A Super Resolution SAR Imaging Algorithm Based on Adaptive Kalman Filter for Land Consolidation

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Abstract: Synthetic aperture radar (SAR) remote sensing, with its advantages of all-weather coverage, all day/night acquisitions, cloud penetration, and signal independence of the solar illumination angle, can be applied to land cover classification and land consolidation, especially in some regions where optics and infrared remote sensing do not work well. A limitation of its using for land consolidation is the available spatial resolution. To increase the resolution, we propose a super-resolution imaging algorithm based on adaptive Kalman Filter procedure. The method strongly improves the resolution by using prior knowledge, which is a scientific breakthrough in the case that the traditional pulse compression constrains the improvement of SAR spatial resolution. It is an optimal method in the sense of mean square error and its computation cost is lower than the traditional Kalman Filter algorithm. Simulation results demonstrate the effectiveness of the proposed method.

Key-Words: Synthetic Aperture Radar, Adaptive Kalman Filter, high resolution, Land Consolidation, SNR, Mean Square Error

1. Introduction
SAR (Synthetic aperture radar) is an active microwave remote sensing imaging radar, which can obtain abundant electromagnetic information from ground objects all day/all night and all weather, can penetrate some special objects and detect the shapes of ground objects. Its unique capabilities greatly propel and improve the SAR applications to many fields, including land cover classification and land consolidation. But there are some uncertain factors [1] in SAR imaging which influence the quality of SAR image classification, and obstruct interpretation and applications of SAR images. For land consolidation application, a main restrained factor is the spatial resolution of SAR images.

Many earlier works have been done to improve SAR spatial resolution [2, 3, 4, 5]. But these works greatly depend on the work mode and hardware set of the imaging radar. Although some delightful achievements have been gained, the resolution can not reach its physical utmost or the optics remote sensing resolution level. Here a key problem is how to process the data from a radar standpoint.

To enhance the resolution beyond the matched filter classical limit, Guglielmi et al. [6] applied super-resolution methods to SAR data. They demonstrated two classes of image reconstruction methods: deterministic regularization and stochastic regularization. Goodman et al. [7, 8] has investigated some stochastic regularization methods for SAR processing of satellite constellations. Some stochastic regularization techniques, such as maximum likelihood estimate and minimum mean-squared error (MMSE) filtering, were introduced, and experiments showed that stochastic regularization techniques can provide better geometric resolution than the traditional matched filtering. But these methods require huge additional computation complexity. An iterative implementation of the minimum mean squared error solution was
developed [9]. Although it improved the computation efficiency, it required calculating the inverse for huge matrix repeatedly, thus the complex computation and time consuming were still huge.

To conquer the above problems, we develop a new Kalman Filter (KF) scheme integrating the matched filter to obtain high resolution radar image. Traditional Kalman Filter has the limitation of the stringent requirement on precise a priori knowledge of the system models and noise properties, and uncertainty in the covariance parameters of the process noise (Q) and the observation errors (R) may significantly degrade the filtering performance. For the application of land consolidation, the noise levels may change in different spatial zones of the study areas. To scale the noise without artificial or empirical parameters, this paper proposes a new adaptive Kalman Filter process to replace the traditional Kalman Filter algorithm. The most distinct advantage of this proposed scheme is that adaptive Kalman Filter fully utilizes the data to eliminate measurement error due to clutter and to enhance the resolution beyond the matched filter classical limit.

The remainder of this paper is organized as follows. Section 2 develops the mathematical model of SAR land echo signal. Section 3 describes the algorithm of the adaptive Kalman Filter. In Section 4, we report the experimental results and perform some comparisons with traditional methods. Finally, Section 5 concludes this paper.

2. Signal Model of Land Echo

To use the prior knowledge, now the system signal mode is formulated firstly.

