Projection Method for Order Reduction of Analog Circuits

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Abstract: - The efficient simulation of large–scale dynamic systems needs a systematic procedure for order reduction of the original circuit. The paper combines the projection method with Krylov subspaces to obtain robust and accurate order reduction techniques. First of all, the algorithm finds a basis matrix onto Krylov subspaces, using a block Arnoldi process, and then constructs a reduced-order model via projection technique. It is shown that we can handle the MIMO systems using the semi-state method and Krylov projection method. An illustrative example is given and some conclusions are pointed out.

Key-Words: - Order reduction, Krylov subspaces, Projection method, Analog circuits, MIMO systems.

1 Introduction

Behavioral models capture certain functional properties without relying on specific structural representations. Using behavioral models a significant speedup of higher level simulation is achived. Also, they allow early design verification during top-down circuit design improving design efficiency [1-3, 10]. The most important objective in analog circuit design is establishing the relationship between the input-output variables. This relationship depends on the device type and geometry, phenomenon and behavior, and on the circuit configuration, so that, especially in the large-scale system case, for an efficient simulation, an equivalent reduced size model of the original circuit is needed. There are different approaches of this problem: some of them are based on the sensitivity computation in order to simplify the circuit structure by noncritical element elimination, others reduce the insignificant terms in the circuit function expression, or the internal nodes of the circuit by equivalent electrical transforms; there are approximation methods based on Taylor series, on Padé Approximations, on Lagrange Polynomials, on Spline functions etc. No matter what is the principle they are build on, the aim of all these methods is getting an approximated transfer function, with a reduced number of poles. The reduced model can be then used for circuit response prediction in time or frequency domain, in a predicted range of the signal frequencies. The choice of one order reducing method or other one depends on the specific problem we have to solve it.

The major problems involved in obtaining reliable reduced-order models are [1-3]: a good accuracy with enough small size, numerical stability and the necessity to keep the original system passivity. The most recently algorithms used for the order reduction of the large-scale analog circuits could be classified in two categories [10 - 12]:

- Methods based on the explicit (direct) matching of the moments to a reduced model – like *Asymptotic Waveform Evaluation* – AWE [10];
- Methods based on implicit (indirect) moment matching – developed in Krylov space – like *Padé via Lanczos* (PVL) and *Arnoldi algorithm*;
- projection method with Krylov subspaces.

Developed by Pillage and Rohrer in 1990, AWE is the standard method for the analysis of large scale analog circuits. This technique uses the Padé approximation based on explicit moment matching to extract the dominant poles and the residues of the circuit. It consists in two steps:

- Moment computation – meaning computation of the coefficients of MacLaurin series of the transfer function $H(s)$;
- Moment matching – matching these moments with those of a reduced model using the Padé approximation.

The reduced-order model is characterized by an approximate transfer function $\hat{H}(s)$ expressed by a number of rational terms equal with the number of the approximated poles.

In this paper we present the basic concepts to establish the background of the projection method, where we combine projection methods with Krylov subspaces to obtain robust and accurate order reduction techniques. An example is used to illustrate the procedure.

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2 Projection Method for Order Reduction

Iterative projection methods have long been used in linear system solutions and have recently become very usefull for model order reduction [11,12]. We shall introduce the basic concepts to establish the background of this method.

2.1 Krylov subspace definition

A subset of a vector space is called a *subspace*. Let be given a vector set $\boldsymbol{V}_n = \{\boldsymbol{v}_1, \boldsymbol{v}_2, .., \boldsymbol{v}_n\}$; the set of all linear combinations of these vectors is a subspace referred to as the *span* of $\boldsymbol{V}_n$:

$$\begin{aligned}&\mathrm{span}\{\boldsymbol{V}_n\} = \mathrm{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_n\} = \\ &= \left\{ \boldsymbol{x} \,\middle|\; \boldsymbol{x} = \sum_{k=1}^{n} \beta_k \boldsymbol{v}_k \right\}\end{aligned} \tag{1}$$

where $\beta_k, k = \overline{1,n}$, are real numbers. If the $\boldsymbol{v}_k, k = \overline{1,n}$ are linearly independent, then each vector of $\mathrm{span}\{\boldsymbol{V}_n\}$ admits a unique expression as a linear combination of the $\boldsymbol{v}_k$'s. The set $\boldsymbol{V}_n$ is called a **basis** of the span $\{\boldsymbol{V}_n\}$.

