Risk-Reduction by Relaxing Priority Constraint for Tranches in Needs-based CDO

TATSUYA NAKAE and KENJI BABA
Systems Development Laboratory
Hitachi, Ltd.
890, Kashimada, Saiwai, Kawasaki, Kanagawa
JAPAN

NORIHISA KOMODA
Graduate School of
Information Science and Technologies
Osaka University
2–1, Yamadaoka, Suita, Osaka
JAPAN

Abstract: Needs-based collateralized debt obligation (CDO) satisfies constraints, such as the priority of tranches and investor needs for the merchantability of tranches, while creating capital-loss risk that is attributable to the excess and the deficiency of redemption money. We analyzed how relaxing the priority constraint for tranches decreases the risk of capital-loss and decreases the standard deviation in the amount of gain or loss. Relaxing the priority constraint enables us to assign redemption money to each tranche more flexibly and to reduce the amount of difference between the repayment money from underlying obligations and the redemption money. Experimental tests indicate that raising the degree of relaxation for the constraint reduces the standard deviation in the amount of gain or loss to one-eighth of its original value, which is worth the guarantee fee paid to the risk receiver.

Key–Words: collateralized debt obligation, investor needs, risk minimization, Pareto optimization

1 Introduction

Market-oriented indirect financing has recently attracted a lot of attention, and the market forcollateralized debt obligation (CDO), which is one form of market-oriented indirect financing, continues to expand[1]. We have been developing a method of designing CDO that conforms to investment parameters, which are credit capability and the issue amount required by investors[2, 3]. In our method, the probability distribution of the gain or loss obtained by incorporating investor needs shows that low loss occurs more than 90 percent of the time and that extremely high loss occurs with a low probability. This means that risk takers cannot be paid unless they deal with a significant number of CDO-issue businesses. Therefore, risk taker candidates are limited to government-affiliated facilities.

We developed a method of designing CDOs with less variance in the amount of gain or loss, which stabilizes the profit estimate for risk takers. Relaxing a priority constraint imposed on tranches, which are securities comprising a CDO, makes it possible to distribute redemption money to each tranche more flexibly. We analyzed the relations among the degree of constraint relaxation, the degree of decrease in the variance of the gain or loss, and the degree of decrease in the largest amount of loss during a prototype experiment.

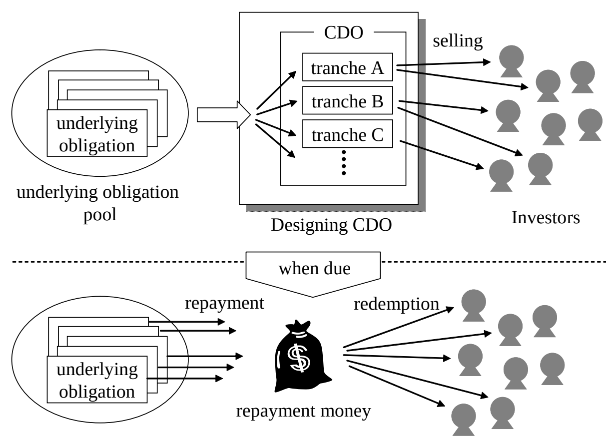

Figure 1: From designing to redeeming CDO.

2 Needs-based CDO Design Method

The flow from designing to redeeming a CDO is shown in Fig. 1. A CDO is comprised of an underlying obligation pool that is an aggregate of obligations such as loans or bonds, and it is constructed from tranches, i.e., several different securities. Designing a CDO means restructuring an underlying obligation pool with some tranches. Each tranche made by restructuring is split into small units and then sold to investors. When it is due, the repayment money collected from the obligors of the underlying obligations

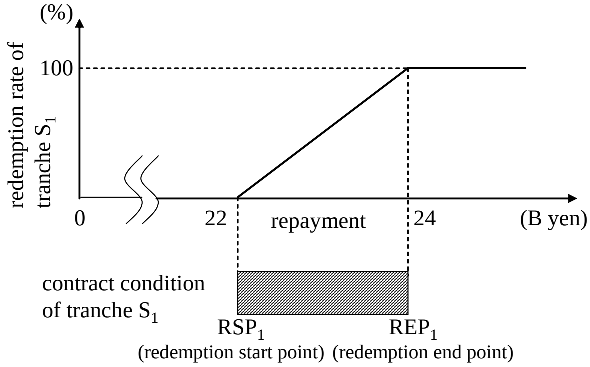

Figure 2: Relations among contract condition, repayment value, and redemption rate.