Assuming a radar transmits signal \( s(t) \), the signal measured by the radar receiver at time \( r \), due to an unit scatterer located at \( r \) can be represented as:

\[
p(t, r) = s(t - \tau_r)
\]

where \( r \) is the position vector describing surface location of the unit scatterer relative and \( \tau_r \) is the propagation time delay.

From a view point of SAR, time \( t \) can be considered as two part: fast time \( t_f \) and slow time \( t_s \), and the transmitted signal can be described with \( t_f \). For a common SAR system, the transmitted signal is often chirp scaling signal. Let the chirp rate be \( k_r \), the pulse duration \( T_r \), and the carrier frequency be defined as \( \omega_c \), then the transmitted signal is:

\[
s_r(t_f) = \exp[j(\omega_c t_f + k_f t_f^2)] \text{rect}\left(\frac{t_f}{T_r}\right)
\]

where, \( \text{rect}(\cdot) \) is defined as:

\[
\text{rect}\left(\frac{t_f}{T_r}\right) = \begin{cases} 1 & -\frac{T_r}{2} \leq t_f \leq \frac{T_r}{2} \\ 0 & \text{others} \end{cases}
\]

During the fast time, the movement of the system can be ignored, thus, the propagation delay \( \tau_r \) can be approximated to \( \tau_{t,s} \). So the signal is:

\[
p(t, r) = p(t_s, t_f, r) = \exp[j(-\omega_c \tau_{t,s} + k_f (t_f - \tau_{t,s}))^2] \cdot \text{rect}\left(\frac{t_f - \tau_{t,s}}{T_r}\right)
\]

Let the transmitting antenna gain be a constant, the total measurement taken by the receiver due to all illuminated scatterers is:

\[
s_r(t) = s_r(t_s, t_f) = \int \gamma(r) \cdot p(t, r) dr + n(t_s, t_f)
\]

where \( \gamma(r) \) is the back reflectivity at \( r \).

Since the transmitted signal is constrained both in bandwidth and in time, Equation (5) sampled at time \( t_{sn} \) and \( t_{fm} \) can be approximated with discrete samples as:

\[
s_r(t_{sn}, t_{fm}, r_i) = \sum_{i} \gamma(A_i) \cdot p(t_{sn}, t_{fm}; A_i) \Delta A + n(t_{sn}, t_{fm})
\]

where \( \Delta A \) refers to unit scatter area, and is a constant, \( i \) is the index of different area.

Let

\[
S_r = \left[ s_r(t_{s1}, t_{f1}) \ldots s_r(t_{s1}, t_{fn}) \ s_r(t_{s2}, t_{f1}) \ldots s_r(t_{s2}, t_{fn}) \right]^T
\]

\[
\gamma = [\gamma(A_1) \ \gamma(A_2) \ \cdots \ \gamma(A_D)]^T
\]
where $D$ is the number of different areas of size $\Delta A$, $sN_a$ is the sampled number during the slow time interval, $fN_r$ is the sampled number during the fast time interval, and $(\cdot)^T$ denotes the transpose operation.

Then Equation (6) can be represented using matrix-vector notation:

$$S_r = P_{\gamma} + N$$  \hspace{1cm} (11)$$

### 3. Implementation of Adaptive Kalman Filter Algorithm

Our adaptive Kalman Filter process includes two main phases: compressing part of the data by the matched filtering and applying the adaptive Kalman filtering to the rest of the data. This section shows the processing of our developed adaptive Kalman Filter algorithm.

#### 3.1 Matched Filtering Processing

To improve the computation performance, part of the data are selected and compressed with the matched filter firstly in our process scheme.

How to divide the data up between the matched filter and Kalman Filter? In fact the data can be selected in any manner as long as the matched filtering process limits the number of non-zero pixels to less than the number of measurements available for the Kalman filtering.