Given an nxn matrix $\boldsymbol{A}$ and a vector $\boldsymbol{r}$, the Krylov subspace Kr($\boldsymbol{A}$,$\boldsymbol{r}$, q) is defined as

$$\mathrm{Kr}(\boldsymbol{A}, \boldsymbol{r}, q) = \mathrm{span}\{\boldsymbol{r}, \boldsymbol{A}\boldsymbol{r}, \boldsymbol{A}^2\boldsymbol{r}, ..., \boldsymbol{A}^{q-1}\}. \tag{2}$$

Consider an nxq rectangle matrix $\boldsymbol{V}_q$ whose columns form bases for the subspace spanned by the Krylov sequence $\{\boldsymbol{r}, \boldsymbol{A}\boldsymbol{r}, \boldsymbol{A}^2\boldsymbol{r}, ..., \boldsymbol{A}^{q-1}\}$, that is

$$\mathrm{colsp}\boldsymbol{V}_q = \mathrm{Kr}(\boldsymbol{A}, \boldsymbol{r}, q), \tag{3}$$

where colsp$\boldsymbol{V}_q$ denotes the *column space* of $\boldsymbol{V}_q$. Equation (3) is equivalent to saying that for each k = 0, 1, ..., q – 1 a q-dimensional column vector β_k exists such that

$$\boldsymbol{A}^k \boldsymbol{r} = \boldsymbol{V}_q \beta_k . \tag{4}$$

We consider a linear circuit with a single input (excitation) and semi-state (MNA) description of {$\boldsymbol{G}$, $\boldsymbol{W}$, $\boldsymbol{b}$}. If we define $\boldsymbol{A} = -\boldsymbol{G}^{-1}\boldsymbol{W}$ and $\boldsymbol{r} = \boldsymbol{G}^{-1}\boldsymbol{b}$, it is straightforward to show that the moment vectors of this circuit are given by $\boldsymbol{A}^k\boldsymbol{r}$, k = 0, 1, ... Assume that a basis matrix $\boldsymbol{V}_q$ is generated for the Krylov subspace Kr($\boldsymbol{A}$, $\boldsymbol{r}$, q). Equation (4) clearly shows how the columns of $\boldsymbol{V}_q$ are related to the circuit moments. Any moment vector can be expressed as a linear combination of the Krylov vectors. These vectors contain some information. However, the Krylov vectors contain much less numerical noise compared to the circuit moments because during the generation of a Krylov vector the effects of lower-order moment vectors are implicitly substracted.

The Krylov subspace in (2) is defined for a single starting vector. Similarly, given an nxn matrix $\boldsymbol{A}$ and an nxm matrix $\boldsymbol{R}$, the *block Krylov subspace* is defined as

$$\mathrm{Kr}(\boldsymbol{A}, \boldsymbol{R}, q) = \mathrm{span}\{\boldsymbol{R}, \boldsymbol{A}\boldsymbol{R}, ..., \boldsymbol{A}^{j-1}\boldsymbol{R}\} \tag{5}$$

where $j = q/m$. If q/m does not result as an integer, we set $j = \lfloor q/m \rfloor$ (The $\lfloor . \rfloor$ operator is the truncation to the nearest integer towards zero) and define the Krylov subspace as

$$\begin{aligned}&\mathrm{Kr}(\boldsymbol{A}, \boldsymbol{R}, q) = \mathrm{span}\{\boldsymbol{R}, \boldsymbol{A}\boldsymbol{R}, ..., \boldsymbol{A}^{j-1}\boldsymbol{R}, \\ &\boldsymbol{A}^j\boldsymbol{r}_1, \boldsymbol{A}^j\boldsymbol{r}_2, ..., \boldsymbol{A}^j\boldsymbol{r}_l\}\end{aligned}, \tag{6}$$

where $\boldsymbol{r}_i$ is the ith column vector of $\boldsymbol{R}$, and $l = q - jm$. For the sake of simplicity, however, we will always assume that q/m is an integer.