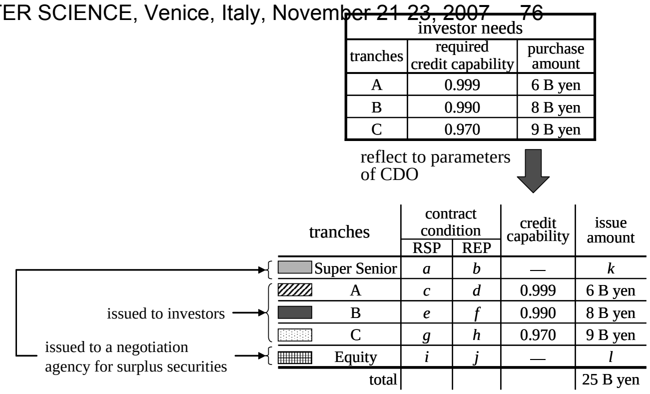

Figure 3: Incorporating investor needs into CDO.

is assembled, and the redemption money is paid to the investors in amounts depending on the amount of repayment money.

In our method of designing a CDO, we set a contract condition, an issue amount, and a credit capability for each tranche. A contract condition has two parameters: redemption start point and redemption end point. They prescribe what percent of the face value of the tranche is redeemed on each amount of redemption money. Using redemption start point RSP, redemption end point REP, and the amount of repayment money, V, the redemption rate, D, of the tranche is represented as follows:

$$D = \begin{cases} 0(\%) & (V < RSP) \\ \frac{V-RSP}{REP-RSP} \times 100(\%) & (RSP < V < REP) \\ 100(\%) & (REP < V) \end{cases}$$

In the following, the redemption start point and the redemption end point of tranche S_k $(k = 0, 1, \ldots, n+1)$ are described as RSP_k and REP_k, where n is the number of the following tranches for investors.

Figure 2 shows a relation between redemption money and redemption rate for tranche S_1 in which $RSP_1 = 22$ B yen and $REP_1 = 24$ B yen.

The contract conditions are constrained by ***constraint* (1)** as follows:

$$REP_k < RSP_{k+1} (k = 0, 1, \ldots, n+1)$$

This constraint causes priority relations to develop among tranches. That is, unless S_k is redeemed 100 percent, S_l $(l = k+1, k+2, \ldots, n+1)$, which are lower priority tranches than S_k, are not redeemed at all.

The issue amount represents the amount of the face value of the tranche. The sum of the issue amounts is equal to the sum of the face values of the underlying obligations.

The credit capability is the expected redemption of principal when the amount of principal is 1. Once a contract condition is decided, the credit capability is uniquely determined using a probability density function representing the repayment money. However, when the credit capability is decided first, the contract condition cannot be uniquely determined, though it is, to some extent, restricted. The probability density function can be derived based on the default ratios for the underlying obligations by using an approximate model, such as a binomial expansion model, a CreditRisk+ model, or a Monte Carlo simulation-based model[4, 5, 6, 7].

Our method of designing a CDO involves designing tranches based on investor needs, which are the investors' intent to purchase CDOs. The investor needs are listed in the table at the top of Fig. 3 and have a required credit capability and a required issue amount for each tranche. In this method, the types of, credit capabilities of, and issue amounts of tranches in the investor needs are shifted directly into the tranches for investors (listed in the table at the bottom of Fig. 3).

In addition, we created a super senior debt (SS debt) and an equity debt as tranches for a negotiation agency for surplus securities. The SS debt has the highest credit capability and lowest profitability of all types of tranche, and the equity debt has the lowest credit capability and highest profitability of all types of tranche. The total profitability of CDOs can be controlled by changing their credit capabilities and issue amounts.

In the following section, S_0, $S_1, S_2, \ldots, S_n$, and S_{n+1} correspond to SS debt, tranches for investors, and equity debt.

