Optimum Shape in Brick Masonry Arches under Dynamic Loads

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Abstract: -Due to importance and application of arches in historical structures, arch shape optimization has been discussed.The objective of this study is to determine brick masonry arches under dynamic loads. In this paper, considerable attention is given to arches, their importance, modeling stages, dynamic analysis and arch optimization using ANSYS11 software. A multiple stage analysis framework was conducted for semicircular arch:

1- The study of optimum shape for semicircular arch on the base of minimize of arch weight.
2- Determination of linear and nonlinear analysis limits by increase of density.
3- The study of optimum shape in semicircular arch by linear and nonlinear analysis.

All of these stages have been conducted for obtuse angel arches,(steep, normal and diminished), four- centered pointed arch, tudor arch, ogee arch, equilateral arch, catenary arch, lancet arch, four-centered arch (normal, diminished and steep). The main purpose has been study of arch optimum shape in three spans (4, 5 and 6m) for minimize of weight: Finally, according to the results, the optimum shape in arches under dynamic load has been determined.

Key-words: - optimum shape- arch- masonry- dynamic load- linear analysis- non linear analysis- tensile stress.

1 Introduction

Before, arch was defined as a part of circle or bow. If we want to define it, we can say it is a curve surface for covering, that it’s span is higher than it’s depth .Overall, arches are classified to three groups:

1- circular arches and similar to that
2- obtuse angle arches
3- decorative arches

Time dynamic analysis is an analytical method to determine responses in each time section, especially for earthquake that a structure is under accelerations of earth motion (accelerograph) in the base level. In this model, structure dynamic response is function of time and calculated by number integral in equation of structure motion. [1, 10]

2 Modeling, analysisand optimizatio of arch shape

Arch modeling has been conducted by ANSYS11 software. Also dynamic analysis has been conducted by north-south horizontal accelerations of Elcentro earthquake in 1940.In this earthquake the time, maximum acceleration, maximum velocity and maximum displacement were 31.98 sec, 0.31g, 33 cm/sec and 21.4cm, respectively. The element which used in this analysis was SOLID 65. Arch shape optimization emphasized on the minimizing of arch weight. So, the base and top thickness, maximum tensile stress and weight of structure have been defined as design variable, state variable and objective function, respectively Optimization has been conducted in Design Optimum Processing. [8]

2-1 Geometrical modeling:

According to optimization of design variables, such as base thickness (t0) and top thickness (t1) as parameters, all of key points are defined as follow. [9]

In order to study of this material, semicircular arch is defined by key points as parameters (fig.1).

Point 1: (0, 0) Point (2): (R, 0) Point3: (-R, 0)
Pint4: (0, R)
Point 5(R+t0, 0) Point6: (-R-t0, 0)
Point 7: (0, R+t1)

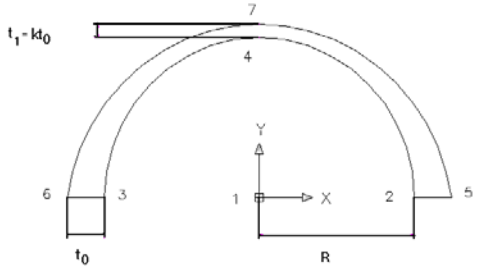

fig. 1: semicircular arch

In arch modeling, the tolerance increases because the thickness decreases from base to top. We should remember that in modeled arch, the thickness decrease from base (t0) to top (t1) linearly. Also, arch thickness in direction of length axis is 20 cm. The motion of support nodes is zero, and dynamic force has no effect on them. Also, brick masonry is made by brick and mortar as homogenous material (table 2). The efficient factors in inelastic nonlinear analysis show in (table 2). [7]

Table 1: Brick masonry specification

density(ρ) kg/m^3	1460 [2]
Elastic modulus N/m^2	5×10^8 [3]
Allowable tension stress(f_t) N/m^2	0.5×10^5 [2,3,4]
Poisson ratio (υ)	0.17[4]

Table 2: Effective coefficient in non elastic and nonlinear analysis

motion coefficient for open crack	0.1 [5]
motion coefficient for close crack	0.9 [5]
allowable tension stress N/m^2 (f_t)	5×10^4 [2,3,4]
allowable compressive stress N/m^2 (f_c)	5×10^5 [2,3,4]

3 Evaluation of optimum shape in semicircular arch

The analysis conducted for semicircular arch in five spans: 4,5,6,7 and meters (Table3).