Consider an nxq rectangular matrix $\boldsymbol{V}_q$, whose columns form bases for the subspace spanned by Krylov sequence {$\boldsymbol{R}$, $\boldsymbol{AR}$, ..., $\boldsymbol{A}^{j-1}\boldsymbol{R}$}, that is,

$$\mathrm{colsp}\boldsymbol{V}_q = \mathrm{Kr}(\boldsymbol{A}, \boldsymbol{R}, q). \tag{7}$$

Thus, qxm matrices $\boldsymbol{\beta}_i$ exist such that:

$$\boldsymbol{A}^i\boldsymbol{R} = \boldsymbol{V}_q\boldsymbol{\beta}_i,\ i = 0,1, ..., j-1. \tag{8}$$

Analogous to the single input case, equation (8) shows the relation between the Krylov matrix and the block moments of a linear circuit with multiple excitations, described by the equations [1 - 9]:

$$\boldsymbol{W}\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} + \boldsymbol{G}\boldsymbol{x} = \boldsymbol{B}\boldsymbol{y}_{in}(t);\ \boldsymbol{u}_{out}(t) = \boldsymbol{L}^{\mathrm{t}}\boldsymbol{x}, \tag{9}$$

where: $\boldsymbol{G}$ and $\boldsymbol{W}$ are the nxn MNA circuit matrices, representing the conductance and dinamic elements, respectively; $\boldsymbol{x}$ – is the vector of MNA variables of size n; $\boldsymbol{y}_{in}(t)$ – represents the vector of input excitations of size n_i; $\boldsymbol{B}$ is the nxn_i matrix corresponding to the coefficients of the input vector $\boldsymbol{y}_{in}(t)$ to the MNA vector $\boldsymbol{x}$; $\boldsymbol{u}_{out}(t)$ – represents the output vector of size n_o, and $\boldsymbol{L}$ is the nxn_o selection matrix, mapping $\boldsymbol{u}_{out}(t)$ to the MNA vector $\boldsymbol{x}$.

If we define the n_ixn_o transfer function matrix $\boldsymbol{H}(s)$ as

$$\boldsymbol{H}(s) \overset{\mathrm{d}}{=} \frac{\boldsymbol{U}_{out}(s)}{\boldsymbol{Y}_{in}(s)}, \tag{10}$$

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where $\boldsymbol{Y}_{in}(s)$ and $\boldsymbol{U}_{out}(s)$ are the Laplace transforms of $\boldsymbol{y}_{in}(t)$ and $\boldsymbol{u}_{out}(t)$, respectively, from (9) and (10) it follows that

$$\boldsymbol{H}(s)=\boldsymbol{L}^{\mathrm{t}}(s\boldsymbol{W}+\boldsymbol{G})^{-1}\boldsymbol{B}\,. \tag{11}$$

If we asume that $\boldsymbol{G}$ is invertible, defining

$$\boldsymbol{A}=-\boldsymbol{G}^{-1}\boldsymbol{W} \text{ and } \boldsymbol{R}=\boldsymbol{G}^{-1}\boldsymbol{B}\,, \tag{12}$$

we can rewrite the double matrix MNA description in (9) in the form of a single matrix reprezentation as it follows:

$$\boldsymbol{x}=\boldsymbol{A}\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}+\boldsymbol{R}\boldsymbol{y}_{in}(t);\ \boldsymbol{u}_{out}(t)=\boldsymbol{L}^{\mathrm{t}}\boldsymbol{x}\,. \tag{13}$$

In this case, the transfer function matrix becomes

$$\boldsymbol{H}(s)=\boldsymbol{L}^{\mathrm{t}}(\boldsymbol{I}-s\boldsymbol{A})^{-1}\boldsymbol{R}\,. \tag{14}$$

Equation (14) can also be written as

$$\boldsymbol{H}(s)=\frac{\boldsymbol{L}^{\mathrm{t}}\mathrm{adj}(\boldsymbol{I}-s\boldsymbol{A})\boldsymbol{R}}{\det((\boldsymbol{I}-s\boldsymbol{A}))}\,. \tag{15}$$

2.2 Krylov Vector Computation

The Krylov vector generation in linear circuits is similar to moment generation, therefore, all of the techniques for moment generation can also be used for efficient Krylov sequence computation.