The contract conditions of the tranches for investors are restricted by the required credit capabilities in the investor needs. Specifically, the credit ca-

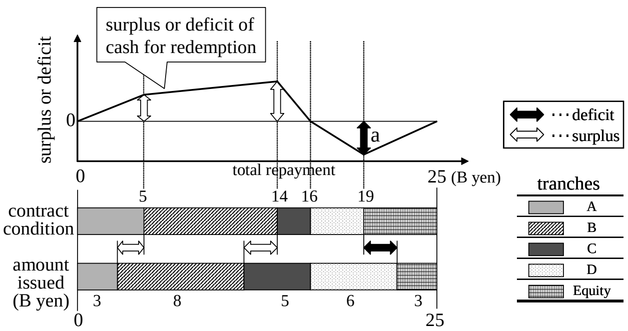

Figure 4: Surplus or deficit of redemption money.

Table 1: Input parameters.

Obligation Pool	
total face value	25 billion yen
probability density function of repayment	normal distribution average: 0.975 * total face value standard deviation: 0.009 * total face value
yield	1.049
correlation	Obligation has no correlation.
collect rate	0%

Investor Needs		
tranches	credit capability (yield)	purchase amount
A	99.90% (1.0103)	6 billion yen
B	99.00% (1.0222)	8 billion yen
C	97.50% (1.0426)	9 billion yen

guarantee fee	120 million yen

pabilities calculated from the probability density function of the total repayment and the contract condition must be equal to the required credit capability (***constraint* (2)**).

The amount of redemption money for investors is calculated based on the amount of repayment money from the underlying obligations and the contract conditions of the tranches when the contract expires. Therefore, the amount of the repayment and that of the redemption are possibly mismatched. Figure 4 shows an example of this. Note that when the total repayment is 5 B yen, the contract condition indicates that only tranche A is redeemed; however, the issue amount of tranche A is 3 B yen, so the amount of repayment exceeds that of redemption by 2 B yen (= 5 B yen − 3 B yen). Conversely, when the total repayment is 19 B yen, the amount of repayment is below that of redemption by 3 B yen. Those risks, which are caused by a surplus or a deficit of redemption money, are transferred to a guarantee institution with risk premiums.

We use "the largest loss" (or 100% VaR) as a measure for representing the amount of risk taken by the guarantee institution. This is expressed as the length of a, as shown in Fig. 4. Minimizing the largest loss is the objective. At the same time, we need to make the profit-and-loss (PL) expectation of a guarantee institution equal to the risk premium (*constraint* (3)).

We define the CDO that has the minimum "largest loss" in all CDOs under the three constraints mentioned above as "the best CDO." We decide the best CDO by following two steps.

(a) Derive all CDOs that fulfill the constraints.

(b) Derive the largest loss for each CDO, and then, identify the least of them and the CDO that has it.

3 Problem

For the CDO design method mentioned in the previous section, the variance in the PL of the guarantee institution is not a constrained value while the expectation is. Therefore, the degree of variance in the PL for the optimized solution is high, and thus, the guarantee institution needs a significant amount of capital to ensure a steady income. In the following section, we discuss this problem in greater detail.

Figure 5 shows the probability distributions for the PL of a guarantee institution, investors, and a negotiation agency for surplus securities for the best CDO derived using the above-mentioned CDO design method under general conditions. The details of the conditions are listed in Table 1. Although the probability distribution of the PL for investors varies depending on which tranche the investors purchase, we do not show this in Fig. 5, rather, we show the probability distribution for each sum of PL for all investors.

Clearly from its PL, the guarantee institution loses less than 600 million yen about 94% of the time while it gains more than 1.5 billion yen about 3% of the time. "More than 1,500 million yen" is comprised of extremely large values, e.g. 2,000 million yen of 1% and 4,000 million yen of 1%. As a result, the expectation of the PL is restored to +120 million yen, which corresponds to the risk premium.

Although the expectation is positive, the guarantee institution loses money with a high degree of probability. Guaranteeing the CDO is like purchasing a ticket in a profitable lottery, so the guarantee institution cannot ensure a stable income unless it engages in a lot of securitization business. Thus, the guarantee institution is confined to being either a governmental enterprise or a nongovernmental enterprise commanding a large amount of capital.

