\rho_f}{\rho}A=-\int_0^{2\pi}\tau_{rz}^{\mathrm{I}}\Big|_{|z|=R}d\theta \quad (9)$$

The each side of Eq. (9) can be expressed as

$$-k^2\mu\pi R\frac{\rho_f}{\rho}A=a+\sum_{n=-\infty}^{\infty}A_n\times a_n$$
$$\int_0^{2\pi}\tau_{rz}^{\mathrm{I}}\Big|_{r=R}d\theta=b+\sum_{n=-\infty}^{\infty}A_n\times b_n \quad (10)$$

Therefore, Eq. (10) is transformed to the following forms:

$$\sum_{n=-\infty}^{\infty}A_n\left(a_n-b_n\right)=b-a \quad (11)$$

Multiplying both sides of Eq. (11) by $e^{-im\theta}$ and integrating over the interval $(-\pi,\pi)$, we have:

$$\sum_{n=-\infty}^{\infty}A_n f_{mn}=f_m\ \left(m=0,\pm1,\pm2,\cdots\right) \quad (12)$$

Eq. (12) is a set of infinite algebraic equations to determine the coefficients A_n.

4 Scattering of SH-waves by the removable rigid cylindrical inclusion and the crack

In a complete elastic half space, the incident steady SH-wave $W^{(i)}$ would be reflected from the interface, and the reflected wave $W^{(r)}$ is also SH-wave. Both angle of incidence and reflection have the same value α_0, as shown in Fig. 1. In the complex plane, $W^{(i)}$ and $W^{(r)}$ can be given by:

$$W^{(i)}=W_0\exp\left\{\frac{ik}{2}\left[\left(z-ih_1\right)e^{-i\alpha_0}+\left(\bar{z}+ih_1\right)e^{i\alpha_0}\right]\right\} \quad (13)$$

$$W^{(r)}=W_0\exp\left\{\frac{ik}{2}\left[\left(z-ih_1\right)e^{i\alpha_0}+\left(\bar{z}+ih_1\right)e^{-i\alpha_0}\right]\right\} \quad (14)$$

The scattering wave $W^{(s)}$ excited by the removable rigid cylindrical inclusion in the half space can be described by Eq. (7), in which A_n are also unknown coefficients. And the process of solving A_n is the same as that of the Green's function discussed in this paper.

Then, the total wave field in domain I is:

$$W=W^{(i)}+W^{(r)}+W^{(s)} \quad (15)$$

The corresponding stress can be written as:

$$\tau_{\theta z}=\tau_{\theta z}^{(i)}+\tau_{\theta z}^{(r)}+\tau_{\theta z}^{(s)} \quad (16)$$

For an arbitrary point in the elastic half space, the stresses generated by the incident wave $W^{(i)}$, the reflected wave $W^{(r)}$ and the scattering wave $W^{(s)}$ can be solved. On the same point, the additional stresses are loaded which have the same magnitude as the solved stresses mentioned above but opposite direction, then the total stresses of this point vanished. Therefore, a pair of forces with the same magnitude and opposite direction is loaded along the region where the crack want to be set, the ultimate forces on that region are zero. Thus that region can be thought as a crack. The total wave field in domain I is:

$$W^{(t)} = W^{(i)} + W^{(r)} + W^{(s)} - \int_{(b,-h_3)}^{(2a+b,-h_3)} \tau_{\theta z} G dz' \quad (17)$$

5 Results and Analysis

In this part, some numerical examples are examined based on the above theoretical derivation. The specific results of scattering of SH-wave by the removable rigid cylindrical inclusion and the crack in half space, the ground motion of the horizontal surface is provided. For various parameters, which include the wave number of incident wave k_1R , the incident angle α_0 , the length of crack 2a, the position angle of crack β_0 , the ratio of the distance of the center of the inclusion to the horizontal interface to the radius of the inclusion h_1/R and the ratio of the distance of the center of the inclusion to the crack tip to the radius of the inclusion h_2/R , shown in Fig. 1, the effects of them on the ground motion of the horizontal surface are discussed.

(1) Fig.2 shows the variation of the ground motion $|W^{(t)}|$ when $h_1/R = 1.1$, $h_2/R = 1.1$, $\alpha_0 = \pi/2$. When the rigid cylindrical inclusion is immobilised($\rho^* = 1000$),there is no shake near the rigid cylindrical inclusion. The ground motion is 1.7 at the place of $|X/R| = 10.0$, and the numerical value is also small.

(2) Fig.3—Fig.6 show the distribution of the ground motion of the horizontal surface. With the different incident angle α_0 , there is no change of the ground motion. It indicates that the influence of the incident angle is little.

(3) In Fig.7, it can be seen that the ground motion varies with the increasing of h_2/R when $h_1/R = 2$, $KR = 0.5$, $\rho^* = 2$, $2a = 2$, $b = -1$, $\alpha_0 = \pi/2$, $\beta_0 = 0$ .The ground motion above the inclusion are difference. The ground motion far away from the inclusion are the same.