The block moments of a linear circuit are generated recursively using the following algorithm:

**Recursive scheme for the block moment*
**calculation:*

$\boldsymbol{G}\boldsymbol{M}_0=\boldsymbol{B}$

for $j = 1, 2, ..., q-1$ (16)

$\quad\boldsymbol{G}\boldsymbol{M}_j=\boldsymbol{W}\boldsymbol{M}_{j-1}$

end

Thus, for a linear circuit with n_p ports, in order to generate q block moments, we need to carry out qn_p forward and back substitutions in addition to a single LU factorization of the usually sparse matrix $\boldsymbol{G}$.

Analog, the Krylov vector blocks are obtained from a similar recursive procedure:

**Recursive procedure for Krylov vector*
**generation*:

$\boldsymbol{G}\boldsymbol{M}_0=\boldsymbol{B}$

$\boldsymbol{V}_0=orth(\boldsymbol{M}_0)$

for $i = 1, 2, ..., q-1$

$\quad\boldsymbol{G}\boldsymbol{M}_i=\boldsymbol{W}\boldsymbol{M}_{i-1}$ (17)

$\quad\boldsymbol{V}_i^{temp}=\boldsymbol{M}_i-\sum_{k=1}^{i}\boldsymbol{V}_{i-k}\left(\boldsymbol{V}_{i-k}^{\mathrm{t}}\boldsymbol{M}_i\right)$

$\quad\boldsymbol{V}_i=orth\left(\boldsymbol{V}_i^{temp}\right)$

end

Procedure (17) is a condensed and mathematically equivalent version of the PRIMA algorithm [12]. In terms of the circuit matrix operations the computational cost of the recursive procedure given in (17) is equivalent to that of (16).

2.3 Projection method description

Consider a linear system

$$\boldsymbol{Ax}=\boldsymbol{b} \tag{18}$$

where $\boldsymbol{A}$ is nxn real matrix. *Projection techniques* extract an approximative solution of the above system from a search subspace K of dimension q so that q constraints are satisfied. Generally, these constraints involve q independent orthogonality conditions. For example, the rezidual vector $\boldsymbol{Ax}-\boldsymbol{b}$ is constrained to be orthogonal to q linearly independent vectors. In this way another subspace L of dimension q is defined. Such constraints are known as *Petrov-Galerkin* conditions.

There are two classes of projection methods: if the subspace K is the same as L, the projection is said to be *orthogonal*; otherwise, it is an *oblique* projection. For the linear dynamic systems, the projection is associated with matrix transformations. For example, we consider multi-input, multi-output linear dynamic systems that can be used for multiport interconnect macromodeling, and are discribed by the equations (13). Consider two q-dimensional subspace K and let L be an nxq matrix whose column vector form a basis of K. Similarly let M_q be an nxq matrix whose column vector form a basis of L, i.e.

$$colspV_q=K\ ;\quad colspM_q=L \tag{19}$$

A reduced order model for the system (13) via projection has the following form:

$$\begin{gathered}\boldsymbol{M}_q^{\mathrm{t}}\boldsymbol{V}_q\boldsymbol{x}_q=\boldsymbol{M}_q^{\mathrm{t}}\boldsymbol{A}\boldsymbol{V}_q\frac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t}+\boldsymbol{M}_q^{\mathrm{t}}\boldsymbol{R}\boldsymbol{y}_{in}(t)\\ \boldsymbol{u}_{out}(t)=\boldsymbol{L}^{\mathrm{t}}\boldsymbol{V}_q\boldsymbol{x}_q.\end{gathered} \tag{20}$$

Since the approximation order q is smaller than the number of original variables, n, the system (20) is a reduced-order approximation of the original in (13), and the output response $\boldsymbol{u}_{out}$(t) is an approximation of the actual output response $\boldsymbol{u}_{out}$(t) in (13). In projection terms, the qxq matrix $\boldsymbol{M}_q^{\mathrm{t}}\boldsymbol{A}\boldsymbol{V}_q$ is the projection of $\boldsymbol{A}$ onto the subspace spanned by $\boldsymbol{V}_q$, and orthogonal to the subspace spanned by $\boldsymbol{M}_q$. In the same way, one can think that the solution vector is approximated by another solution vector, but in the subspace K,