Therefore, the amount of variance in the PL should be reduced to enable the guarantee institution

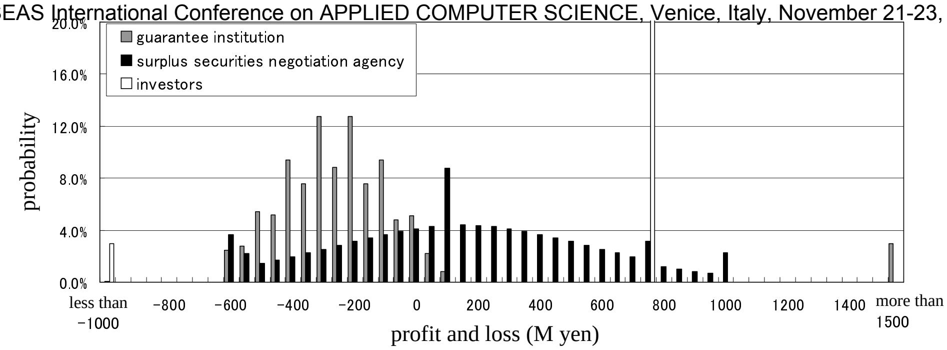

Figure 5: Probability distribution of profit or loss.

to earn a stable income. In Fig. 5, the standard deviation is 2,440, which is about 20 times the risk premium 120 (M yen). The goal is to reduce the SD to 2 times the risk premium.

4 Minimizing The Variance

If sacrificing the largest loss enables us to reduce the amount of variance in the PL to the target value, we can only resolve the issue by adding one constraint: the amount of variance in the PL must be less than a specified value. We therefore derived the best CDO by changing the objective function to a variance of the PL and by maintaining the other constraints.

Using the input data listed in Table 1, we obtained the best CDO with the amount of variance in the PL equaling 2,100 (and the largest loss of 960 M yen). This variance is 18 times the risk premium of 120 and is far from the target value.

Figure 6 shows a comparison of two contract conditions, one of which is derived when the objective function represents the largest loss, and the other is derived when it represents the amount of variance of PL. In this figure, we represent the contract conditions of five types of tranches on five separate levels.

With respect to every tranche, the distance between the RSP and the REP (hereinafter called the "width of contract condition") when the objective function represents the amount of variance in the PL, is wider than that when the objective function represents the largest loss. This indicates that the wider the contract conditions, the smaller the amount of variance in the PL.

However, in Fig. 6, the width of the contract condition for each tranche is already expanded as far as it can go, and it almost touches one of its neighbor tranches. Due to the constraint described in section 2, which restricts the overlapping of tranches, the widths of contract conditions cannot be spread any more, thus the variance of PL cannot be reduced any longer.

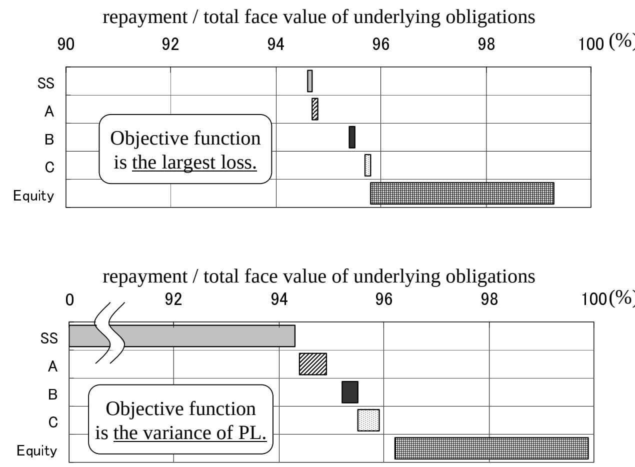

Figure 6: Comparison of contract conditions.

5 Relaxing The Priority Constraint

5.1 Redefinition of The Priority Constraint

It is difficult to reduce the amount of variance in the PL to the target value under the framework of the CDO design method we have developed.