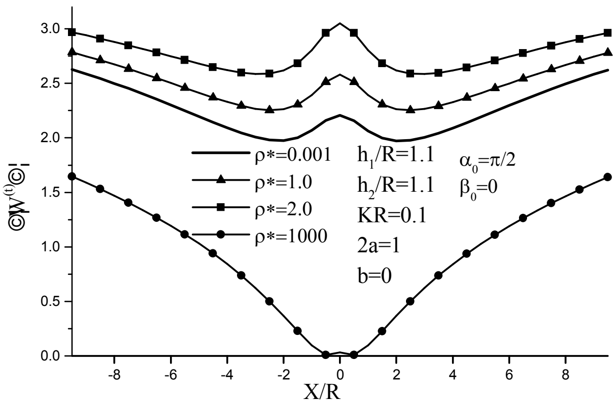

Fig.2 Variation of ground motion $|W^{(t)}|$ with X/R

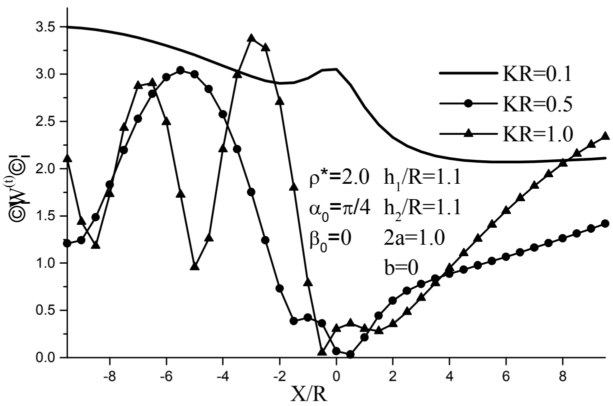

Fig.3 Variation of ground motion $|W^{(t)}|$ with X/R

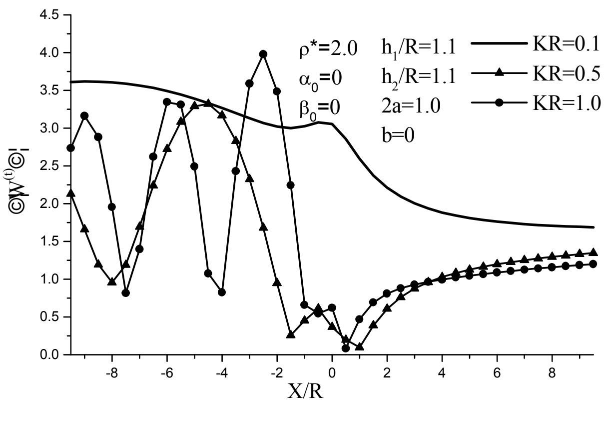

Fig.4 Variation of ground motion $|W^{(t)}|$ with X/R

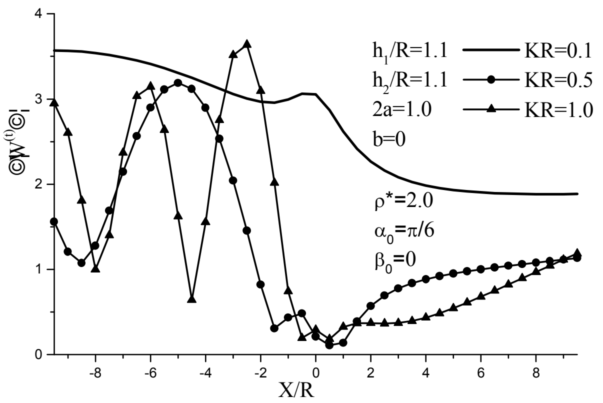

Fig.5 Variation of ground motion $|W^{(t)}|$ with X/R

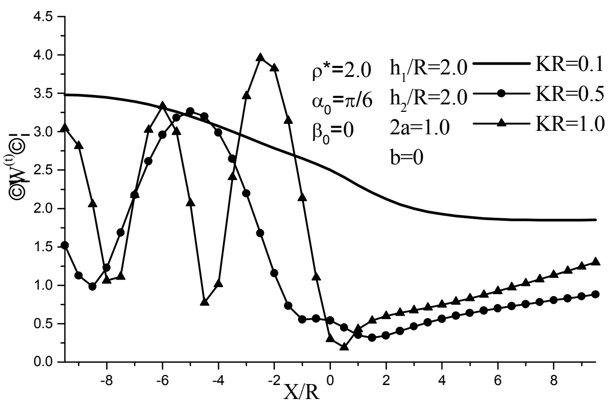

Fig.6 Variation of ground motion $|W^{(t)}|$ with X/R

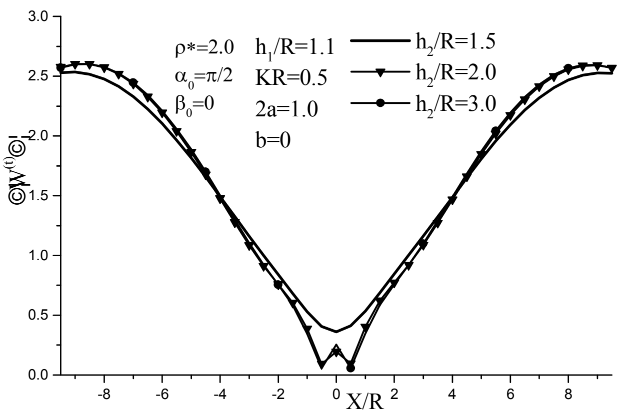

Fig.7 Variation of ground motion $|W^{(t)}|$ with X/R

6 Summary

In this paper, by using the technique of crack-division, a new method is given to solve the problem of the ground motion of the horizontal surface for incident SH wave. By using the method an example is solved, and some new conclusion is given. The method in the paper could be used to study some other correlative problem.

Acknowledgment

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