$$\boldsymbol{x} = \boldsymbol{V}_q \boldsymbol{x}_q \,. \tag{21}$$

Similarly, the original system in (9) can be reduced with the double matrix projection (reducing $\boldsymbol{G}$ and $\boldsymbol{W}$ separately):

$$\begin{aligned} &\boldsymbol{M}_q^{t}\boldsymbol{G}\boldsymbol{V}_q\boldsymbol{x}_q + \boldsymbol{M}_q^{t}\boldsymbol{W}\boldsymbol{V}_q\frac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t} = \boldsymbol{M}_q^{t}\boldsymbol{B}\boldsymbol{y}_{in}(t) \\ &\boldsymbol{u}_{out}(t) = \boldsymbol{L}^{t}\boldsymbol{V}_q\boldsymbol{x}_q. \end{aligned} \tag{22}$$

The approximate solution is sought in the subspace K = span(V_q) and the residual is orthogonal to the subspace L = span$(\boldsymbol{M}_q)$, so that, it satisfies the Petrov-Galerkin conditions.

There are two projection mehods onto Krylov subspace: bock Lanczos and block Arnoldi. We shall develope a projection method onto Krylov subspace similarly to the Arnoldi procedure. We introduce projections onto Krylov subspaces. In this case, the subspace Krylov K = span(V_q) is identical to the Krylov subspace L = span$(\boldsymbol{M}_q)$ ($L \equiv K$). For this reason the equations (20) and (22) become

$$\begin{cases} \boldsymbol{V}_q^{t}\boldsymbol{V}_q\boldsymbol{x}_q = \boldsymbol{V}_q^{t}\boldsymbol{A}\boldsymbol{V}_q\dfrac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t} + \boldsymbol{V}_q^{t}\boldsymbol{R}\boldsymbol{y}_{in}(t) \\ \boldsymbol{u}_{out}(t) = \boldsymbol{L}^{t}\boldsymbol{V}_q\boldsymbol{x}_q \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \boldsymbol{V}_q^{t}\boldsymbol{V}_q\boldsymbol{x}_q = \boldsymbol{A}_q\dfrac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t} + \boldsymbol{R}_q\boldsymbol{y}_{in}(t) \\ \boldsymbol{u}_{out}(t) = \boldsymbol{L}_q^{t}\boldsymbol{x}_q \end{cases}, \tag{23}$$

where $\boldsymbol{A}_q = \boldsymbol{V}_q^{t}\boldsymbol{A}\boldsymbol{V}_q, \boldsymbol{R}_q = \boldsymbol{V}_q^{t}\boldsymbol{R}, \boldsymbol{L}_q^{t} = \boldsymbol{L}^{t}\boldsymbol{V}_q;$

and

$$\begin{cases} \boldsymbol{V}_q^{t}\boldsymbol{G}\boldsymbol{V}_q\boldsymbol{x}_q + \boldsymbol{V}_q^{t}\boldsymbol{W}\boldsymbol{V}_q\dfrac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t} + \boldsymbol{V}_q^{t}\boldsymbol{B}\boldsymbol{y}_{in}(t) \\ \boldsymbol{u}_{out}(t) = \boldsymbol{L}^{t}\boldsymbol{V}_q\boldsymbol{x}_q \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \boldsymbol{G}_q\boldsymbol{x}_q + \boldsymbol{W}_q\dfrac{\mathrm{d}\boldsymbol{x}_q}{\mathrm{d}t} = \boldsymbol{B}_q\boldsymbol{y}_{in}(t) \\ \boldsymbol{u}_{out}(t) = \boldsymbol{L}_q^{t}\boldsymbol{x}_q \end{cases}, \tag{24}$$

where: $\boldsymbol{G}_q = \boldsymbol{V}_q^{t}\boldsymbol{G}\boldsymbol{V}_q, \boldsymbol{W}_q = \boldsymbol{V}_q^{t}\boldsymbol{W}\boldsymbol{V}_q, \boldsymbol{B}_q = \boldsymbol{V}_q^{t}\boldsymbol{B},$

and $\boldsymbol{L}_q^{t} = \boldsymbol{L}^{t}\boldsymbol{V}_q$, respectively.