Here, we select $1/3$ data for the matched filter. A subset of the data is chosen:

$$S_{\text{sr}} = \begin{bmatrix} s_r(t_{s1}, t_{f1}) \cdots s_r(t_{s1}, t_{f1}) \cdots s_r(t_{sr}, t_{fr}) \end{bmatrix}^T$$ \hspace{1cm} (12)$$

#### 3.2 Adaptive Kalman Filtering Processing

Now it’s time to use the adaptive Kalman Filter for the rest of the data to get images with high performance.

The Kalman Filter technique formulates a linear discrete system by two equations: the state equation and the measurement equation. For a SAR system, the state equation of a Kalman filtering can be given as:

$$\gamma(i) = A(i)\gamma(i - 1) + n_p(i)$$ \hspace{1cm} (16)$$

The measurement equation for this case can be written as:

$$S_r(i) = P(i)\gamma(i) + n_o(i)$$ \hspace{1cm} (17)$$
where \( i \) is the iteration number or the section of data and the data can be divided into \( I \) smaller vectors, so \( i \) varies from 1 to \( I \), \( n_p(i) \) is the process noise and expresses the uncertainty in the modeling of the expected variation, \( n_o(i) \) represents the measurement errors that occur at each observation time. The noise statistic will satisfy:

\[
E[n_p(i)] = E_p, \quad E[n_p(i), n_p(j)] = R_p \delta_{ij}\\
E[n_o(i)] = E_o, \quad E[n_o(i), n_o(j)] = R_o \delta_{ij}\\
E[n_p(i), n_o(J)] = 0
\]

where \( E(\bullet) \) denotes the expectation function.

Usually it is assumed that the state vector comprising of the scattering coefficients keeps approximately constant with respect to time, space, and frequency over the extent of the radar measurement. So the state transition matrix \( A(i) \) will be an identity matrix, i.e., \( A(i) = I \) and the state equation of Equation (16) can be rewritten as:

\[
\gamma(i) = \gamma(i-1) + n_p(i)
\]

To improve the robustness of the adaptive filtering algorithm, the noise statistic is scaled here.

\[
\begin{align*}
\hat{E}_p(i) &= (1 - \frac{1}{l}) \hat{E}_p(i-1) + \frac{1}{l} [\gamma(i) - \gamma(i-1)] \\
\hat{R}_p(i) &= (1 - \frac{1}{l}) \hat{R}_p(i-1) + \frac{1}{l} [K(i) w(i) w(i)^T K(i) + \tilde{K}_g(i) - \tilde{K}_g(i-1)] \\
\hat{E}_o(i) &= (1 - \frac{1}{l}) \hat{E}_o(i-1) + \frac{1}{l} [S_r(i) - P(i) \gamma(i)] \\
\hat{R}_o(i) &= (1 - \frac{1}{l}) \hat{R}_o(i-1) + \frac{1}{l} [w(i) w(i)^T - P(i) \tilde{K}_g(i) P(i)^T]
\end{align*}
\]

(20)

So the adaptive Kalman Filter is:

\[
\tilde{\gamma}(i) = \tilde{\gamma}(i) + K(i) w(i)
\]

(21)

where \( w(i) \) is termed as the innovation and \( K(i) \) is the Kalman gain matrix, they are expressed as follows:

\[
w(i) = S_r(i) - P(i) \gamma(i) - \hat{E}_o(i-1)
\]

\[
K(i) = \tilde{K}_g(i) P(i)^T [P(i) \tilde{K}_g(i) P(i)^T + \hat{R}_o(i-1)]^{-1}
\]

\[
\tilde{\gamma}(i) = \hat{\gamma}(i) + \hat{E}_p(i-1)
\]

\[
\tilde{K}_g(i) = \tilde{K}_g(i-1) + \hat{R}_g(i-1)
\]

\[
\tilde{K}_g(i) = (I - K(i) P(i)) \tilde{K}_g(i)
\]

(22)

The steps are repeated until all the radar measurements are used, and then the estimation is the result. The initial \( \tilde{\gamma}(0), \tilde{K}_g(0) \) have to be set to initiate the filtering process. When implementing the adaptive Kalman Filter, we take the result of the first phase into account and set the initial state carefully to initiate the filtering process. The initial \( \tilde{\gamma}(0) \) will be set as the result of the filtering in the matched filtering. The initial \( \tilde{K}_g(0) \) is computed based on the results of the matched filtering and Equation (22).