We therefore propose a new priority constraint. We changed the traditional priority constraint: $REP_k < RSP_{k+1} (k = 0, 1, \ldots, n+1)$, which restricts the overlapping of tranches, into the following expression:

$$RSP_k < RSP_{k+1},\ REP_k < REP_{k+1} \quad (k = 0, 1, \ldots, n+1)$$

Figure 7 shows patterns of overlapping contract conditions, one of which meets the new constraint (*True*) and the others which do not (*False*). Because the new constraint allows for overlapping, as shown in (a), a subordinate tranche can be partly redeemed, even though a preferential tranche is not fully redeemed. Note, however, the redemption rate of the preferential tranche remains higher than that of

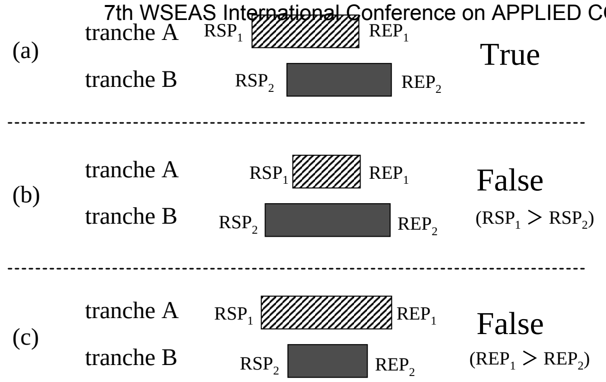

Figure 7: Relaxed priority constraint.

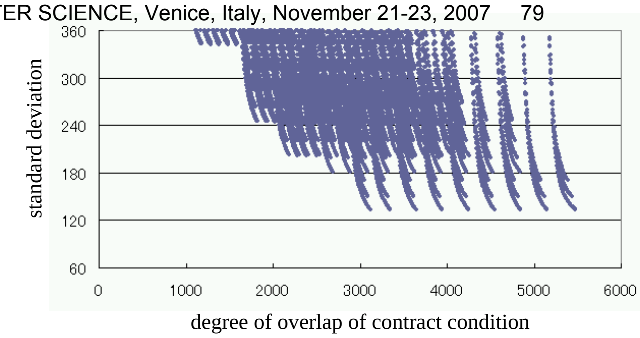

Figure 8: Relaxing priority constraint decreases amount of variance.

the subordinate one. Subordinate tranches are never partly redeemed as long as higher priority tranches are not redeemed at all (case (b)), and subordinate tranches are never fully redeemed as long as higher priority tranches are not redeemed completely (case (c)).

Relaxing the priority constraints changes the financial characteristics of CDOs. The associated decrease in the advantage of high-priority tranches over the other tranches is not helpful, especially for investors who want high-priority tranches. However, relaxing the priority constraints also has an advantage: investors get more opportunity to partly redeem their investments when the repayment money remains low. In other words, the stability of the products increases (and the associated risk decreases).

5.2 Variance Reduction by Relaxing the Constraint

We designed CDOs based on the specifications listed in Table 1 to determine whether the amount of variance in the PL decreases when the priority constraint mentioned in the previous section is used. We derived not only the best CDO, but all CDOs that satisfy the constraints, and we plotted them in the graph whose horizontal and vertical axes represent the amount of overlap in contract conditions and the standard deviation of PL.

The amount of overlap in the contract conditions, H, is defined as the sum of the lengths by which a REP of each preferential tranche exceeds the RSP of the other subordinate tranches. This is expressed mathematically as:

$$H = \sum_{k=0}^{n} \sum_{l=k+1}^{n+1} f(REP_k, RSP_l)$$

$$f(REP_k, RSP_l) = \begin{cases} REP_k - RSP_l & (REP_k > RSP_l) \\ 0 & (REP_k \leq RSP_l) \end{cases}$$

The result is shown in Fig. 8. It indicates that the larger the overlap between the contract conditions is, the smaller the amount of variance in the PL. This suggests that relaxing the priority constraints enables us to create many CDOs whose standard deviation of PL is less than 240, twice as much as the risk premium.

5.3 Pareto Optimal Solutions

We now discuss how to minimize the largest loss under the restriction, which limits the standard deviation of the PL to less than twice as much as the risk premium.

