Introducing an innovative mathematical method to predict the bankruptcy risk. Measures for the financial markets stability

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Abstract: - This paper advances innovative measurement models for bankruptcy risk and develops a framework for further research in the use of latent multilevel static and dynamic models in modeling bankruptcy decision making. A brief presentation of literature is given, with an emphasis on the most recent developments in this field. The econometric model underlying the fuzzy sets theory applied to the measurement of bankruptcy risk is particularly examined here. The innovative part of the paper is the application of a new methodology to the analysis of bankruptcy. This model, called Item Response Theory (IRT), is a traditional technique in psychometrics and brings a set of advantages when being applied to the analysis of latent measures by a set of observable binary indicators, as is the bankruptcy risk. At the end, the paper concludes on the need for more collaboration among researchers in business, computer science and law, in order to develop new interdisciplinary measurement tools in finance.

Key-words: - Bankruptcy risk, Fuzzy sets, Item response theory, Financial ratio

1 Introduction

In finance, the bankruptcy risk is the risk that a company will be unable in future to meet its debt obligations. Both the equity- and bondholders must consider this risk when investing in a company. The bankruptcy risk is close to the credit score, which is calculated and sold by credit bureaus.

In recent years, bankruptcy prediction has gained a great attention for practitioners and academicians, who realized that the asymmetric information between firms and banks relies on the credit market failure. Therefore, the analysis of the bankruptcy risk is one of the most important tasks for all stakeholders in any company (Berk and DeMarzo, 2007). The leaders are always interested in the company’s ability to meet the payments over the life of a loan. Auditors investigate whether the financial problems that the company experience at present are likely to continue as a going concern. Managers and their teams are interested in anticipating all problems that may occur at a moment in future in order to prevent them at present. For all of them, anticipating the bankruptcy risk is equally important. Also, the problem of predicting as accurate as possible financial performances and bankruptcy risk became important tools in reducing borrower’s moral hazard. But modeling the bankruptcy should be adapted to each situation, industry and country. For instance, in Japan, the measures of a company’s social importance and the strength of its bank relationship may be more important at financially crucial moments than accounting information (Suzuki and Wright, 1985).

This paper is concerned with examining the econometric models explaining the cases of bankruptcy and predicting the risk of falling in bankruptcy. All econometric statistical methods used in the measurement of bankruptcy risk basically aggregate some financial ratios into an index of bankruptcy, at a static or dynamic level. Our paper is innovative in the sense that it proposes a new methodology to measure the bankruptcy risk.
This model recognizes the latent nature of bankruptcy and account for this particular aspect in calculating its summarizing index. For comparison purposes, the paper also introduces other models that may be used to aggregate the financial ratios into an index.

2 Models predicting the bankruptcy risk

In literature, a number of models predicting the bankruptcy risk have been created over time. They are classified according to three main approaches: accounting analytical approach, option theoretical approach and statistical approach. Within the statistical approach, there are a number of statistical models assessing the risk of financial failure, such as: linear or quadratic discriminant analysis, probit regression analysis, logistic regression analysis and neural network analysis. All statistical models are based on a score \( Z \), which is derived as a linear combination of a set of financial ratios.

\[
Z = a_1R_1 + a_2R_2 + a_3R_3 + \ldots + a_n
\]

(1)

Where, \( R_1, R_2, R_3 \ldots R_{n-1} \) are the financial ratios and \( a_1, a_2, a_3 \ldots a_n \) are the coefficients.

The most famous model is the Altman Z-score (Altman, 1960), which uses five financial ratios to determine the likelihood of bankruptcy among companies. Altman combines the ratios by using the Multiple Discriminant Analysis. The ratios are derived using 8 financial indicators from a company’s financial statements. The financial indicators are:

- EBIT (earnings before income and taxes) and net sales, from the Income Statement.
- Total assets, market value of equity, total liabilities, current assets, current liabilities and retained earnings from the Balance Sheet.

The empirical evidence has proved over time that the Altman model really predicted the 72% of corporate bankruptcies within two years. According to the model, the lower the score, the higher the odds of bankruptcy is.

The Altman model is different upon the type of company under analysis. Originally, the Z-score model was designed for the public companies, but in time a model for private companies was also created. In the model for public companies, the scores above 3 indicate a healthy company. Scores between 1.8 and 3 locate the company in a grey area, while scores below 1 indicate the bankruptcy risk. When the model is adapted for a private company, a score of 1.23 or below suggests that bankruptcy is likely.

