Educational and Technical Issues in Teaching Resonance Phenomena in the Theory of Electrical Engineering

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Abstract: - The paper has a purpose to show some abilities for investigation of resonance phenomena in study circumstances, when we use developed for this purpose models and circuits. The use of these circuits in the laboratory model will make investigation of the resonance circuits equivalent and closer to the results of analytical methods and conclusions.

Key-Words: - Resonance phenomena, Gyrator, Generator of current, Power amplifier, Resonance of “all” frequencies, Electrical inductivity

1 Introduction

One good methodical opportunity for demonstration of facilities and peculiarity, which occur during the sinusoidal steady state in ordinary electrical circuits, is experimental investigation and demonstration of electrical resonance in such circuits [1], [2], [3], [4], [6]. Most probably it doesn't exists a theoretical course in which there is not included such an experimental investigation. Laboratory results and theoretical explanations on different types of resonance characteristics of voltage and currents, of equivalent impedance or conductance, of reactive resistance or conductance, of phase argument, unsteady change in characteristic of circuit- inductive or capacitive is very good opportunity for students to understand better these characteristic peculiarities. Sequential experimental investigation of resonance phenomena of cascade or parallel type helps in addition to distinguish and form the needed mathematical knowledge for analysis of this kind of circuits. Graphical depiction of resonance function makes more clear and hardens knowledge, and also develops in one more clear way relevant habits. Demonstration of physical meaning of quality factor, wave resistance or wave conductance and their calculation according made measurements, shows “sight existence” of used in theory definitions.

Unfortunately when such of laboratory exercises are given often there are additional difficulties, which overcome will permit more accurate experimental results closer to the relevant analytical results and conclusions. The use of frequency generators with lower class of accuracy bring in account output resistance of generator in determination of relevant electrical quantities. Most often, during the experiment function of output voltage of frequency generator is noticed. This requires bigger attention and certain skills for more accurate picture of resonance function with constant voltage of the input of the circuit. The use of the source of current, needed for investigation of relevant dual resonance functions, sometimes leads to change in the constant value of the current, due to the lack of good characteristics of the sources. Realization of the frequency investigation in given frequency area requires linear coil with significant value of the inductivity and significant dimensions. Given difficulties (great values of the inductance and accurate variable decade resistance in parallel brunches) exist as well experiment to demonstrate the case of resonance in “all” frequencies which, if successful, will be memorable demonstration.

The given paper has a purpose to show some abilities for investigation of resonance phenomena in study circumstances, when we use developed for this purpose models and circuits. The use of these circuits in the laboratory model will make investigation of the resonance circuits equivalent and closer to the results of analytical methods and conclusions.

2 Laboratory setting and opportunities for possible investigation

In order to receive more identical results with analytical results when we investigate different resonance circuits in our department we have developed the following electrical devices:

- **Power amplifier** - its purpose is to keep constant value of the output voltage of the used in the setting generator, with changing of the generated frequency in the investigated frequency area. We have in account the amplifier to be adapted for
switching on to different kinds of frequency generators, which include sound frequency bandwidth of the investigation.

- **Generator of current** - its purpose is to generate sinusoidal electric current which is with constant value for a given range in the investigated frequency. It is used for investigation of resonance characteristics when the current in the circuit in the given frequency area has a constant value. We use circuit which assures more stable indexes of the generator.

- **Electrical inductivity** - made with the use of active device gyrator. Its purpose is to be used in the investigated electrical circuits, instead of standard electrical linear coil. The given circuit allows achievement of artificial inductivity in the interval 10mH – 1H. This will allow work with a big value of the inductivity when we have small volume and better quality factor of the coil, necessary for example for laboratory investigation of resonance of “all” frequencies.

Mentioned devices are investigated with models and computer simulations, and as well like an experimental pattern. Circuits, results and characteristics for them are given in the relevant part of the paper. With these we can make better investigation of different electrical resonance phenomena in sinusoidal steady state.

For the purpose of the teaching course of Theoretical electrotechnique of the department of Theoretical electrotechnique in TU-Sofia most appropriate is to make in the laboratory exercise completely the following opportunities:
- Investigation of voltage resonance in cascade RLC circuit;
- Investigation of parallel resonance in parallel RLC circuit;
- Investigation of resonance in “all” frequencies in parallel one-port.