Considering the trade-off relation between the amount of overlap in contract conditions and amount of variance in the PL described in the previous section, we anticipate that the amount of overlap in the contract conditions and the largest loss also have a trade-off relation. Thus, we derived a Pareto optimal solution, which uses these two parameters as objective functions, for our method.

We experimentally derived the Pareto optimal solution by using the underlying obligations, the investor needs, and the risk premium listed in Table 1. CDOs that not onlysatisfied the constraints, but also had standard deviations of PL less than twice the risk premium, are derived cyclopaedically, and are plotted in a graph whose horizontal and vertical axes represent the amount of overlap in contract conditions and the largest loss.

The result is shown in Fig. 9. As is expected, it indicates that the larger the overlap between contract conditions is, the smaller the largest loss. The minimum value of the largest loss is 99 million yen, which corresponds to the CDO that has one of the largest amounts of overlap in the contract conditions. Fig. 10 shows the probability distribution of PL for the guarantee institution, the negotiation agency for surplus securities, and the investors on that CDO. The PL of those three participants stays within a range from

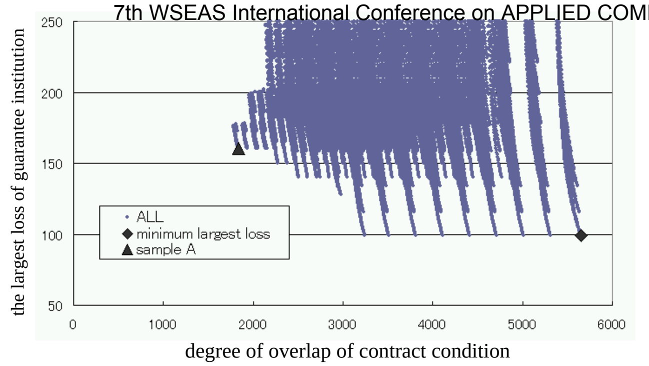

Figure 9: Pareto optimal solution.

−400 million yen to +800 million yen. We have also calculated the probability distribution of PL for sample A, one of the Pareto optimal solutions in Fig. 9. It is shown in Fig. 11. Compared to those that are under the condition of the non-relaxed priority constraint (Fig. 5), the variances of PL of not only the guarantee institution but also the other two decrease for sample A. The contract conditions for sample A are shown in Fig. 12.

This experiment has revealed that relaxing the priority constraint reduces the standard deviation of the PL to the target value and also reduces the largest loss and the variance in the PL of the investors, although the PL distribution varies based on which Pareto optimal solution is selected.

6 Conclusion

We have revealed that relaxing the constraint concerning priority relations among tranches, which are securities comprising CDOs, can reduce the amount of variance in profit or loss (or the risk of PL) generated by issuing CDOs at levels corresponding to the risk premium paid to a guarantee institution.

In the conventional CDO design method, unless all preferential tranches are completely redeemed, lower priority tranches are not redeemed at all, while in our method, a newly defined priority constraint enables the lower priority tranches to be partly redeemed even though preferential tranches have not been fully redeemed. As a result, for a guarantee institution that receives surplus/deficit money, the chance of having a large gain/loss declines. Thus, the amount of variance in the PL decreases. We conducted a CDO design experiment based on the hypothesis that the largest loss of the guarantee institution which is an objective function on the conventional method has a negative correlation with a relaxation degree of the priority constraint. That experiment have indicated that there are some Pareto optimal solutions which objective functions are the largest loss and the relaxation degree, and standard deviations of PL on those solutions are less than twice as much as the risk premium, and are also below one eighth of those on the conventional method.

References:

[1] F. J. Fabozzi: *Bond Markets, Analysis, and Strategies*, Pearson Education, Inc. (2003).

[2] T. Nakae, T. Moritsu, K. Baba, and N. Komoda: "A Design and Evaluation of Needs-Based Collateralized Debt Obligation (CDO)," *WSEAS Transactions on Information Science and Applications*, Issue 1, Vol. 3, pp. 105–112 (2006).

[3] T. Nakae, K. Baba, T. Moritsu, and N. Komoda: "Fast calculation method for fine-tuned design of need-based collateralized debt obligation (CDO)," *IEEJ Transactions on Electrical and Electronic Engineering*, Vol. 2, Issue 3, pp. 378–386 (2007).