Table3: specification of optimum shape for semicircular arch with various spans.

Span Length	*4(m)*	*5(m)*	*6(m)*	*7(m)*	*8(m)*
$t_0(m)$	.8328	.973	1.2154	1.4828	1.6208
$t_1(m)$	.2763	.28182	.297	.31879	.36388
k	.3317	.2896	.2443	.2149	.2245
t_0/R	.4164	.3892	.4051	.4236	.4052
t_1/R	.1381	.1127	.099	.091	.0909
$\overline{W}/H$	.4347	.917	5.68	.435	.8064
$(\sigma_t)_{\max}$ N/m^2	50982	48072	52815	51600	48430

3.1 Evaluation of different arch and their optimum shape

Here, in addition to semicircular arch, the obtuse angel, four centered pointed, tudor ogee arch, equilateral catenary, four centered, lanced arches have been studied. Analyzed arches were studied in three spans: 4, 5 and 6 meters. In each span, dynamic force, maximum tension stress, arch optimum dimensions and stability factor are calculated. Also, Obtus angel, four centered pointed tudor and ogee arch, arches have been analyzed in 3 levels: normal, diminished and steep (Table4).

Table 4: Comparison of optimum arches

L(m)		$t_0(m)$	$t_1(m)$	K	$\overline{W}/H$	$(\sigma_t)_{\max}$
Equilateral arch	**4**	.82923	.2073	.2499	.4876	46137
	5	1.0769	.2776	.2577	1.955	53033
	6	1.2125	.32458	.2676	.708	52903
Fourcentered arch	**4**	1.0875	.32358	.2975	2.2	52845
	5	1.0945	.34641	.3165	.39	51515
	6	1.1457	.35342	.3079	.63	50091

Continue of Table 4: Comparison of optimum arches

L(m)			$t_0(m)$	$t_1(m)$	K	$\overline{W}/H$	$(\sigma_t)_{\max}$
Catenary arch		**4**	.8969	.21984	.2451	.464	4790
		5	.99269	.27688	.2789	.872	4523
		6	1.1539	.28849	.2500	2.54	47095
Lancet arch		**4**	.96243	.18058	.1876	.4	5359
		5	1.06	.2095	.197	.7842	4629
		6	1.132	.2843	.214	.492	5076
Ogee arch	diminished	**4**	.83438	.39919	.4784	.41	49629
		5	.81818	.34175	.4176	.661	4658
		6	.80817	.24095	.2981	2.35	4668
	normal	**4**	.81414	.19308	.237	3.44	5368
		5	.8389	.22744	.2711	.557	5057
		6	.98287	.36179	.3680	1.145	5303
	st	**4**	1.3931	.3143	.2256	1.78	4890
		5	1.2725	.32409	.2546	.6	5270
		6	1.2126	.32669	.2694	.878	4536
Tudor arch	diminished	**4**	1	.3	.3	.38	47049
		5	.96541	.22347	.2314	.52	5384
		6	.81758	.20173	.2467	2.46	4547
	normal	**4**	.94988	.21925	.2308	.602	4659
		5	1.0553	.26254	.2487	2.93	4923
		6	1.1021	.33083	.3001	7.71	4990
	steep	**4**	1	.3	.3	1.018	4525
		5	1.0055	.21145	.2102	.428	4696
		6	1.0081	.20728	.2056	.746	5399

Continue of Table 4: Comparison of optimum arches

L(m)			$t_0(m)$	$t_1(m)$	K	$\overline{W}/H$	$(\sigma_t)_{\max}$
Obtuse anyec arch	diminished	**4**	1	.3	.3	.428	51732
		5	1.0692	.32387	.3029	6.32	47999
		6	1.1662	.32977	.2827	.807	45882
	normal	**4**	1.0975	.25091	.2286	1.49	51981
		5	1.1472	.30751	.268	5.72	53113
		6	1.1606	.31979	.275	.193	51373
	steep	**4**	.96942	.1798	.1854	.55	45853
		5	1.0975	.25091	.2286	.135	53922
		6	1.1769	.30722	.261	7.3	52566
Four centered pointed arch	diminish	**4**	.83728	.24854	.2968	.887	46341
		5	1.1309	.32538	.2877	1.156	50859
		6	1.1472	.33751	.2942	3.94	47815
	normal	**4**	1.0682	.27979	.2619	4.62	48692
		5	.98693	.34854	.353	5.69	45980
		6	.98287	.36943	.3758	.471	53175
	steep	**4**	.89212	.34194	.3832	.32	47463
		5	.9222	.3546	.386	.589	47367
		6	.98992	.37287	.376	5.01	49506