With these we do not exhaust opportunities for the use of developed devices when we investigate resonance phenomena - for example when we investigate encompass and bar filters, in resonances in parallel one-ports with two or three conservative elements and so on.

### 2.1 Investigation of voltage resonance in series RLC

Series connection circuit is shown in Fig.1. Resistance is included in the circuit in order to measure the current with the voltage drop on it. To investigate the resonance on the input voltage \( U=\text{const} \) the sound generator is connected to the input of the circuit with the use of developed power amplifier. In order to make experiments with input current \( I=\text{const} \) to the input of the circuit is connected the developed generator-source of current.

\[
\begin{align*}
\text{Fig. 1: A circuit for investigation of series resonance.}
\end{align*}
\]

It is practical and methodical interest the receiving with measurement and understanding of:
- a) Resonance frequency \( f_P \) and inductivity \( L \) with known capacitance \( C \);
- b) the functions \( I(f), U_R(f), U_L(f) \) and \( U_C(f) \) with \( U=\text{const} \);
- c) the functions \( U(f), U_R(f) \) and \( U_L(f) \) with \( I=\text{const} \);
- d) the function of full resistance \( Z = Z(f) \) with \( U(f)/I(f) \);
- e) the value of the quality factor \( Q \) of the circuit and comparing of the results received with:
  - values \( U_{in} \) and \( U_{lp} \) , respectively \( U_{cp} \) measured in resonance;
  - resistance \( R \) and \( X_{lp} \), respectively \( X_{cp} \), calculate in resonance;
  - \( Q = \frac{\rho}{R} \), when the wave resistance is \( \rho = \sqrt{\frac{L}{C}} \);
  - frequency bandwidth;
- f) collection of quality factor \( Q \) when we use capacitors with low and high level of losses, depending on the frequency bandwidth.

### 2.2 Investigation of parallel resonance in parallel GLC circuit

A circuit for investigation of parallel resonance is shown in Fig.2. The resistance \( R \) of the resistor is chosen in such a way so we can measure the common current \( I \) using its voltage drop \( U_R \). For investigation of the circuit when we have voltage on the parallel junction \( U=\text{const} \) and current \( I=\text{const} \) respectively we connect to the circuit the voltage generator and the power amplifier or current generator. For the presence of resonance in the circuit we can judge from the values of voltage \( U \) and current \( I \) from Fig. 2.

From practical and methodical interest is obtaining using measurement and interpretation of:

- a) resonance frequency \( f_P \) and inductivity \( L \) with given capacitance \( C \);
b) determination of parasitic conductivity of the capacitor from the expression \( G_C = \frac{I_p}{U_p} \) (electric coil is consider as one without losses). Using capacitors with low losses (for example polypropylene capacitors) decrease the value of \( G_C \), obtained from this calculation;

c) functions \( I(f) \), \( I_L(f) \) and \( I_C(f) \) with \( U = \text{const} \), since \( I_L(f) \) and \( I_C(f) \) are calculated on the basis of fulfilled \( U = \text{const} \);

d) functions \( U(f) \), \( I_L(f) \) and \( I_C(f) \) with \( I = \text{const} \) \((U_0(f) = \text{const} \) for the respective value of the current from the current source), \( I_L(f) \) and \( I_C(f) \) are calculated on the basis of the measured \( U(f) \);

e) function of full conductance \( y = y(f) \) with \( I(f)/U(f) \);

f) value of the quality factor \( Q \) of the circuit and collation of the results obtained with:
- currents \( I \) and \( I_{LP} \), respectively \( I_{CP} \);
- conductance \( G \) and \( B_{LP} \), respectively \( B_{CP} \);

\[ Q = \frac{y}{G}, \text{where wave conductance is } y = \sqrt{\frac{C}{L}}; \]
- frequency bandwidth.

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### Fig. 2: A circuit for investigation of parallel resonance.