The ratios that Altman included into the model are:

- Working capital/ Total assets
- Retained assets/ Total assets
- EBIT/ Total Assets
- Market value of equity/ Book value of total liabilities
- Sales/ Total assets

The ratios above are weighted differently for the public and private companies, as resulted from the discriminant analysis. Up to now, the Altman models continue to be the most famous and representative statistical model summarizing the bankruptcy risk.

3 Stages of designing an econometric model calculating the bankruptcy risk

In case that analysis is run at a longitudinal level, both balanced and unbalanced datasets may be used, with the condition that the “good” and “bad” firms must be equally represented (50% good firms and 50% bad firms) and randomly selected. The measurement is relevant only when companies from the same industry and of the same size are analyzed.

Most of the models predicting the bankruptcy risk are based on the discriminant analysis. Initially, the dataset must include two groups of variables: one group of “good” or “healthy” companies and one group of “bad” or “at risk” companies. The sample of “bad” firms should include minimum 30 companies for results to have statistical validity. Financial ratios are calculated for each company and then the linear combination that best separate the two groups is also determined.
The first step in designing an econometric model to predict bankruptcy is to decide upon which financial ratios should be selected as to better describe the financial performance. The financial indicators and ratios selected should discriminate at a large extent between the two groups of companies. Computer analysis should reveal which ratios are consistently and significantly different between the two groups of companies. This selection process can be done by running descriptive analyses for each rate and upon each group of companies.

When using the discriminant analysis, the independent variables explaining the financial performance or bankruptcy are the financial ratios. They are recoded as binary variables, in the same way as the dependant variables which represent the exposure to the bankruptcy risk. The explanatory indicators may reflect different aspects of financial performance, like: profitability, solvability, liquidity, leverage, turnover, size, capitalization and efficiency. The dependent variable, i.e. the financial score Z, is recoded with 0 representing a high risk of bankruptcy and 1 indicating a good financial situation and a strong stability for that company.

Some papers introduce innovative instruments like the Stochastic Frontier Approach (Becchetti and Sierra, 2003) or the fuzzy sets.

4 Applying the fuzzy sets theory to the measurement of bankruptcy risk

In the framework of the fuzzy sets theory, the bankruptcy risk can be analyzed through a membership function. This function establishes a link between each company and the sets of “healthy” or “risky” firms. The membership function can be constructed in many forms. Ionita and Stoica (2006) define for each ratio (that they call as “influence factor”) a fuzzy subset in a discrete set theory. They separate the ratios into two groups of static and dynamic ratios. For example, for the ratio of the annual loss ph to the annual profit Ph, the membership function (µ) can be determined as it follows:

When \( P_h \geq P_{h \text{ min}} \):

\[
\mu \left( \frac{P_h}{P_{h \text{ min}}} \right) \quad \left( e^{\frac{-k_i P_h}{P_{h \text{ min}}}} \right)
\]

When \( P_h < P_{h \text{ min}} \):

\[
\mu \left( \frac{P_h}{P_{h \text{ min}}} \right) \quad \left( e^{\frac{-k_i P_h}{P_{h \text{ max}}}} \right)
\]

After determining all membership functions (for all ratios) in the same way as presented above, in the static and also in the dynamic form, the functions will be aggregated by using a multiplying procedure. The aggregation process will generate the global membership function of financial performance (\( \mu_c \)) and the global membership function of financial risk (\( \mu_r \)). The financial risk function is obtained by reversing the ratios used in the determination of financial performance chances.

\[
\mu_c = \prod_{i=1}^{n} e^{-k_i^* R_i^*} \times e^{-k_i^{**} R_i^{**}}
\]

Where,

- \( n \) – the number of financial ratios
- \( R_i^* \) - financial ratios in a static version
- \( k_i^* \) - importance coefficients in a static version
- \( R_i^{**} \) - financial ratios in a dynamic version
- \( k_i^{**} \) - importance coefficients in a dynamic version

By comparing \( \mu_c \) and \( \mu_r \) one can predict the state of bankruptcy.