Therefore, the transfer function matrix in the reduced form has the following expression:

$$\boldsymbol{H}_q(s) = \boldsymbol{L}_q^{t}\left(\boldsymbol{I}_q - s\boldsymbol{A}_q\right)^{-1}\boldsymbol{R}_q \,, \tag{25}$$

when we use the MNA description in the form of a single matrix representatin (23), and

$$\boldsymbol{H}_q(s) = \boldsymbol{L}_q^{t}\left(s\boldsymbol{W}_q + \boldsymbol{G}_q\right)^{-1}\boldsymbol{B}_q \tag{26}$$

when we use the double matrix MNA description in (24).

Given an RLC circuit with the MNA (semi-state) formulation (9), the following algorithm finds for rthe begining a basis matrix $\boldsymbol{V}_q$ onto Krylov subspaces using a block Arnoldi process and then constructs a reduced-order model via projection in the form of (24). The basic algorithm is as follows:

1. Generate, using the program SEMAG – **Se**mi-state **Ma**trix **G**eneration [6], the matrices $\boldsymbol{W}$, $\boldsymbol{G}$, $\boldsymbol{B}$, and $\boldsymbol{L}^{t}$. These matrices are generated in symbolic or numeric form.
2. Compute the bock moments $\boldsymbol{M}_0$, $\boldsymbol{M}_1$, ..., $\boldsymbol{M}_{q-1}$, using the recursive procedure (16).
3. Using the recursive procedure (17) compute the Krylov vector blocks, an nxq matrix

$$\boldsymbol{V}_q = \left[\boldsymbol{V}_0, \boldsymbol{V}_1, ..., \boldsymbol{V}_{q-1}\right] \,. \tag{27}$$

4. Compute, using the relation (26), the transfer function matrix in reduced form.

3 Example

Consider the neural network shown in Fig. 1.

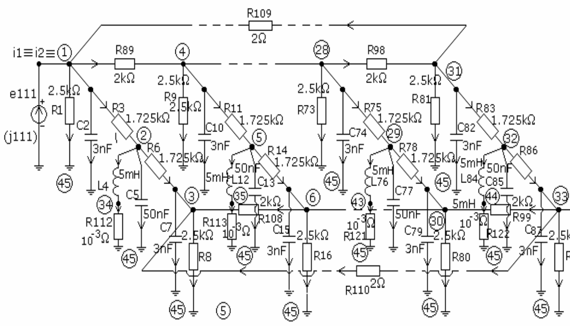

Fig. 1. Neural network

We want to find the transfer function matrix in the reduced form, using the projection method presented in Section 2,

$$\boldsymbol{F}(s) = \begin{bmatrix} A_{30\_45,33\_45}(s) = \dfrac{U_{C87}(s)}{E_{111}(s)} \\ Z_{30\_45,33\_45}(s) = \dfrac{U_{C87}(s)}{J_{111}(s)} \end{bmatrix}. \tag{28}$$

Bode characteristics (magnitude-frequency and argument-frequency) of the transfer functions of the original circuit, *Aoi_vs*, together with the approximated ones: *Aoi5_sem, Aoi7_sem, and*

Aoi9_sem (*Zoi_vs*, together with the approximated ones: *Zoi5_sem*, *Zoi7_sem*, *and Zoi9_sem*) are represented in Figures 2, 3 and 3 (Figures 3 and 4).

The subscripts have the following meanings: *vs* – state equations are used, *sem* – semi-state equations (MNE) are used, and the number is the order of the denominator.

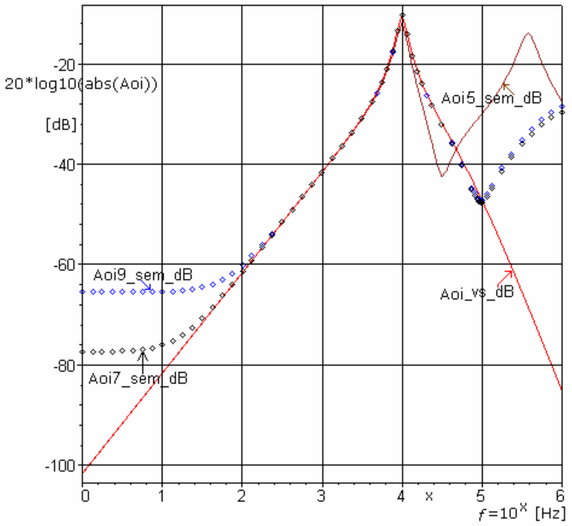

Fig. 2. Magnitude-frequency characteristics for *A*.