4. Experimental Results

Presented experimental results are aimed at showing the performance of the proposed filtering algorithm with respective to three aspects: 1) the spatial resolution, 2) the error criterion, and 3) the computing speed. To demonstrate the resolution performance straightly, in this section, a simulated scene with 5 uniformly spaced dots placed on the earth surface is considered, and the raw data sets have been generated using the signal model mentioned before.

Images in Fig. 1 show the filtering results obtained by the matched filter, the traditional Kalman Filter and the adaptive Kalman Filter respectively for various SNRs. The SNRs are: low SNR (-20dB), moderate SNR (0dB) and high SNR.
(20dB). It can be seen that for all the SNR scenarios considered, Kalman Filter gives better estimates than the matched filter in terms of resolution performance. Especially in the high SNR situation, Kalman Filter minimizes correlation with other pixels in order to reduce the error due to clutter, but the matched filter is seen to be clutter limited. Even though the matched filter estimate of the scattering coefficients improves with SNR, it is still unable to describe the image of a dot object with an area less than 5 pixels in any case. In the low-SNR case, the adaptive Kalman Filter scales the filter to rely on target statistics and outperforms the traditional Kalman Filter result. The results also prove that the matched filter is optimal in the sense of the output signal-to-noise ratio.

To assess performance of different algorithms numerically, simulation results in terms of the error criterion will be taken into account. The error criterion is the mean-squared error (MSE) of the pixel magnitudes normalized by the image's mean-squared pixel magnitude:

$$MSE = \frac{(\hat{\gamma} - \gamma)^H(\hat{\gamma} - \gamma)}{\gamma^H\gamma}$$  \hspace{1cm} (23)

Fig. 1 Comparison of the Matched filter, the traditional Kalman Filter and the adaptive Kalman Filter performance versus SNR

Fig. 2 shows the variation of the Normalized MSE as a function of input SNR. It shows that the adaptive Kalman Filter has the lowest error at both low SNR and high SNR. But at moderate SNR, the MSE of the adaptive Kalman Filter is higher because of the using of matched Kalman filtering for part data. For land consolidation application, the SNR is usually about 20dB, so the adaptive Kalman Filter will bring less error to images.

Fig. 2 MSE performance of the Matched filter, MMSE filter, the traditional Kalman Filter and the adaptive filter versus SNR

Fig. 3 Processing speed of the Matched filter, MMSE filter, the traditional Kalman Filter and the adaptive Kalman Filter

Another important advantage of the developed algorithm is its ability of decreasing the processing load inherent in the MMSE filter and the traditional KF. Fig. 3 shows this improvement obtained. The processing time for different number of sampling dots is shown. The processing is done in Matlab on a PC with 512M RAM and a 2.93 GHz processor. It
can be seen that as the number of radar measurements increases, the processing time for MMSE increases huge fold. But in the case of the KF, there is only a slight change in the processing time. Most important, the adaptive KF takes about half of processing time of the traditional KF, especially when the number of sampling measurements is increased. The results validate the developed adaptive KF.

5. Conclusion
SAR system enables to obtain all day/all night and all weather information for land consolidation. But land consolidation application requires high resolution images. To obtain high resolution SAR images, a signal space representation of the radar was presented and facilitated the discussion of reconstruction filter algorithms. The developed adaptive Kalman Filter procedure was applied to the simulated radar data. Results were also presented and demonstrated the feasibility of the high resolution SAR. In future, further tests will be performed by using real measured data on spaceborne SAR platform.

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