### 2.3 Investigation of resonance in parallel one-port

Resonance in the one-port from Fig.3 comes when we have the following frequency:

\[
f_p = \frac{1}{2\pi} \sqrt{\frac{L - CR_1^2}{L^2C - LC^2R_2^2}} = f_o \sqrt{\frac{\rho^2 - R_1^2}{\rho^2 - R_2^2}},
\]

where \( f_o = \frac{1}{\sqrt{LC}} \) and \( \rho = \sqrt{\frac{L}{C}}. \)

For the presence of resonance in the circuit we judge from the values of the voltage \( U \) and current \( I \) from Fig.3. It’s proven analytically, that when we have equality \( R_1 = R_2 = \rho \) in the circuit we have resonance for every frequency of the input sinusoidal signal.

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One opportunity for achieving of such resonance is laboratory task with the following steps:

**Step 1:** With known from the examined already two cases of resonance value of inductance \( L \) and unknown value of the capacitance \( C \) we have \( R_1 = R_2 \approx 0 \). For this purpose connected in the circuit resistors are shunted. Under these circumstances we can determine experimentally the resonance frequency \( f_o \), and from this value to calculate capacitance \( C \).

**Step 2:** With connected in the both brunches resistors with appropriate chosen value we accomplish the equality \( R_1 = R_2 \). With such equality the resonance frequency of the circuit is equal of the one that is found \( f_o \). Equality of the both resistances is achieved with adjustment of two helicoidally potentiometers cascade connected to the resistors. The goal is to achieve again resonance in the circuit for frequency \( f = f_o \). With this step we report the presence of different resistance in the connective conductors and the losses in the conservative elements.

**Step 3:** We change the frequency of the input signal and for new frequency with smooth adjustment of used variable capacitor to \( C \) we change \( \rho = \frac{R}{\sqrt{C}} \) until we achieve the equality \( R_1 = R_2 = \rho \). The equality is settled with occurring of the same values of the voltage and current, achieved in 2. In this way the circuit is adjusted in resonance for all frequencies. With constant by effective value sinusoidal supply voltage, current in the circuit keeps its maximum (resonance) value for the different frequencies. This value is determined from the equation \( I(f) = \frac{U_{in}(f)}{R + 1/G_e} = \text{const} \).
3 Auxiliary circuits developed—schematics and characteristics

The following auxiliary electronic circuits have been developed [5]:

3.1 Power amplifier

It has the function to preserve a constant RMS value of the output voltage of the functional generator, when the frequency is changed in the frequency range $f = \frac{20 \text{ Hz}}{20 \text{ kHz}}$, and a resonant circuit is connected as a load to the generator.

This additional electronic stage is necessary as the output impedance of the commonly used functional generators is $50 \Omega$. The resonance circuits used in the laboratory have impedance $Z = 100 \div 10000 \Omega$ in the investigated frequency range. Therefore, the generator output voltage will not be constant, because its output impedance and the loading resonance circuit will form a frequency dependent voltage divider. To avoid the generator output voltage variation with the frequency, an additional amplifier stage is necessary – power amplifier having very low output impedance ($<1 \Omega$). In this case the investigated resonance circuit will not load substantially the generator output and its voltage will remain constant.

To implement the power amplifier, the integrated circuit TDA2050 has been used. It represents an IC - 32W-power amplifier for the audio frequency range - Fig. 4. It has short-circuited output protection and thermal protection. The irregularity of the frequency response is less than 3 dB in the frequency range $f = \frac{20 \text{ Hz}}{20 \text{ kHz}}$. The input impedance of the circuit is $22k\Omega$.

![Power Amplifier Schematic](image)

**Fig 4:** 32W-power amplifier for the audio frequency range.

3.2 Current source

It has the function to convert the constant RMS sinusoidal voltage of a voltage generator to a sinusoidal current with constant RMS value. This conversion must be performed for a range of load impedance $R_L = 0 \div 10k\Omega$, for the frequency range $f = \frac{20 \text{ Hz}}{20 \text{ kHz}}$. The RMS value of the output current must be in the range $I = 0.01 \div 1 \text{ mA}$. Using this current source allows to avoid manually tuning the current value in the resonance circuit at every frequency by regulating the output voltage of the generator and looking for specified constant voltage over a serial resistor with known value.