[4] Credit Suisse Financial Products: "Credit Risk+" (1997).

[5] A. Ieda, K. Marumo, and T. Yoshiba: "A Simplified Method for Calculating the Credit Risk of Lending Portfolios," *Monetary and Economic Studies*, Vol. 18, No. 2 (2000).

[6] M. Gordy: "A Risk-factor Model Foundation for Ratings-based Bank Capital Rules," *Journal of Financial Intermediation*, Vol. 12, Issue 3, pp. 199–232 (2003).

[7] M. Pykhtin and A. Dev: "Analytical Approach to Credit Risk Modeling," *Risk*, Vol. 15, No. 3, pp. s26–s32 (2002).

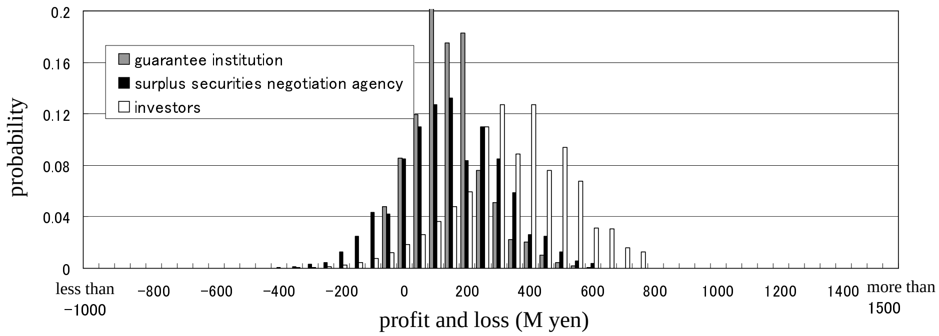

Figure 10: Probability distribution of profit or loss (minimum largest loss).

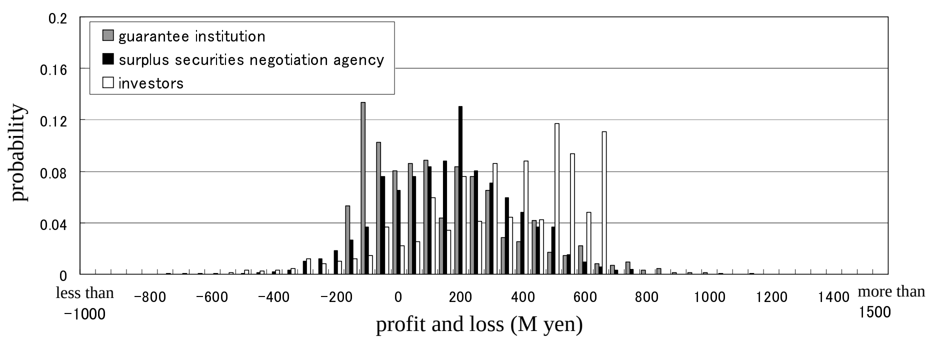

Figure 11: Probability distribution of profit or loss (sample A).

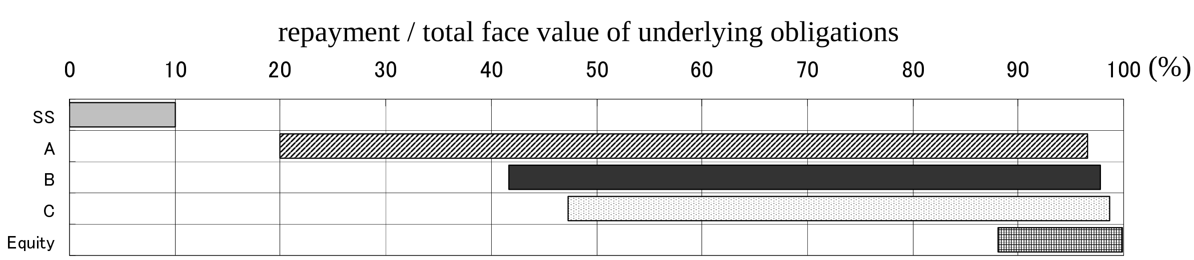

Figure 12: Contract condition for sample A.