- If \( \mu_c > \mu_r \), it means that the financial situation is favorable and the bankruptcy risk is low. A big difference between \( \mu_c \) and \( \mu_r \) suggest a very comfortable situation for the company.
- If \( \mu_c < \mu_r \), the company is at risk of bankruptcy. A large difference indicates a high risk.
- If \( \mu_c = \mu_r = 0.5 \), the situation is still unfavorable, the bankruptcy risk exists, but it still may be prevented by adequate measures.
5 Integrating the analysis of bankruptcy risk into the framework of the Item Response Theory (IRT)

Bankruptcy risk may be seen as a latent variable in the sense that it cannot be directly observed, but can be derived from other indicators e.g. financial ratios, which are observed and directly measured. Traditionally, the bankruptcy risk is measured as the weighted sum of financial indicators. But this approach does not take into account the fact that bankruptcy is a latent variable that can involve large measurement errors.

Item response theory, also known as latent trait theory, belongs to the latent class analysis and is widely used in the educational and psychological measurement. For example, in psychometrics, is considered as the most famous econometric technique to measure abilities and attitudes. Over time, due to the advantages that this model brings in comparison with the traditional models, it has been also applied to other social settings, such as demography, deprivation and biology, being often referred to as an interdisciplinary methodology.

In fact, the IRT was built on the basis of the critics formulated towards the classical test theory (CTT). The central core of both CTT and IRT is the “item analysis”¹. But even though the superiority of IRT over CTT is largely recognized in the measurement community, the empirical evidence in this sense is rather weak (Fan, 1998). This is the reason that exploring and applying the IRT to different fields and data may improve its underlying econometric models and may reveal new insights.

The IRT has been developed in three forms: one parameter, two parameter- and three parameter- IRT. In the one parameter- IRT, the model allows inferring one parameter for company called latent score, which is derived from a set of dichotomous observable indicators, and one parameter for the financial indicators (which is called the difficulty parameter). The other forms of the IRT models bring other parameters for the financial indicators: the discrimination and the guessing parameters. The IRT can be viewed as a generalized linear model (Skroandal and Rabe-Heschet, 2004), where the conditional probability to get a “good” or “bad” value for a financial ratio given the latent score Z is specified by a link function, e.g. linear, probit or logit. Thus, the IRT relates both the characteristics of financial ratios and the characteristics of companies to the probability of providing a particular value, i.e. good or bad financial performance (Lord, 1980).

This paper is innovative in two aspects, compared to the previous research on bankruptcy. First, the IRT does not only calculate the Z score of bankruptcy risk, as all statistical models do, but it also calculates two parameters called difficulty and discrimination for each financial ratio. These “indicator” parameters are useful for an optimal selection of indicators explaining bankruptcy. Second, this model takes into account the latent nature of bankruptcy and incorporates it into the model through the error term.

Let R_ij be the binary indicator j=1..m explaining the bankruptcy risk of company i=1..n. The indicators are continuous financial ratios, which are initially selected as to better discriminate among companies. This selection can be done by applying the discrimination analysis or various descriptive statistics. The indicators are then dichotomized, with 1 indicating the absence of bankruptcy risk and 0 indicating the presence of bankruptcy risk. The aim is to first calculate a summarizing index of bankruptcy risk that we call the Z score and second to calculate difficulty and discrimination parameters² for each indicator. The difficulty parameters allow deriving a scale of financial bankruptcy and also ranking the indicators of bankruptcy risk.

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¹ In our analysis the item is the financial indicator describing the bankruptcy risk. The analysis is therefore centered on the aggregation of indicators into a score Z.

² The difficulty parameter may be derived in both one and two- parameter IRT, while the discrimination parameter can be determined only in the two parameter- IRT.
The general form of the one parameter IRT model is:

\[ R_{ij}^* = \beta_j + Z_i^* + \epsilon_{ij} \]  
(5)

\[ R_{ij} = 1 \text{ if } R_{ij}^* > 0 \text{ and } R_{ij} = 0 \text{ otherwise.} \]

Where, \( \beta_j \) represents the difficulty parameter of the financial ratio \( j \), \( Z_i^* \) is the score of bankruptcy risk for the company \( i \) and \( \epsilon_{ij} \) is the error term. The error terms are assumed to have zero mean and a fixed and common variance.

In the two parameter IRT model, we can also get a second parameter for the financial ratios \((\lambda_j)\) which is called discrimination parameter. This reflects how well the financial ratios discriminate among companies.

\[ R_{ij}^* = \beta_j + Z_i^* \lambda_j + \epsilon_{ij} \]  
(6)

\[ R_{ij} = 1 \text{ if } R_{ij}^* > 0 \text{ and } R_{ij} = 0 \text{ otherwise.} \]

If we treat \( Z_i^* \) as random individual effects, then we can estimate the difficulty parameters by the standard maximum likelihood. The latent scores \( Z \) can be estimated by Empirical Bayes methods. In the two parameter- IRT, the assumption of equi-correlation between financial ratios is relaxed. This is allowed by the discrimination parameter \( \lambda_j \). For model identification, one has to assume that \( \lambda_j = 1 \).