To implement the current source, a popular schematic having two operational amplifiers (OpAmps) has been used – Fig. 5. In the specific case, 2 OpAmps from the IC TL084 have been used. To work normally as a current source, the elements of circuit must follow the condition:

$$R_1R_4 = R_2R_3$$

The regulating equation between the controlling sinusoidal input voltage and the sinusoidal output current is:

$$I_{RL} = \frac{V_{in}}{R_5}$$

At RMS value of the input voltage $V_{in} = 2 \text{ V}$ and value of the resistor $R_5 = 20k\Omega$, the output sinusoidal
current will have a RMS value of \( I_{RL} = 0.1 \text{mA} \), which is very suitable for the investigated resonance circuits.

![Fig. 5: Current source implementation.](image)

### 3.3 Electronic inductor

It is implemented using a gyrator, an electronic circuit [7], build on two OpAmps from the IC TL084 – Fig.6. The implemented electronic circuit allows obtaining an artificial inductor having inductance in the range \( 10\text{mH} \div 1\text{H} \), with very low size, linear properties and low losses (it is possible to obtain quality factor \( Q = 200 \)).

![Fig. 6: A gyrator circuit (framed with a dash line) for implementation of an inductor.](image)
In Fig. 6 the gyrator (framed with a dash line) consists of the two OpAmps, the resistors $R_1, R_2, R_3, R_5$, and the capacitor $C_4$.

It is known, that the input impedance of this kind of gyrator is given by the formula:

$$Z_{in} = \frac{Z_3 Z_5 Z_2}{Z_2 Z_4}$$

If $Z1=R1$, $Z2=R2$, $Z3=R3$, $Z5=R5$ and $Z_4 = \frac{1}{j\omega C_4}$, the input impedance will be

$$Z_{in} = \frac{R_1 R_3 R_5}{R_2} j\omega C_4 = j\omega L$$

therefore, the inductance value is given by the formula:

$$L = \frac{R_1 R_3 R_5 C_4}{R_2}$$

At the values of the resistors in Fig. 6, $R_1 = R_2 = R_3 = R_5 = 10 \, \text{k}\Omega$ and $C_4 = 1 \, \text{nF}$, the inductance is found to be

$$L = \frac{R_1 R_3 R_5 C_4}{R_2} = 0.1 \, \text{H}$$

The electronic inductor implemented by the gyrator, together with the resistor $R_6$ and the capacitor $C_1$, forms a series resonance circuit. For $C_1 = 10 \, \text{nF}$, the resonance frequency will be

$$f_0 = \frac{1}{2\pi \sqrt{L C_1}} = 5032.9 \, \text{Hz}$$

Since the wave resistance is

$$\rho = \sqrt{\frac{L}{C_1}} = 3162 \, \Omega$$

the quality factor $Q$ of the circuit is

$$Q = \frac{\rho}{R_6} = \frac{3162}{1000} = 3.162$$

The frequency response of the circuit of Fig. 6, obtained using PSpice, is shown on Fig. 7.

![Fig. 7: The frequency response of the circuit of Fig. 6, obtained using PSpice.](image)

From this frequency response, the resonance frequency $f_0 = 5014 \, \text{Hz}$ and the quality factor $Q = 3.21$ are found. The differences between the analytically obtained values and these obtained by PSpice modeling are less than 1.6 %.

The drawbacks of the “electronic inductor”, used in this paper, are:
a) One of the circuit’s leads must be grounded.
b) The “electronic inductor” cannot let pass direct current – this means, that such an inductor cannot be used in smoothing filters for rectifier circuits.
c) The “electronic inductor” cannot work at high frequencies, because of the diminishing gain of the operation amplifiers when the frequency is increased. The “electronic inductor” is mainly used for working frequencies below 100-200 kHz.

During the investigation of the circuit shown on Fig.3 we used the same model of inductor, which is given on Fig.6. The results from the simulations are illustrated on Fig.8.

All the circuits, described here, are modeled using PSpice and experimented in practice using real circuits. The results obtained are very close.

4 Conclusion
We have developed laboratory models of power amplifier, generator of current and electronic inductivity, using gyrator. These devices are meant to be used during investigation of resonance phenomena. The circuits of electronic devices have been modeled using computer simulations with PSpice and using experimental models, as well. We have given also ideas for their use in laboratory circumstances for the needs of the teaching course of Theoretical Electrical Engineering.

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