3.2 Determination of limits in linear and non linear analysis by increase of density

3.2.1 Evaluation and comparison of linear and nonlinear limits in semi circular and obtuse angel arches by density factor

In this part, linear and nonlinear analysis of semicircular arches with span of 5m and obtuse angle arch with span of 4 m has been studied. Also, the density is applied to evaluation of linear and nonlinear analysis. This was also noticed that in which limits the maximum tension stress (the arch optimization factor) can change (table 5). [6]

Table 5: Comparison between linear and nonlinear limits by density factor

	$\rho = 1460\ kg/m^3$		ρ	1.5ρ	2ρ	3ρ	4ρ
Semicircular arch	Linear Analysis	$(\sigma_t)_{\max}$	212921	148307	94944	60169	48072
	Non Linear Analysis		225149	148307	94944	60169	48072
Obtus angel arch	Linear Analysis	$(\sigma_t)_{\max}$	856833	267317	248307	211944	183337
	Non Linear Analysis		593918	267317	248307	211944	183337

According to results of test and error (table 2), if density is higher than 4ρ, the response of linear and nonlinear stress is different. So for linear analysis, increase of density to 4ρ is ineffective.

3.2.2 Evaluation and comparison of optimum shape in semicircular and obtus angle arch by linear and non linear analysis

The optimum shape of semicircular arch and obtus arch with spans of 4m have been calculated by linear and nonlinear analysis and density of 4ρ .Then the results compared to the optimum shape of semicircular and obtus by linear analysis and density of ρ (Table6). [8]

Table 6: Comparison of optimum shape in semicircular and Obtus angle arches with of 4m spans by linear and nonlinear analysis

density		Kind of analysis	t_0	t_1	k
Semicircular arch	ρ	Linear Analysis	.8328	.2763	.3317
	ρ	Non Linear Analysis	.8328	.2763	.3317
	4ρ	Linear Analysis	1.3	.2921	.2247
	4ρ	Non Linear Analysis	1.541	.3344	.2168
Obtuse angel arch	ρ	Linear Analysis	.9694	.1798	.1854
	ρ	Non Linear Analysis	.9694	.1798	.1854
	4ρ	Linear Analysis	1.332	.3	.2269
	4ρ	Non Linear Analysis	1.609	.3886	.241

Continue of Table 6: Comparison of optimum shape in semicircular and Obtus angle arches with of 4m spans by linear and nonlinear analysis.

density		Kind of analysis	W	H	$\overline{W}/H$	$(\sigma_t)_{\max}$
Semicircular arch	ρ	Linear Analysis	917.2	1057.8	.4347	50982
	ρ	Non Linear Analysis	917.2	1057.8	.4347	50982
	4ρ	Linear Analysis	5641.1	4052	.69	51700
	4ρ	Non Linear Analysis	6681	4471	.747	53873
Obtuse angel arch	ρ	Linear Analysis	1188	1079.3	.552	45853
	ρ	Non Linear Analysis	1188	1079.3	.552	45853
	4ρ	Linear Analysis	5781	5012	.576	52853
	4ρ	Non Linear Analysis	6483	5221	.62	53541

6 Conclusion

Considering to optimum shape in arches under dynamic load, several conclusions can be surmised from the results as follow:

1-With increase of masonry density, the difference between maximum tensile stress in linear and nonlinear analysis reveals. It means that the increase of density to 4ρ for linear and non linear analysis is ineffective.

2- The limit for increase of base thickness in linear and nonlinear analysis for $4\rho : \rho$ is 36 to 93%.

3- The limit for increase of top thickness in linear and nonlinear analysis for $4\rho : \rho$ is 66 to 116%.

4-Increase of ϖ / H in linear and nonlinear analysis for $4\rho : \rho$ is 12%.

5- Increase of arch base thickness in nonlinear analysis of 4ρ to linear analysis of 4ρ is 21%.

6- Increase of arch top thickness in linear analysis of 4ρ to linear analysis of 4ρ is 30%.

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