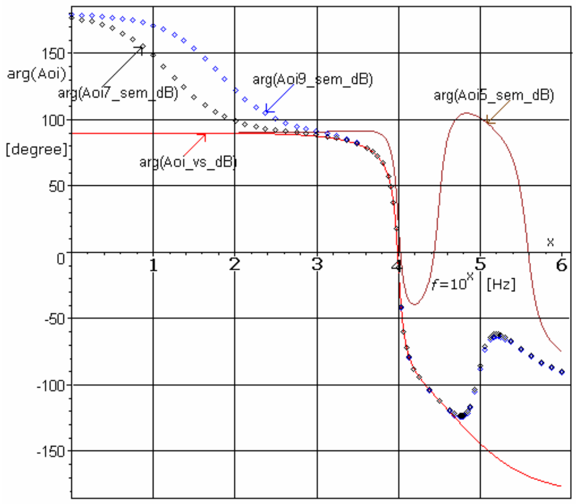

Fig. 3. Argument – frequency characteristics for *A*.

The complexity order of the circuit in Figure 1 is 43. The macromodels of 5, 7 and 9 order have been computed by the projection technique based on the semi-state equations presented above. The Bode characteristics of the voltage gain and of the transfer impedance show that the ones correponding to the 7 order model are the closest to the the exact characteristics (obtained by the state equation method). In the frequency range of interest all Bode characteristics overlay.

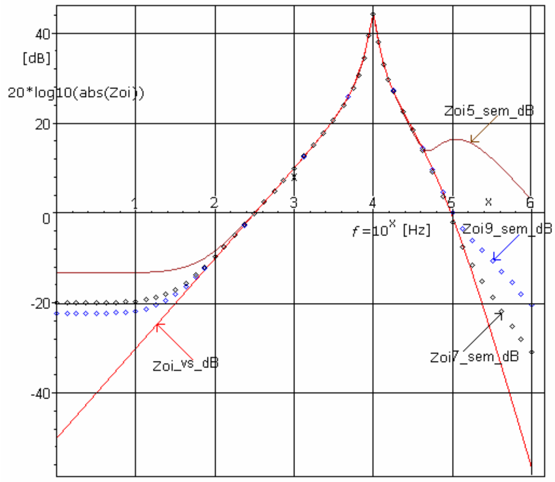

Fig. 4. Magnitude – frequensy characteristics for *Z*.

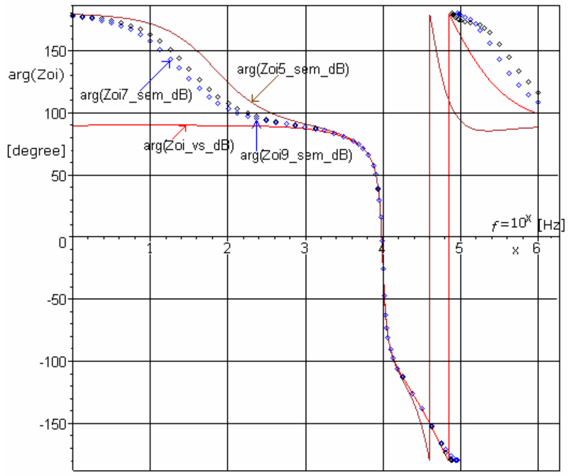

Fig. 5. Argument - frequency characteristics for *Z*.

4 Conclusion

Based on the semi-state equation formulation a technique for behavioral model generation by the projection method with Krylov subspaces is developed in order to obtain an accurate approximation of the original transfer function. Performing this procedure a set of reduced models were generated. Krylov vectors, bases for Krylov subspaces, contain the same information as the moments, but numerically they are better conditioned. Thus, their use allows us to obtain very a high accuracy of the reduced-order models. Comparing the characteristics of the different approximations of the transfer functions, the best approximation in the frequency range of interest can be selected. Moreover, after applying a synthesis procedure, a symbolic transfer function in reduced form is available for the equivalent reduced-size

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circuit. For a network function there is an optimum order of the behavioral macromodel. Over this order the Krylov vector can lose the orthogonality so that the macromodel becomes instable and loses the passivity.

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