By adding a second indicator parameter, the two parameter- IRT should fit the dataset better than the one parameter IRT, as shown up by the likelihood ratio test. Still, there are cases that the one parameter IRT may be preferred, like when studying the cumulative nature of financial performance. A particular form of the one parameter IRT model is the Rasch model (Rasch, 1966). In the Rasch model, the latent scores \( Z \) are treated as fixed parameters and the error term has a logistic distribution. When the Rasch assumptions are met, the simple aggregation of financial indicators (the un-weighted sum score) is a sufficient statistic of the latent score of bankruptcy risk. Although the model presents this nice propriety it is very restrictive in the sense that it is difficult to find indicators respecting the Rasch condition.

As regards the empirical application of IRT, unlike other methodologies, the IRT allows dealing with unbalanced data structure and missing data. This can be seen as an advantage of the IRT, in comparison with other traditional summarizing methods.

Not only defining and measuring the bankruptcy risk is important, but also analyzing its determinants. One particular aspect that is often neglected is that the symptoms and causes of bankruptcy are different. The IRT model can be seen as a MIMIC model (Multiple Cause Multiple Indicator Cause), in the sense that it studies both the risk of bankruptcy and the determinants of bankruptcy through a single model. This is a real advantage over traditional methods, because it avoids separating the analysis of bankruptcy and its determinants in two different stages. In the IRT the determinants can be examined by incorporating a structural equation into the measurement component of the IRT. The structural equation introduces the covariates which explain the risk of bankruptcy.

\[ Z_i^* = \alpha V_i + \xi_i \]  
(7)

Where, \( V_i \) is the vector of covariates and \( \xi_i \) is a normally distributed error term, with mean zero and fixed variance.

Furthermore, the standard IRT method can be extended to treat the bankruptcy as a tendency or a direction the company is taking, not just a static risk value (dynamic or longitudinal IRT). As a static value, the financial ratios will indicate a high bankruptcy risk only when they reach the “red” thresholds, which may be too late. Predicting that a company is not closing, but heading to bankruptcy may be more helpful for long-term involvements (like credits and loans arrangements).

There are many modern statistical computing packages that allow running IRT models, such as: Mplus, Splus, R, SAS and STATA. In STATA for instance, the GLLAMM and GLLAPRED modules estimate generalized linear latent and mixed models by maximum likelihood (Rabe-Hesketh, S. Skrondal, A. and...
Pickles A., 2004). IRT represents only a small part of this group of models.

6 Conclusions

Bankruptcy and prediction models are largely used in auditing, large corporate transactions, investment decision making and in the judiciary setting. Improving and innovating the analysis and measurement of bankruptcy risk is highly required, especially in the context of globalization, which may have negative effects especially on the financial markets. Predicting the financial risks may prevent not only bankruptcies but also the transmission of financial crisis by the banking channels, due to the global economy.

Among the models designed with identifying bankruptcy risk, statistical models provide some advantages, such as:
- By using statistical models one would get more accurate and “friendly” results, in comparison with using a set of contradictory financial ratios.
- It predicts the financial vulnerability of a company, based on past financial data dynamics. It therefore avoids the stakeholder’s subjective judgments.

We have shown in the paper that the use of the IRT in the analysis of bankruptcy risk is innovative and gives positive insights to minimizing the risks on the financial markets. The positive effect of this method on the financial market stability is explained by the fact that it calculates “indicator” parameters and generates “indicator” rankings, which are very useful in the credit analyses. We have also shown that the IRT brings a set of advantages in comparison with the traditional models. First, it calculates the Z score of bankruptcy risk, taking into account the measurement errors and the latent nature of bankruptcy. Second, it also calculates difficulty and discrimination parameters for each financial ratio. This allows ranking, analyzing and selecting the financial ratios in order to better define and measure bankruptcy risk.

The bankruptcy risk is a latent measure, i.e. is not observable, which makes more difficult its estimation. Innovative models, recognizing the complexity of this financial risk and its latent nature are therefore needed. A stronger collaboration among researchers in different areas, such as economics, business, computer science and mathematics is therefore needed.

References